First Observation of $D^+ \to a_0(980)\rho$ and $D^+ \to a_0(980)+f_0(500)$ in $D^+ \to \pi^+\pi^+\pi^-\eta$ and $D^+ \to \pi+\pi^0\pi^0\eta$ Decays*

M. Ablikim (麦迪娜)[1] M. N. Achasov[4,c] P. Adlarson[81] X. C. Ai (艾小聪)[87] C. S. Akondi[31A,31B] R. Aliberti[39] A. Amoroso[80A,80C] Q. An (安琪)[77,64#] Y. H. An[87] Y. Bai (白羽)[62] O. Bakina[40] Y. Ban[50,h] H.-R. Bao (包浩然)[70] X. L. Bao (保先岚)[49] V. Batozskaya[1,48] K. Begzsuren[35] N. Berger[39] M. Berlowski[48] M. B. Bertani[30A] D. Bettoni[31A] F. Bianchi[80A,80C] E. Bianco[80A,80C] A. Bortone[80A,80C] I. Boyko[40] R. A. Briere[5] A. Brueggemann[74] H. Cai (蔡浩)[82] M. H. Cai (蔡铭航)[42,k,l] X. Cai (蔡啸)[1,64] A. Calcaterra[30A] G. F. Cao (曹国富)[1,70] N. Cao (曹宁)[1,70] S. A. Cetin[68A] X. Y. Chai (柴新宇)[50,h] J. F. Chang (常劲帆)[1,64] T. T. Chang (常甜甜)[47] G. R. Che (车国荣)[47] Y. Z. Che (车逾之)[1,64,70] C. H. Chen (陈春卉)[10] Chao Chen (陈超)[1] G. Chen (陈刚)[1] H. S. Chen (陈和生)[1,70] H. Y. Chen (陈弘扬)[20] M. L. Chen (陈玛丽)[1,64,70] S. J. Chen (陈申见)[46] S. M. Chen (陈少敏)[67] T. Chen (陈通)[1,70] W. Chen (陈旺)[49] X. R. Chen (陈旭荣)[34,70] X. T. Chen (陈肖婷)[1,70] X. Y. Chen (陈心宇)[12,g] Y. B. Chen (陈元柏)[1,64] Y. Q. Chen[16] Z. K. Chen (陈梓康)[65] J. Cheng (程捷)[49] L. N. Cheng (程利娜)[47] S. K. Choi[11] X. Chu (初晓)[12,g] G. Cibinetto[31A] F. Cossio[80C] J. Cottee-Meldrum[69] H. L. Dai (代洪亮)[1,64] J. P. Dai (代建平)[85] X. C. Dai (戴鑫琛)[67] A. Dbeyssi[19] R. E. de Boer[3] D. Dedovich[40] C. Q. Deng[78] Z. Y. Deng (邓子艳)[1] A. Denig[39] I. Denisenko[40] M. Destefanis[80A,80C] F. De Mori[80A,80C] X. X. Ding (丁晓萱)[50,h] Y. Ding (丁勇)[44] Y. X. Ding (丁源钦)[32] Yi. Ding[38] J. Dong (董静)[1,64] L. Y. Dong (董燎原)[1,70] M. Y. Dong (董明义)[1,64,70] X. Dong[82] M. C. Du (杜蒙川)[1] S. X. Du (杜书先)[87] Shaoxu Du (杜少旭)[12,g] X. L. Du (杜小龙)[12,g] Y. Q. Du (杜一琦)[82] Y. Y. Duan[60] Z. H. Duan (段宗欢)[46] P. Egorov[40,a] G. F. Fan (樊高峰)[46] J. J. Fan[20] Y. H. Fan (范宇晗)[49] J. Fang (方建)[1,64] Jin Fang (方进)[65] S. S. Fang (房双世)[1,70] W. X. Fang (方文兴)[1] Y. Q. Fang[1,64#] L. Fava[80B,80C] F. Feldbauer[3] G. Felici[30A] C. Q. Feng (封常青)[77,64] J. H. Feng (冯俊华)[16] L. Feng[42,k,l] Q. X. Feng (冯千禧)[42,k,l] Y. T. Feng (冯玥潼)[77,64] M. Fritsch[3] C. D. Fu (傅成栋)[1] J. L. Fu (傅金林)[70] Y. W. Fu (傅亦威)[1,70] H. Gao[70] Y. Gao (高扬)[77,64] Y. N. Gao (高原宁)[50,h] Y. Y. Gao[32] Yunong Gao (高语浓)[20] Z. Gao (高枝)[47] S. Garbolino[80C] I. Garzia[31A,31B] L. Ge (葛玲)[62] P. T. Ge (葛潘婷)[20] Z. W. Ge[46] C. Geng (耿聪)[65] E. M. Gersabeck[73] A. Gilman[75] K. Goetzen[13] J. Gollub[3] J. B. Gong (龚家宝)[1,70] J. D. Gong (龚家鼎)[38] L. Gong (龚丽)[44] W. X. Gong (龚文煊)[1,64] W. Gradl[39] S. Gramigna[31A,31B] M. Greco[80A,80C] M. D. Gu (顾萌冬)[55] M. H. Gu (顾旻皓)[1,64] C. Y. Guan (关春懿)[1,70] A. Q. Guo (郭爱强)[34] H. Guo (郭昊)[54] J. N. Guo (郭嘉宁)[12,g] L. B. Guo (郭立波)[45] M. J. Guo (国梦娇)[54] R. P. Guo (郭如盼)[53] X. Guo (郭旭)[54] Y. P. Guo (郭玉萍)[12,g] Z. Guo (郭震)[77,64] A. Guskov[40,a] J. Gutierrez[29] J. Y. Han (韩靖宜)[77,64] T. T. Han[1] X. Han (韩许)[77,64] F. Hanisch[3] K. D. Hao (郝科迪)[77,64] X. Q. Hao (郝喜庆)[20] F. A. Harris[71] C. Z. He (何承芝)[50,h] K. K. He (何凯凯)[17,46]

CPC Accepted

Received 29 October 2026; Accepted 2 November 2026

* The BESIII Collaboration thanks the staff of BEPCII (https://cstr.cn/31109.02.BEPC) and the IHEP computing center for their strong support. This work is supported in part by National Key R&D Program of China under Contracts Nos. 2023YFA1606000, 2023YFA1606704; National Natural Science Foundation of China (NSFC) under Contracts Nos. 11635010, 11935015, 11935016, 11935018, 12025502, 12035009, 12035013, 12061131003, 12192260, 12192261, 12192262, 12192263, 12192264, 12192265, 12221005, 12225509, 12235017, 12342502, 12361141819; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; the Strategic Priority Research Program of Chinese Academy of Sciences under Contract No. XDA0480600; CAS under Contract No. YSBR-101; 100 Talents Program of CAS; The Institute of Nuclear and Particle Physics (INPAC) and Shanghai Key Laboratory for Particle Physics and Cosmology; ERC under Contract No. 758462; German Research Foundation DFG under Contract No. FOR5327; Istituto Nazionale di Fisica Nucleare, Italy; Knut and Alice Wallenberg Foundation under Contracts Nos. 2021.0174, 2021.0299, 2023.0315; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Research Foundation of Korea under Contract No. NRF-2022R1A2C1092335; National Science and Technology fund of Mongolia; Polish National Science Centre under Contract No. 2024/53/B/ST2/00975; STFC (United Kingdom); Swedish Research Council under Contract No. 2019.04595; U. S. Department of Energy under Contract No. DE-FG02-05ER41374

# These authors contributed equally as the first authors

©2026 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

K. L. He (何康林)[1,70] F. H. Heinsius[3] C. H. Heinz[39] Y. K. Heng (衡月昆)[1,64,70] C. Herold[66]
P. C. Hong[38] G. Y. Hou (侯国一)[1,70] X. T. Hou (侯贤涛)[1,70] Y. R. Hou[70] Z. L. Hou (侯治龙)[1]
H. M. Hu (胡海明)[1,70] J. F. Hu (胡继峰)[61,j] Q. P. Hu (胡启鹏)[77,64] S. L. Hu (胡圣亮)[12,g] T. Hu (胡涛)[1,64,70]
Y. Hu (胡誉)[1] Y. X. Hu (胡宇轩)[82] Z. M. Hu (胡忠敏)[65] G. S. Huang (黄光顺)[77,64] K. X. Huang[65]
L. Q. Huang (黄麟钦)[34,70] P. Huang (黄盼)[46] X. T. Huang (黄性涛)[54] Y. P. Huang (黄燕萍)[1]
Y. S. Huang (黄永盛)[65] T. Hussain[79] N. Hüsken[39] N. in der Wiesche[74] J. Jackson[29] Q. Ji (纪全)[1]
Q. P. Ji (姬清平)[20] W. Ji (季旺)[1,9] X. B. Ji (季晓斌)[1,70] X. L. Ji (季筱璐)[1,64] Y. Y. Ji (吉钰瑶)[1]
L. K. Jia (贾乐康)[70] X. Q. Jia (贾晓倩)[54] D. Jiang (姜地)[1,70] H. B. Jiang[82] P. C. Jiang[50,h]
S. J. Jiang (蒋思婧)[10] X. S. Jiang (江晓山)[1,64,70] Y. Jiang (蒋艺)[70] J. B. Jiao (焦健斌)[54] J. K. Jiao[38]
Z. Jiao (焦铮)[25] L. C. L. Jin (金梁程龙)[1] S. Jin (金山)[46] Y. Jin (金毅)[72] M. Q. Jing (荆茂强)[1,70]
X. M. Jing (景新媚)[70] T. Johansson[81] S. Kabana[36] X. L. Kang (康晓琳)[10] X. S. Kang (康晓珅)[44]
B. C. Ke (柯百谦)[87] V. Khachatryan[29] A. Khoukaz[74] O. B. Kolcu[68A] B. Kopf[3] L. Kröger[74]
L. Krümmel[3] Y. Y. Kuang (匡洋洋)[78] M. Kuessner[3] X. Kui (奎贤)[1,70] N. Kumar[28] A. Kupsc[48,81]
W. Kühn[41] Q. Lan (兰强)[78] W. N. Lan[20] T. T. Lei (雷天天)[77,64] M. Lellmann[39] T. Lenz[39]
C. Li (李翠)[51] C. H. Li (李春花)[45] C. K. Li (李春凯)[47] Chunkai Li[21] Cong Li[47] D. M. Li (李德民)[87]
F. Li (李飞)[1,64] G. Li (李刚)[1] H. B. Li (李海波)[1,70] H. J. Li (李惠静)[20] H. L. Li[87] H. N. Li (李衡讷)[61,j]
H. P. Li (李昊鹏)[47] Hui Li (李慧)[47] J. N. Li (李佳宁)[32] J. S. Li[65] J. W. Li (李井文)[54] K. Li (李科)[1]
K. L. Li (李凯璐)[42,k,l] L. J. Li (李林健)[1,70] Lei Li (李蕾)[52] M. H. Li (李明浩)[47] M. R. Li (李明润)[1,70]
M. T. Li (李梦婷)[54] P. L. Li (李佩莲)[70] P. R. Li (李培荣)[42,k,l] Q. M. Li (李启铭)[1,70] Q. X. Li[54]
R. Li (李燃)[18,34] S. Li[87] S. X. Li (李素娴)[12] S. Y. Li[87] Shanshan Li (李珊珊)[27,i] T. Li (李腾)[54]
T. Y. Li (李天佑)[47] W. D. Li (李卫东)[1,70] W. G. Li (李卫国)[1#] X. Li (李旭)[1,70] X. H. Li (李旭红)[77,64]
X. K. Li (李小康)[50,h] X. L. Li (李晓玲)[54] X. Y. Li (李晓宇)[1,9] X. Z. Li (李绪泽)[65] Y. Li[20]
Y. G. Li (李彦谷)[70] Y. P. Li (李雁鹏)[38] Z. H. Li (李宗豪)[42] Z. J. Li (李志军)[65] Z. L. Li[87]
Z. X. Li (李振轩)[47] Z. Y. Li (李紫阳)[85] C. Liang (梁畅)[46] H. Liang (梁昊)[77,64] Y. F. Liang (梁勇飞)[59]
Y. T. Liang (梁羽铁)[34,70] G. R. Liao (廖广睿)[14] L. B. Liao (廖立波)[65] M. H. Liao (廖明华)[65]
Y. P. Liao (廖一朴)[1,70] J. Libby[28] A. Limphirat[66] C. C. Lin[60] C. X. Lin (林创新)[34]
D. X. Lin (林德旭)[34,70] T. Lin (林韬)[1] B. J. Liu (刘北江)[1] B. X. Liu[82] C. Liu (刘成)[38]
C. X. Liu (刘春秀)[1] F. Liu (刘芳)[1] F. H. Liu (刘福虎)[58] Feng Liu (刘峰)[6] G. M. Liu (刘国明)[61,j]
H. Liu (刘昊)[42,k,l] H. B. Liu (刘宏邦)[15] H. M. Liu (刘怀民)[1,70] Huihui Liu (刘汇慧)[22]
J. B. Liu (刘建北)[77,64] J. J. Liu[21] K. Liu (刘凯)[42,k,l] K. Y. Liu (刘魁勇)[44] Ke Liu (刘珂)[23]
Kun Liu (刘坤)[78] L. Liu (刘亮)[42] L. C. Liu (刘良辰)[47] Lu Liu (刘露)[47] M. H. Liu[38]
P. L. Liu (刘佩莲)[54] Q. Liu (刘倩)[70] S. B. Liu (刘树彬)[77,64] T. Liu (刘桐)[1] W. M. Liu[77,64]
W. T. Liu (刘婉婷)[43] X. Liu (刘翔)[42,k,l] X. K. Liu (刘筱康)[42,k,l] X. L. Liu (刘兴淋)[12,g]
X. P. Liu (刘新平)[12,g] X. Y. Liu[82] Y. Liu (刘英)[42,k,l] Y. B. Liu (刘玉斌)[47] Yi Liu (刘义)[87]
Z. A. Liu (刘振安)[1,64,70] Z. D. Liu (刘宗德)[83] Z. L. Liu[78] Z. Q. Liu (刘智青)[54] Z. Y. Liu (刘正扬)[42]
X. C. Lou (娄辛丑)[1,64,70] H. J. Lu (吕海江)[25] J. G. Lu (吕军光)[1,64] X. L. Lu (陆小玲)[16] Y. Lu (卢宇)[7]
Y. H. Lu (卢泱宏)[1,70] Y. P. Lu (卢云鹏)[1,64] Z. H. Lu[1,70] C. L. Luo (罗成林)[45] J. R. Luo (罗家瑞)[65]
J. S. Luo (罗家顺)[1,70] M. X. Luo (罗民兴)[86] T. Luo (罗涛)[12,g] X. L. Luo (罗小兰)[1,64] Z. Y. Lv (吕在裕)[23]
X. R. Lyu (吕晓睿)[70,o] Y. F. Lyu (吕翌丰)[47] Y. H. Lyu (吕云鹤)[87] F. C. Ma (马凤才)[44]
H. L. Ma (马海龙)[1] Heng Ma (马衡)[27,i] J. L. Ma (马俊力)[1,70] L. L. Ma (马连良)[54] L. R. Ma[72]
Q. M. Ma (马秋梅)[1] R. Q. Ma (马润秋)[1,70] R. Y. Ma[20] T. Ma (马腾)[77,64] X. T. Ma (马晓天)[1,70]
X. Y. Ma (马骁妍)[1,64] Y. M. Ma[34] F. E. Maas[19] I. MacKay[75] M. Maggiora[80A,80C] S. Maity[34]
S. Malde[75] Q. A. Malik[79] H. X. Mao[42,k,l] Y. J. Mao (冒亚军)[50,h] Z. P. Mao (毛泽普)[1]
S. Marcello[80A,80C] A. Marshall[69] F. M. Melendi[31A,31B] Y. H. Meng[70] Z. X. Meng (孟召霞)[72]
G. Mezzadri[31A] H. Miao (妙晗)[1,70] T. J. Min (闵天觉)[46] R. E. Mitchell[29] X. H. Mo (莫晓虎)[1,64,70]
B. Moses[29] N. Yu. Muchnoi[4,c] J. Muskalla[39] Y. Nefedov[40] F. Nerling[19,e] H. Neuwirth[74]

CPC Accepted

Z. Ning (宁哲)[1,64] S. Nisar[33] Q. L. Niu (牛祺乐)[42,k,l] W. D. Niu (牛文迪)[12,g] Y. Niu (牛艳)[54]
C. Normand[69] S. L. Olsen[11,70] Q. Ouyang (欧阳群)[1,64,70] S. Pacetti[30B,30C] X. Pan (潘祥)[60]
Y. Pan (潘越)[62] A. Pathak[11] Y. P. Pei (裴宇鹏)[77,64] M. Pelizaeus[3] G. L. Peng (彭高乐)[77,64]
H. P. Peng (彭海平)[77,64] X. J. Peng (彭宣嘉)[42,k,l] Y. Y. Peng (彭云翊)[42,k,l] K. Peters[13,e] K. Petridis[69]
J. L. Ping (平加伦)[45] R. G. Ping (平荣刚)[1,70] S. Plura[39] V. Prasad[38] L. Pöpping[3] F. Z. Qi (齐法制)[1]
H. R. Qi (漆红荣)[67] M. Qi[46] S. Qian (钱森)[1,64] W. B. Qian (钱文斌)[70] C. F. Qiao (乔从丰)[70]
J. H. Qiao (乔佳辉)[20] J. J. Qin (秦佳佳)[78] J. L. Qin (覃嘉良)[60] L. Q. Qin (秦丽清)[14]
L. Y. Qin (秦龙宇)[77,64] P. B. Qin (秦鹏勃)[78] X. P. Qin (覃潇萍)[43] X. S. Qin (秦小帅)[54]
Z. H. Qin (秦中华)[1,64] J. F. Qiu (邱进发)[1] Z. H. Qu[78] J. Rademacker[69] C. F. Redmer[39] A. Rivetti[80C]
M. Rolo[80C] G. Rong (荣刚)[1,70] S. S. Rong (荣少石)[1,70] F. Rosini[30B,30C] Ch. Rosner[19]
M. Q. Ruan (阮曼奇)[1,64] N. Salone[48,q] A. Sarantsev[40,d] Y. Schelhaas[39] M. Schernau[36] K. Schoenning[81]
M. Scodeggio[31A] W. Shan (单葳)[26] X. Y. Shan (单心钰)[77,64] Z. J. Shang (尚子杰)[42,k,l]
J. F. Shangguan (上官剑锋)[17] L. G. Shao (邵立港)[1,70] M. Shao (邵明)[77,64] C. P. Shen (沈成平)[12,g]
H. F. Shen (沈宏飞)[1,9] W. H. Shen (沈文涵)[70] X. Y. Shen (沈肖雁)[1,70] B. A. Shi (施伯安)[70]
Ch. Y. Shi (石超逸)[85,b] H. Shi (史华)[77,64] J. L. Shi (石家磊)[8,p] J. Y. Shi (石京燕)[1] M. H. Shi[87]
S. Y. Shi[78] X. Shi (史欣)[1,64] H. L. Song (宋海林)[77,64] J. J. Song (宋娇娇)[20] M. H. Song (宋铭昊)[42]
T. Z. Song (宋天资)[65] W. M. Song (宋维民)[38] Y. X. Song (宋昀轩)[50,h,m] Zirong Song (宋子荣)[27,i]
S. Sosio[80A,80C] S. Spataro[80A,80C] S. Stansilaus[75] F. Stieler[39] M. Stolte[3] S. S Su (苏闪闪)[44]
G. B. Sun (孙光豹)[82] G. X. Sun (孙功星)[1] H. Sun (孙昊)[70] H. K. Sun (孙浩凯)[1] J. F. Sun (孙俊峰)[20]
K. Sun (孙开)[67] L. Sun (孙亮)[82] R. Sun (孙睿)[77] S. S. Sun (孙胜森)[1,70] T. Sun[56,f]
W. Y. Sun (孙文玉)[55] Y. C. Sun (孙雨长)[82] Y. H. Sun[32] Y. J. Sun (孙勇杰)[77,64] Y. Z. Sun (孙永昭)[1]
Z. Q. Sun (孙泽群)[1,70] Z. T. Sun (孙振田)[54] H. Tabaharizato[1] C. J. Tang (唐昌建)[59] G. Y. Tang (唐光毅)[1]
J. Tang (唐健)[65] J. J. Tang (唐嘉骏)[77,64] L. F. Tang (唐林发)[43] Y. A. Tang (唐迎澳)[82]
L. Y. Tao (陶璐燕)[78] M. Tat[75] J. X. Teng (滕佳秀)[77,64] J. Y. Tian (田济源)[77,64] W. H. Tian (田文辉)[65]
Y. Tian (田野)[34] Z. F. Tian[82] I. Uman[68B] E. van der Smagt[3] B. Wang (王博)[65] Bin Wang (王斌)[1]
Bo Wang (王博)[77,64] C. Wang (王程)[42,k,l] Chao Wang[20] Cong Wang (王聪)[23] D. Y. Wang (王大勇)[50,h]
H. J. Wang (王泓鉴)[42,k,l] H. R. Wang (王皓冉)[84] J. Wang (王杰)[10] J. J. Wang (王家驹)[82]
J. P. Wang (王吉鹏)[37] K. Wang (王科)[1,64] L. L. Wang (王亮亮)[1] L. W. Wang[38] M. Wang (王萌)[54]
Mi Wang (王咪)[77,64] N. Y. Wang (王南洋)[70] S. Wang (王石)[42,k,l] Shun Wang (王顺)[63] T. Wang[12,g]
T. J. Wang[47] W. Wang (王为)[65] W. P. Wang (王维平)[39] X. F. Wang (王雄飞)[42,k,l]
X. L. Wang (王小龙)[12,g] X. N. Wang (王新南)[1,70] Xin Wang (王心)[27,i] Y. Wang (王亦)[1]
Y. D. Wang (王雅迪)[49] Y. F. Wang (王贻芳)[1,9,70] Y. H. Wang (王英豪)[42,k,l] Y. J. Wang (王祎景)[77,64]
Y. L. Wang (王艺龙)[20] Y. N. Wang[49] Yanning Wang (王燕宁)[82] Yaqian Wang (王亚乾)[18]
Yi Wang (王义)[67] Yuan Wang (王源)[18,34] Z. Wang (王铮)[1,64] Z. L. Wang (王治浪)[2]
Z. Q. Wang (王紫祺)[12,g] Z. Y. Wang (王至勇)[1,70] Zhi Wang (王旨)[47] Ziyi Wang[70] D. Wei (魏栋)[47]
D. H. Wei (魏代会)[14] D. J. Wei (魏德杰)[72] H. R. Wei[47] F. Weidner[74] H. R. Wen (文焕然)[34]
S. P. Wen (文硕频)[1] U. Wiedner[3] G. Wilkinson[75] M. Wolke[81] J. F. Wu (吴金飞)[1,9] L. H. Wu (伍灵慧)[1]
L. J. Wu (吴连近)[20] Lianjie Wu[20] S. G. Wu (吴韶光)[1,70] S. M. Wu (吴蜀明)[70] X. W. Wu (吴潇)[78]
Z. Wu (吴智)[1,64] H. L. Xia (夏惠亮)[77,64] L. Xia (夏磊)[77,64] B. H. Xiang (向本后)[1,70] D. Xiao (肖栋)[42,k,l]
G. Y. Xiao (肖光延)[46] H. Xiao (肖浩)[78] Y. L. Xiao (肖云龙)[12,g] Z. J. Xiao (肖振军)[45] C. Xie (谢陈)[46]
K. J. Xie (谢凯吉)[1,70] Y. Xie (谢勇)[54] Y. G. Xie (谢宇广)[1,64] Y. H. Xie (谢跃红)[6] Z. P. Xie (谢智鹏)[77,64]
T. Y. Xing (邢天宇)[1,70] D. B. Xiong[1] C. J. Xu[65] G. F. Xu (许国发)[1] H. Y. Xu (许皓月)[2] M. Xu[77,64]
Q. J. Xu (徐庆君)[17] Q. N. Xu[32] T. D. Xu[78] X. P. Xu (徐新平)[60] Y. Xu (徐月)[12,g] Y. C. Xu (胥英超)[84]
Z. S. Xu (许昭燊)[70] F. Yan (严芳)[24] L. Yan (严亮)[12,g] W. B. Yan (鄢文标)[77,64] W. C. Yan (闫文成)[87]
W. H. Yan (闫文昊)[6] W. P. Yan[20] X. Q. Yan (严薛强)[12,g] Y. Y. Yan[66] H. J. Yang (杨海军)[56,f]
H. L. Yang (杨昊霖)[38] H. X. Yang (杨洪勋)[1] J. H. Yang (杨君辉)[46] R. J. Yang (杨润佳)[20]

CPC Accepted

X. Y. Yang (杨晓艺)[72] Y. Yang (杨莹)[12,g] Y. H. Yang (杨耘赫)[47] Y. M. Yang[87] Y. Q. Yang (杨永强)[10]
Y. Z. Yang (杨颖喆)[20] Youhua Yang[46] Z. Y. Yang (杨忠兵)[78] Z. P. Yao[54] M. Ye (叶梅)[1,64]
M. H. Ye (叶铭汉)[9#] Z. J. Ye (叶子健)[61,j] Junhao Yin (殷俊昊)[47] Z. Y. You (尤郑昀)[65]
B. X. Yu (俞伯祥)[1,64,70] C. X. Yu (喻纯旭)[47] G. Yu[13] J. S. Yu (俞洁晟)[27,i] L. W. Yu (喻丽雯)[12,g]
T. Yu (于涛)[78] X. D. Yu (余旭东)[50,h] Y. C. Yu[87] Yongchao Yu (于勇超)[42] C. Z. Yuan (苑长征)[1,70]
H. Yuan (袁昊)[1,70] J. Yuan (袁菁)[38] Jie Yuan (袁杰)[49] L. Yuan (袁丽)[2] M. K. Yuan (袁铭宽)[12,g]
S. H. Yuan (袁升豪)[78] Y. Yuan (袁野)[1,70] C. X. Yue (岳崇兴)[43] Ying Yue (岳颖)[20] A. A. Zafar[79]
F. R. Zeng (曾凡蕊)[54] S. H. Zeng[69] X. Zeng (曾鑫)[12,g] Y. J. Zeng (曾溢嘉)[1,70] Yujie Zeng (曾宇杰)[65]
Y. C. Zhai[54] Y. H. Zhan (詹永华)[65] B. L. Zhang (张伯伦)[1,70] B. X. Zhang[1#] D. H. Zhang (张丹昊)[47]
G. Y. Zhang (张广义)[20] Gengyuan Zhang (张耕源)[1,70] H. Zhang (张晗)[77,64] H. C. Zhang (张航畅)[1,64,70]
H. H. Zhang (张宏浩)[65] H. Q. Zhang (张华桥)[1,64,70] H. R. Zhang (张浩然)[77,64] H. Y. Zhang (章红宇)[1,64]
Han Zhang (张晗)[87] J. Zhang (张晋)[65] J. J. Zhang (张进军)[57] J. L. Zhang (张杰磊)[21]
J. Q. Zhang (张敬庆)[45] J. S. Zhang (张家文)[12,g] J. W. Zhang (张家文)[1,64,70] J. X. Zhang (张建勇)[42,k,l]
J. Y. Zhang[1] J. Z. Zhang (张景芝)[1,70] Jianyu Zhang[70] Jin Zhang (张进)[52] Jiyuan Zhang (张纪元)[12,g]
L. M. Zhang (张黎明)[67] Lei Zhang (张雷)[46] N. Zhang[38] P. Zhang (张鹏)[1,9] Q. Zhang[20]
Q. Y. Zhang (张秋岩)[38] Q. Z. Zhang (张铨正)[70] R. Y. Zhang (张若愚)[42,k,l] S. H. Zhang (张水涵)[1,70]
S. N. Zhang[75] Shulei Zhang (张书磊)[27,i] X. M. Zhang (张晓梅)[1] X. Y. Zhang (张学尧)[54]
Y. Zhang (张雨)[1] Y. T. Zhang (张亚腾)[87] Y. H. Zhang (张银鸿)[1,64] Y. P. Zhang (张亚鹏)[77,64]
Yu Zhang (张宇)[78] Z. D. Zhang (张正德)[1] Z. H. Zhang (张泽恒)[1] Z. L. Zhang[38] Z. X. Zhang[20]
Z. Y. Zhang (张振宇)[82] Z. Zhang (张哲)[34] Zh. Zh. Zhang[20] Zhilong Zhang (张志龙)[60]
Ziyang Zhang (张子扬)[49] Ziyu Zhang (张子羽)[47] G. Zhao (赵光)[1] J.-P. Zhao (赵俊鹏)[70]
J. Y. Zhao (赵静宜)[1,70] J. Z. Zhao (赵京周)[1,64] L. Zhao (赵玲)[1] Lei Zhao (赵雷)[77,64]
M. G. Zhao (赵明刚)[47] R. P. Zhao (赵若平)[70] S. J. Zhao[87] Y. B. Zhao (赵豫斌)[1,64] Y. L. Zhao (赵艳琳)[60]
Y. P. Zhao (赵祎鹏)[49] Y. X. Zhao (赵宇翔)[34,70] Z. G. Zhao (赵政国)[77,64] A. Zhemchugov[40,a]
B. Zheng (郑波)[78] B. M. Zheng (郑变敏)[38] J. P. Zheng (郑建平)[1,64] W. J. Zheng (郑文静)[1,70]
W. Q. Zheng (郑稳倩)[10] X. R. Zheng (郑心如)[20] Y. H. Zheng (郑阳恒)[70,o] B. Zhong (钟彬)[45]
C. Zhong (钟翠)[20] H. Zhou (周航)[39,54,n] J. Q. Zhou (周嘉奇)[38] S. Zhou (周帅)[6] X. Zhou (周详)[82]
X. K. Zhou (周晓康)[6] X. R. Zhou (周小蓉)[77,64] X. Y. Zhou (周兴玉)[43] Y. X. Zhou (周亦雄)[84]
Y. Z. Zhou (周祎卓)[20] A. N. Zhu (朱傲男)[70] J. Zhu (朱江)[47] K. Zhu (朱凯)[1] K. J. Zhu (朱科军)[1,64,70]
K. S. Zhu (朱康帅)[12,g] L. X. Zhu[70] Lin Zhu (朱林)[20] S. H. Zhu (朱世海)[76] T. J. Zhu (朱腾蛟)[12,g]
W. D. Zhu (朱稳定)[12,g] W. J. Zhu (朱文静)[1] W. Z. Zhu (朱文卓)[20] Y. C. Zhu (朱莹春)[77,64]
Z. A. Zhu (朱自安)[1,70] X. Y. Zhuang (庄新宇)[47] M. Zhuge (诸葛铭)[54] J. H. Zou (邹佳恒)[1] J. Zu[34]

[1]Institute of High Energy Physics, Beijing 100049, People's Republic of China
[2]Beihang University, Beijing 100191, People's Republic of China
[3]Bochum Ruhr-University, D-44780 Bochum, Germany
[4]Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia
[5]Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
[6]Central China Normal University, Wuhan 430079, People's Republic of China
[7]Central South University, Changsha 410083, People's Republic of China
[8]Chengdu University of Technology, Chengdu 610059, People's Republic of China
[9]China Center of Advanced Science and Technology, Beijing 100190, People's Republic of China
[10]China University of Geosciences, Wuhan 430074, People's Republic of China
[11]Chung-Ang University, Seoul, 06974, Republic of Korea
[12]Fudan University, Shanghai 200433, People's Republic of China
[13]GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany
[14]Guangxi Normal University, Guilin 541004, People's Republic of China
[15]Guangxi University, Nanning 530004, People's Republic of China
[16]Guangxi University of Science and Technology, Liuzhou 545006, People's Republic of China
[17]Hangzhou Normal University, Hangzhou 310036, People's Republic of China
[18]Hebei University, Baoding 071002, People's Republic of China
[19]Helmholtz Institute Mainz, Staudinger Weg 18, D-55099 Mainz, Germany
[20]Henan Normal University, Xinxiang 453007, People's Republic of China
[21]Henan University, Kaifeng 475004, People's Republic of China

CPC Accepted

[22]Henan University of Science and Technology, Luoyang 471003, People's Republic of China
[23]Henan University of Technology, Zhengzhou 450001, People's Republic of China
[24]Hengyang Normal University, Hengyang 421001, People's Republic of China
[25]Huangshan College, Huangshan 245000, People's Republic of China
[26]Hunan Normal University, Changsha 410081, People's Republic of China
[27]Hunan University, Changsha 410082, People's Republic of China
[28]Indian Institute of Technology Madras, Chennai 600036, India
[29]Indiana University, Bloomington, Indiana 47405, USA
[30]INFN Laboratori Nazionali di Frascati, (A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy; (B)INFN Sezione di Perugia, I-06100, Perugia, Italy; (C)University of Perugia, I-06100, Perugia, Italy
[31]INFN Sezione di Ferrara, (A)INFN Sezione di Ferrara, I-44122, Ferrara, Italy; (B)University of Ferrara, I-44122, Ferrara, Italy
[32]Inner Mongolia University, Hohhot 010021, People's Republic of China
[33]Institute of Business Administration, University Road, Karachi, 75270 Pakistan
[34]Institute of Modern Physics, Lanzhou 730000, People's Republic of China
[35]Institute of Physics and Technology, Mongolian Academy of Sciences, Peace Avenue 54B, Ulaanbaatar 13330, Mongolia
[36]Instituto de Alta Investigación, Universidad de Tarapacá, Casilla 7D, Arica 1000000, Chile
[37]Jiangsu Ocean University, Lianyungang 222000, People's Republic of China
[38]Jilin University, Changchun 130012, People's Republic of China
[39]Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
[40]Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
[41]Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
[42]Lanzhou University, Lanzhou 730000, People's Republic of China
[43]Liaoning Normal University, Dalian 116029, People's Republic of China
[44]Liaoning University, Shenyang 110036, People's Republic of China
[45]Nanjing Normal University, Nanjing 210023, People's Republic of China
[46]Nanjing University, Nanjing 210093, People's Republic of China
[47]Nankai University, Tianjin 300071, People's Republic of China
[48]National Centre for Nuclear Research, Warsaw 02-093, Poland
[49]North China Electric Power University, Beijing 102206, People's Republic of China
[50]Peking University, Beijing 100871, People's Republic of China
[51]Qufu Normal University, Qufu 273165, People's Republic of China
[52]Renmin University of China, Beijing 100872, People's Republic of China
[53]Shandong Normal University, Jinan 250014, People's Republic of China
[54]Shandong University, Jinan 250100, People's Republic of China
[55]Shandong University of Technology, Zibo 255000, People's Republic of China
[56]Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China
[57]Shanxi Normal University, Linfen 041004, People's Republic of China
[58]Shanxi University, Taiyuan 030006, People's Republic of China
[59]Sichuan University, Chengdu 610064, People's Republic of China
[60]Soochow University, Suzhou 215006, People's Republic of China
[61]South China Normal University, Guangzhou 510006, People's Republic of China
[62]Southeast University, Nanjing 211100, People's Republic of China
[63]Southwest University of Science and Technology, Mianyang 621010, People's Republic of China
[64]State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People's Republic of China
[65]Sun Yat-Sen University, Guangzhou 510275, People's Republic of China
[66]Suranaree University of Technology, University Avenue 111, Nakhon Ratchasima 30000, Thailand
[67]Tsinghua University, Beijing 100084, People's Republic of China
[68]Turkish Accelerator Center Particle Factory Group, (A)Istinye University, 34010, Istanbul, Turkey; (B)Near East University, Nicosia, North Cyprus, 99138, Mersin 10, Turkey
[69]University of Bristol, H H Wills Physics Laboratory, Tyndall Avenue, Bristol, BS8 1TL, UK
[70]University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China
[71]University of Hawaii, Honolulu, Hawaii 96822, USA
[72]University of Jinan, Jinan 250022, People's Republic of China
[73]University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom
[74]University of Muenster, Wilhelm-Klemm-Strasse 9, 48149 Muenster, Germany
[75]University of Oxford, Keble Road, Oxford OX13RH, United Kingdom
[76]University of Science and Technology Liaoning, Anshan 114051, People's Republic of China
[77]University of Science and Technology of China, Hefei 230026, People's Republic of China
[78]University of South China, Hengyang 421001, People's Republic of China
[79]University of the Punjab, Lahore-54590, Pakistan
[80]University of Turin and INFN, (A)University of Turin, I-10125, Turin, Italy; (B)University of Eastern Piedmont, I-15121, Alessandria, Italy; (C)INFN, I-10125, Turin, Italy
[81]Uppsala University, Box 516, SE-75120 Uppsala, Sweden
[82]Wuhan University, Wuhan 430072, People's Republic of China
[83]Xi'an Jiaotong University, No.28 Xianning West Road, Xi'an, Shaanxi 710049, P.R. China
[84]Yantai University, Yantai 264005, People's Republic of China
[85]Yunnan University, Kunming 650500, People's Republic of China
[86]Zhejiang University, Hangzhou 310027, People's Republic of China

CPC Accepted

[87]Zhengzhou University, Zhengzhou 450001, People's Republic of China
[a]Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia
[b]Also at the Functional Electronics Laboratory, Tomsk State University, Tomsk, 634050, Russia
[c]Also at the Novosibirsk State University, Novosibirsk, 630090, Russia
[d]Also at the NRC "Kurchatov Institute", PNPI, 188300, Gatchina, Russia
[e]Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany
[f]Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory for Particle Physics and Cosmology; Institute of Nuclear and Particle Physics, Shanghai 200240, People's Republic of China
[g]Also at Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan University, Shanghai 200443, People's Republic of China
[h]Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People's Republic of China
[i]Also at School of Physics and Electronics, Hunan University, Changsha 410082, China
[j]Also at Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China
[k]Also at MOE Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, People's Republic of China
[l]Also at Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, People's Republic of China
[m]Also at Ecole Polytechnique Federale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland
[n]Also at Helmholtz Institute Mainz, Staudinger Weg 18, D-55099 Mainz, Germany
[o]Also at Hangzhou Institute for Advanced Study, University of Chinese Academy of Sciences, Hangzhou 310024, China
[p]Also at Applied Nuclear Technology in Geosciences Key Laboratory of Sichuan Province, Chengdu University of Technology, Chengdu 610059, People's Republic of China
[q]Currently at University of Silesia in Katowice, Institute of Physics, 75 Pulku Piechoty 1, 41-500 Chorzow, Poland

Abstract: We perform the first amplitude analysis of the singly Cabibbo-suppressed decays $D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta$, using e^+e^- collision data collected with the BESIII detector at the center-of-mass energy of 3.773 GeV, corresponding to an integrated luminosity of 20.3 fb^{-1}. The absolute branching fractions of the $D^+ \to \pi^+\pi^+\pi^-\eta$ and $D^+ \to \pi^+\pi^0\pi^0\eta$ decays are measured to be $(3.20 \pm 0.06_{\rm stat.} \pm 0.03_{\rm syst.}) \times 10^{-3}$ and $(2.43 \pm 0.11_{\rm stat.} \pm 0.04_{\rm syst.}) \times 10^{-3}$, respectively. The decay process $D^+ \to a_0(980)^+ f_0(500)$ is observed for the first time with an unexpectedly large branching fraction. Moreover, we observe the decays $D^+ \to a_0(980)^{+(0)}\rho(770)^{0(+)}$ and measure the ratio $r_{+/0} \equiv \dfrac{\mathcal{B}(D^+ \to a_0(980)^+\rho(770)^0)}{\mathcal{B}(D^+ \to a_0(980)^0\rho(770)^+)}$ for the first time to be $0.55 \pm 0.08_{\rm stat.} \pm 0.05_{\rm syst.}$. These results offer a novel insight into our comprehension of the nature of the $a_0(980)$ and $f_0(500)$ states.

Keywords: Charm meson decays, Amplitude analysis, Exotic states, Light scalar mesons, a$_0$(980), , Final state interactions

DOI: **CSTR:**

CPC Accepted

Although the existence of light scalar mesons, particularly the $f_0(500)$, $f_0(980)$, and $a_0(980)$, has been firmly established experimentally, their classification within the constituent quark model [1] remains a subject of intense debate. These states present significant theoretical challenges when attempting to describe them as conventional quark-antiquark configurations, such as the heavy mass of $a_0(980)$ [1], the observation of the Okubo-Zweig-Iizuka suppressed decay $\phi \to \gamma a_0(980)^0$ [2], and several unexpected large values of branching fraction (BF) for decays involving scalar mesons [3−6]. These intriguing phenomena motivate the development of alternative hypotheses on their internal structure, including compact tetraquark states [7−14], two-meson molecule bound states [15−20], and mixed states [2]. The production of these exotic structures typically relates to final state interactions (FSIs), whose mechanisms are predominantly governed by non-perturbative QCD.

Complementarily, it is known that hadronic decays of charmed mesons receive significant contribution from FSIs, providing an exceptional laboratory for investigating the nature of light scalar mesons. The BESIII collaboration has observed anomalously large BFs for the $D_s^+ \to a_0(980)^{0(+)}\pi^{+(0)}$ [4], $D_s^+ \to a_0(980)^{0(+)}\rho^{+(0)}$ [5], and $D^{0(+)} \to a_0(980)^+\pi^{-(0)}$ decays [6]. Particularly noteworthy is the measured ratio $\mathcal{B}(D^+ \to a_0(980)^+\pi^0)/\mathcal{B}(D^+ \to a_0(980)^0\pi^+) = 2.6 \pm 0.6 \pm 0.3$ [6]. This ratio is predicted to be much smaller than 1 in the $q\bar{q}$ model of $a_0(980)$, since $D \to a_0(980)^+ P$ decays proceed primarily through external W-emission diagrams, which are suppressed by the extremely small coupling of the W^+ to the $a_0(980)^+$ [21]. Examples of the external W-emission topological diagrams of $D^+ \to a_0(980)^+\rho^0$ in the two-quark and tetraquark models are shown in Figs. 1(a) and 1(b), respectively. These observations provide compelling evidence for substantial FSI effecting the $D \to SP$ decays involving scalar mesons [21]. Here S, P, and V denote scalar, pseudo-scalar, and vector states, respectively. Similarly, sizeable contributions from FSIs are anticipated to influence the $D^+ \to SV$ decays. The measurement of both the BFs for $D^+ \to a_0(980)\rho$ decays and the relative ratio $\mathcal{B}(D^+ \to a_0(980)^+\rho^0)/\mathcal{B}(D^+ \to a_0(980)^0\rho^+)$ will offer cru-

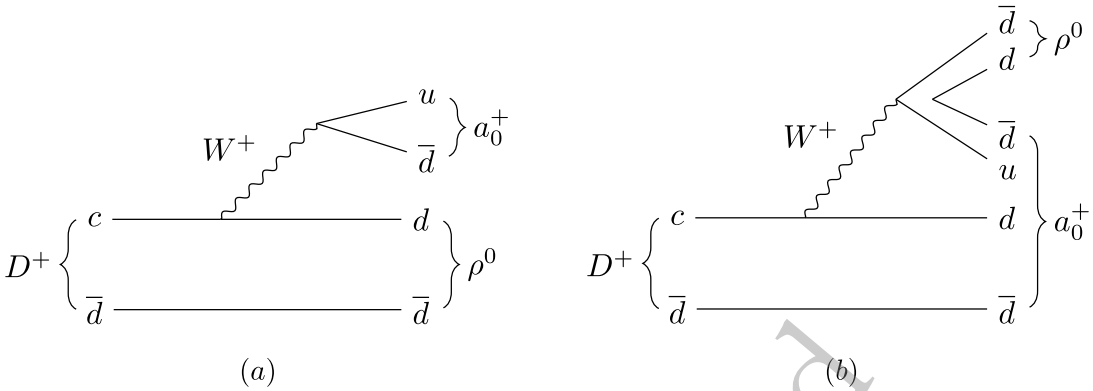


Fig. 1. Topological diagrams for the decay $D^+ \to a_0(980)^+\rho(770)^0$ showing the external W-emission process with $a_0(980)^+$ (a) as a two-quark state, and (b) as a tetraquark state. (b) shows one of the several possible diagrams. Other diagrams contributing to related decays are discussed in Ref. [21].

CPC Accepted

cial insights into the exotic structure of light scalar particles.

Furthermore, although numerous $D_{(s)} \to SP$ decays have been studied [8, 19, 21–25], the $D_{(s)} \to SS$ decay channel remains rarely explored experimentally and theoretically. In a tetraquark scenario, the hadronization of each S requires an additional pair of quark-antiquark. This makes $D_{(s)} \to SS$ more sensitive to the role of FSIs in the production of scalar particles. The intermediate process $D^+ \to a_0(980)^+ f_0(500)$ of $D^+ \to \pi^+\pi^0\pi^0\eta$ and $D^+ \to \pi^+\pi^+\pi^-\eta$ is expected to be governed by the external W-emission, and the BF should be suppressed by the smallness of the $a_0(980)^+$ decay constant under the assumption of $a_0(980)$ as a conventional two-quark meson in Fig. 1(a). However, this decay mode may receive contribution from other diagrams assuming $a_0(980)$ and $f_0(500)$ as tetraquark states. Examples are shown in Fig. 1(b), which could potentially enhance its BF significantly [21]. The discovery of such decays would open a new window into the fundamental dynamics of charmed meson decays, particularly for probing exotic scalar meson configurations and their production mechanisms.

The $\psi(3770)$ decays predominantly into $D\bar{D}$ pairs without any additional hadrons. The excellent tracking, accurate calorimetry, and the large data sample at the $D\bar{D}$ threshold [26] provide an unprecedented opportunity to accurately study the intermediate processes of the $D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta$ decays. Based on a data set corresponding to an integrated luminosity of 20.3 fb^{-1} collected with the BESIII detector [27, 28], this Letter reports the first amplitude analysis and updated BF measurements of $D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta$.

A description of the design and performance of the BESIII detector can be found in Ref. [26]. Simulated data samples are produced with a GEANT4-based [29] software package, which includes the geometric description [30] of the BESIII detector and the detector response. The simulation models the beam energy spread and initial state radiation (ISR) in the e^+e^- annihilations with the generator KKMC [31]. The inclusive Monte Carlo (MC) sample includes the production of $D\bar{D}$ pairs, the non-$D\bar{D}$ decays of the $\psi(3770)$, the ISR production of the J/ψ and $\psi(3686)$ states, and the continuum processes incorporated in KKMC. All particle decays are modeled with EVTGEN [32, 33] using BFs either taken from the Particle Data Group (PDG) [1], when available, or otherwise estimated with LUNDCHARM [34, 35]. Final state radiation from charged final state particles is incorporated using PHOTOS [36]. Charge-conjugate modes are implied throughout this Letter.

The double-tag (DT) method [37] provides high-purity samples for measuring the absolute BFs of D meson decays, eliminating dependence on the integrated luminosity and the $D\bar{D}$ production cross section [38]. The tagged D^- mesons are first reconstructed in one of the five decay modes $K^+\pi^-\pi^-$, $K^+\pi^-\pi^-\pi^0$, $K^0_S\pi^-$, $K^0_S\pi^-\pi^0$, and $K^0_S\pi^-\pi^-\pi^+$, referred to as single-tag (ST) events. An event in which both a signal $D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta$ decay and a tagged D^- is simultaneously found is referred to as a DT event. The selection criteria of the final state particles are the same as Ref. [39].

The D mesons are identified using the energy difference $\Delta E \equiv E_D - E_{\rm beam}$ and the beam-constrained mass $M_{\rm BC} \equiv \sqrt{E^2_{\rm beam}/c^4 - |\vec{p}_D|^2/c^2}$, where $E_{\rm beam}$ is the beam energy, and $\vec{p}_D$ and E_D are the measured momentum and energy of the D candidate in the e^+e^- rest frame, respectively. The $M_{\rm BC}$ and ΔE distributions of the $D^\pm$ signals are expected to be around the known $D^\pm$ mass [1] and zero, respectively. The tagged D^- is reconstructed through the tag modes. In case of multiple candidates, the one with the minimum $|\Delta E|$ is chosen. More details about the ST information can be found in Ref. [40].

Once a ST D^- decay has been identified, a search is made for a D^+ signal decay in the same event. The signal candidates are reconstructed using the remaining charged tracks and neutral showers. The best signal candidate

with the minimum $|\Delta E|$ is selected. All $D^{\pm}$ candidates are required to satisfy $1.863 < M_{\rm BC} < 1.877$ GeV/c^2, and $-0.033\,(-0.052) < \Delta E < 0.032\,(0.038)$ GeV for $\pi^+\pi^{+(0)}\pi^{-(0)}\eta$. For the $\pi^+\pi^-[\pi^0\pi^0]$ and $\pi^{+(0)}\pi^{-(0)}\eta$ combinations, the K^0_S and η' contributions are rejected by requiring their invariant masses to be outside $(0.478, 0.517)\,\text{GeV}/c^2$ [$(0.448, 0.534)\,\text{GeV}/c^2$] and $(0.8, 1.0)\,\text{GeV}/c^2$, respectively. These requirements correspond to at least five times the fitted mass resolution away from the individual nominal mass. For $\pi^+\pi^0\pi^0\eta$ decays, the peaking backgrounds from $\pi^+\pi^0\pi^0\pi^0$ are rejected by requiring no $\pi^+\pi^0\pi^0\pi^0$ combinations in the same candidate event that satisfy both $\Delta E_{\pi\pi^0\pi^0\pi^0} \in (-0.1, 0.1)\,\text{GeV}$ and $M^{\pi\pi^0\pi^0\pi^0}_{\rm BC} \in (1.83, 1.89)\,\text{GeV}/c^2$. This combined selection rejects more than 77% of the background while retaining 98% of the signal efficiency [41]. Furthermore, a kinematic fit is performed with some constraints: (1) the four-momenta of final-state particles are constrained to match the initial e^+e^- system momentum, and (2) the invariant masses of intermediate states (K^0_S, π^0, and tagged D^- on the tag side; π^0 and η on the signal side) are constrained to their known values [1]. The requirements $\chi^2 < 100(50)$ are applied for $\pi^+\pi^{+(0)}\pi^{-(0)}\eta$. Finally, for $\pi^+\pi^{+(0)}\pi^{-(0)}\eta$ channels, there are 3122 (648) DT events obtained for the amplitude analysis with a signal purity of $(89.3\pm0.5)\%$ ($(78.0\pm1.4)\%$) [42]. The amplitude analysis requires a sample with good resolution and all candidates falling within the phase-space boundary. Therefore, a subsequent kinematic fit is performed, which incorporates the previous constraints and introduces an additional constraint on the invariant mass of the signal D^+ meson. The amplitude analysis is then conducted using the four-momenta of particles obtained from this kinematic fit.

An unbinned maximum likelihood method is adopted in the amplitude analysis of the $D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta$ decays. The likelihood function is constructed with a probability density function (PDF) in which the momenta of the four final-state particles are used as inputs. The total likelihood is the product of the likelihoods for all events in the data samples. The total amplitude is modeled as a coherent sum over all intermediate processes $\mathcal{M} = \sum \rho_n e^{i\phi_n}\mathcal{A}_n$, where $\rho_n e^{i\phi_n}$ is the coefficient of the $n^{\rm th}$ amplitude with magnitude ρ_n and phase ϕ_n. The amplitude is symmetrized under the exchange of the two identical $\pi^{+(0)}$ mesons to incorporate the Bose symmetry. The $n^{\rm th}$ amplitude $\mathcal{A}_n$ is given by $\mathcal{A}_n = P^1_n P^2_n S_n F^1_n F^2_n F^3_n$, where the indices 1, 2 and 3 correspond to the two subsequent intermediate resonances and the D^+ meson, respectively, F^i_n is the Blatt-Weisskopf barrier factor [43] and P^i_n is the propagator of the intermediate resonance. The function S_n describes the angular momentum conservation in the decay and is constructed using the covariant tensor formalism [43]. The relativistic Breit-Wigner (RBW) [44] function is used to describe the propagator for the resonances $\eta(1405)$, $f_1(1285)$ and $f_1(1420)$. The resonance $\rho(770)$ is parameterized by the Gounaris-Sakurai [45] lineshape, and the $a_0(980)$ is parameterized by a coupled Flatté formula, and the parameters are fixed to the values given in Ref. [46]. We use the same parameterization to describe $f_0(500)$ as in Ref. [47]. The masses and widths of the intermediate resonances, except for $a_0(980)$ and $f_0(500)$, are taken from the PDG [1].

The background PDF, B, which is derived from the inclusive MC sample using the XGBoost package [48, 49], is then added incoherently to the signal PDF. The details can be found in the supplemental material [42]. As a consequence, the combined PDF can be written as [5]

$$\epsilon R_4 \left[f_s \frac{|\mathcal{M}|^2}{\int \epsilon |\mathcal{M}|^2 d\Phi_4} + (1-f_s)\frac{B_\epsilon}{\int \epsilon B_\epsilon d\Phi_4} \right], \qquad (1)$$

where ϵ is the acceptance function, $B_\epsilon = B/\epsilon$, f_s is the signal purity, and $d\Phi_4 = R_4\, dp_j$ is the volume element of the four-body phase space, with p_j denoting the four-momenta of the final-state particles and R_4 the corresponding phase-space factor. All quantities in Eq. (1), including $\mathcal{M}$, B, ϵ, and R_4, are functions of the final-state four-momenta p_j. The normalization integral in the denominator is determined by an MC technique as described in Ref. [50].

In the data projections, the decay channels $D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta$ exhibit distinct resonance structures: the former shows prominent $a_0(980)^+$ and ρ^0 signals, while the latter displays clear $a_0(980)^0$ and ρ^+ peaks. Therefore, the decays $D^+ \to a_0(980)^+\rho^0$ and $D^+ \to a_0(980)^0\rho^+$ are chosen as the reference amplitudes for the $\pi^+\pi^+\pi^-\eta$ and $\pi^+\pi^0\pi^0\eta$ channels, respectively. We consider some possible amplitudes involving the resonances $f_0(500)$, $f_0(980)$, $f_1(1285)$, $\eta(1295)$, $\eta(1405)$, $\eta(1475)$, $f_1(1420)$ and $f_1(1510)$, as well as other non-resonant components. All contributions with significance larger than 5σ are retained for further analysis. Here, the significance is calculated using the changes of $\ln L$ and the number of degrees of freedom when the fit is performed with and without including the corresponding amplitudes. Following this criterion, six intermediate states are retained in the fit model for both channels. Amplitudes are symmetrized with respect to the two identical $\pi^{+(0)}$ mesons and charge-conjugation constraints through Clebsch-Gordan coefficients (see the supplemental material [42] for details).

The decay amplitudes and the corresponding ϕ_n, fit fraction (FF_n) and BF_n are listed in Table 1. The mass projections of the fit are shown in Fig. 2. For the $n^{\rm th}$ component, its contribution relative to the total BF is quantified by $\text{FF}_n = \int |\rho_n \mathcal{A}_n|^2 d\Phi_4 / \int |\mathcal{M}|^2 d\Phi_4$. We evaluate the goodness-of-fit using the mixed-sample method [51, 52]. This method quantifies the consistency between the data and the signal MC in the multi-dimensional phase space. For each event, a test statistic T is calculated based on the

CPC Accepted

Table 1. Phase (ϕ), fit fraction (FF), and BF of each amplitude. The first and second uncertainties are statistical and systematic, respectively. The intermediate states are reconstructed in the decays $a_0(980) \to \eta\pi$, $f_0(500) \to \pi\pi$, $\rho(770) \to \pi\pi$ and $\eta(1405)/f_1(1285)/f_1(1420) \to a_0(980)\pi$.

Amplitude	ϕ	FF(%)	BF($\times 10^{-4}$)
	$D^+ \to \pi^+\pi^+\pi^-\eta$ *decay:*		
$D^+ \to a_0(980)^+\rho(770)^0$	0 (fixed)	$14.7 \pm 1.1 \pm 1.0$	$4.7 \pm 0.4 \pm 0.3$
$D^+ \to a_0(980)^+ f_0(500)$	$5.5 \pm 0.1 \pm 0.2$	$23.2 \pm 2.3 \pm 2.2$	$7.5 \pm 0.7 \pm 0.7$
$D^+ \to \eta(1405)\pi^+$	$2.4 \pm 0.1 \pm 0.1$	$11.2 \pm 1.4 \pm 1.5$	$3.6 \pm 0.5 \pm 0.5$
$D^+ \to f_1(1285)\pi^+$	$4.9 \pm 0.1 \pm 0.3$	$17.3 \pm 0.9 \pm 0.8$	$5.5 \pm 0.3 \pm 0.3$
$D^+ \to f_1(1420)\pi^+$	$6.2 \pm 0.1 \pm 0.1$	$16.4 \pm 1.6 \pm 2.5$	$5.2 \pm 0.5 \pm 0.8$
$D^+ \to [a_0(980)^\pm\pi^\mp]_{S-wave}\pi^+$	$3.5 \pm 0.1 \pm 0.3$	$34.3 \pm 2.8 \pm 3.2$	$11.0 \pm 0.9 \pm 1.0$
	$D^+ \to \pi^+\pi^0\pi^0\eta$ *decay:*		
$D^+ \to a_0(980)^0\rho(770)^+$	0 (fixed)	$35.0 \pm 3.5 \pm 2.6$	$8.5 \pm 0.9 \pm 0.6$
$D^+ \to a_0(980)^+ f_0(500)$	$0.4 \pm 0.1 \pm 0.0$	$34.6 \pm 3.1 \pm 1.0$	$8.4 \pm 0.8 \pm 0.3$
$D^+ \to \eta(1405)\pi^+$	$2.6 \pm 0.3 \pm 0.2$	$4.3 \pm 1.7 \pm 0.7$	$1.0 \pm 0.4 \pm 0.2$
$D^+ \to f_1(1285)\pi^+$	$2.9 \pm 0.2 \pm 0.3$	$12.8 \pm 1.9 \pm 0.7$	$3.1 \pm 0.5 \pm 0.2$
$D^+ \to f_1(1420)\pi^+$	$5.9 \pm 0.2 \pm 0.3$	$17.8 \pm 3.0 \pm 1.6$	$4.3 \pm 0.8 \pm 0.4$
$D^+ \to [a_0(980)^0\pi^0]_{S-wave}\pi^+$	$4.4 \pm 0.2 \pm 0.2$	$11.0 \pm 2.6 \pm 0.9$	$2.7 \pm 0.6 \pm 0.2$

CPC Accepted

fraction of nearest neighbours that originate from the other sample. The pull of T from its expected value under the hypothesis of a perfect fit is then computed. For the $\pi^+\pi^{+(0)}\pi^{-(0)}\eta$ channel, we find a pull value of 0.51 (0.57), demonstrating good fit quality.

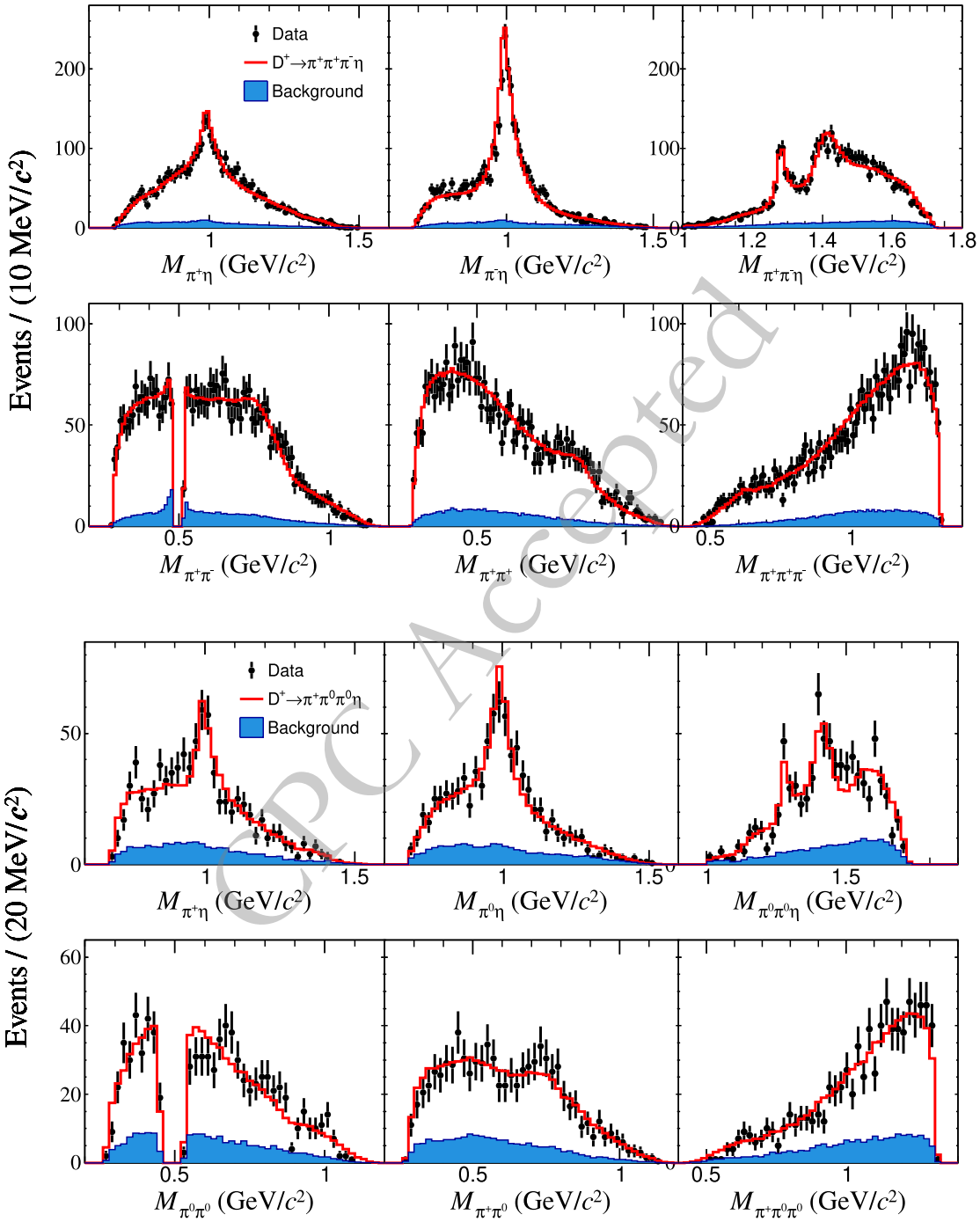
The systematic uncertainties in the amplitude analysis arise from the following sources: (I) the amplitude model, including the mass and coupling constants of the $a_0(980)^{\pm(0)}$, which are varied within the uncertainties reported in Ref. [46], the masses and widths of the $\eta(1405)$, $f_1(1285)$, and $f_1(1420)$ states, varied according to the uncertainties listed in the PDG [1], and the $f_0(500)$ line shape, which is altered by using an alternative RBW function with its mass and width fixed to 526 MeV/c^2 and 535 MeV, respectively, as in Ref. [53]; (II) the effective barrier radii for D^+ and other intermediate states (R value), estimated by varying the nominal values by ± 1 GeV^{-1}; (III) experimental effects related to the detector response, evaluated using the same method as described in Ref. [54]; (IV) the fit bias, assessed by fitting 600 signal MC samples of the same size as the data sample: the results show good agreement between the fitted and input parameters of the amplitude model, with no significant bias observed; (V) the background estimation: the uncertainty associated with the background sample size has been studied by varying the purity within its statistical uncertainty, while the uncertainty from the peaking background [42] has been assessed by using an alternative sample with their fractions varied within the BF uncertainties. These systematic uncertainties are estimated separately by taking the difference between the values of ϕ_n and FF$_n$ obtained by the alternative and the baseline fits. The total systematic uncertainties are obtained by adding each term in quadrature, and the details are listed in the supplemental material [42].

Furthermore, we measure the BFs of $D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta$ with the DT technique [37], using the same five tag modes and selection criteria as in the amplitude analysis. The BFs of the signal decays are determined by

$$\mathcal{B} = \frac{N_{\rm DT}}{\sum_\alpha (N^\alpha_{\rm ST}\epsilon^\alpha_{\rm DT}/\epsilon^\alpha_{\rm ST})\mathcal{B}_{\rm sub}}, \quad (2)$$

where α denotes the tag mode, $N^\alpha_{\rm ST}$ and $N_{\rm DT}$ are the ST and total DT yields, respectively, and $\mathcal{B}_{\rm sub}$ is the product of the BFs of all intermediate particles in the decay chain. The efficiencies $\epsilon^\alpha_{\rm ST}$ and $\epsilon^\alpha_{\rm DT}$ are obtained from MC simulation, with $\epsilon^\alpha_{\rm DT}$ evaluated using the amplitude model of the signal decays. All quantities—$N^\alpha_{\rm ST}$, $\epsilon^\alpha_{\rm ST}$, and $\epsilon^\alpha_{\rm DT}$—are detailed in the supplemental material [42], where the 2D fit procedure used to extract $N_{\rm DT}$ is also described. Finally, we obtain the absolute BFs with the highest precision to date: $\mathcal{B}(D^+ \to \pi^+\pi^+\pi^-\eta|_{\rm non-}{}_{\eta'}) = (3.20 \pm 0.06_{\rm stat.} \pm 0.03_{\rm syst.}) \times 10^{-3}$ and $\mathcal{B}(D^+ \to \pi^+\pi^0\pi^0\eta|_{\rm non-}{}_{\eta'}) = (2.43 \pm 0.11_{\rm stat.} \pm 0.04_{\rm syst.}) \times 10^{-3}$, which are consistent with the previous measurements [41].

The systematic uncertainties in the BF measurement are summarized in the supplemental material [42] and discussed below. The uncertainty in the ST D^- yield reflects the uncertainty in the fit to the $M_{\rm BC}$ distributions



CPC Accepted

Fig. 2. (color online) Mass projections of the fit results. The plots containing identical π^+ or π^0 are added into one projection. The data are represented by points with error bars and the fit results by the red curves. The blue curves indicate the background contribution estimated with inclusive MC samples.

and is assessed by varying the signal and background shapes. The uncertainties in the tracking or PID of $\pi^\pm$ are studied with control samples of DT-hadronic events. The uncertainty associated with the η and π^0 reconstruction are both assigned by studying a control sample of $D^0\to K^-\pi^+\pi^0$ decays. The possible difference between data and MC simulation is accounted for by examining the $\Delta E_{\rm sig}$ cut efficiency after applying a double-Gaussian smearing to the signal MC sample. The observed efficiency variation is taken as the systematic uncertainty. The uncertainty from the amplitude model is determined by varying the amplitude model parameters based on their error matrix. For the 2D fit, the mean and width of the convolved Gaussian function are varied by $\pm1\sigma$ for the sig-

nal shape; the ARGUS end-point for the background shape is varied by ±0.2 MeV; the fixed yields of peaking background decays are varied by $\pm1\sigma$ according to their quoted BFs [1, 42]. The uncertainty due to $\mathcal{B}(\pi^0/\eta \to \gamma\gamma)$ is taken from the PDG [1]. The effect of the limited MC sample size is also included.

In summary, using an e^+e^- collision data corresponding to an integrated luminosity of 20.3 fb^{-1} collected with the BESIII detector at $\sqrt{s} = 3.773$ GeV, the amplitude analysis of the singly Cabibbo-suppressed decays $D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta$ is performed for the first time, improving the absolute BFs by a factor of three in precision [1]. The absolute BFs of intermediate states are calculated as $\mathcal{B}_i = \mathrm{FF}_i \times \mathcal{B}(D^+ \to \pi^+\pi^{+(0)}\pi^{-(0)}\eta)$ and listed in Table 1.

The $D \to SS$ process $D^+ \to a_0(980)^+ f_0(500)$ has been observed for the first time in both channels, with a significance exceeding 10σ. The measured BF reaches an unexpectedly large level of 10^{-3}, which may arise either from enhanced direct production due to tetraquark components in the light scalars [21, 55, 56] or from triangle rescattering dynamics [57].

Moreover, in the $D \to SV$ sector, we report the first observation of both $D^+ \to a_0(980)^+\rho(770)^0$ and $D^+ \to a_0(980)^0\rho(770)^+$ decays with statistical significances exceeding 10σ. The ratio $\dfrac{\mathcal{B}(D^+ \to a_0(980)^+\rho(770)^0)}{\mathcal{B}(D^+ \to a_0(980)^0\rho(770)^+)}$ is first measured to be $0.55 \pm 0.08_{\mathrm{stat.}} \pm 0.05_{\mathrm{syst.}}$. Combined with the earlier BESIII result in $D \to a_0\pi$ decays [6], this ratio points to substantial FSI effects. Taken together, these results provide essential inputs to pin down the internal structure of the $a_0(980)$ and $f_0(500)$, and a testing ground for non-perturbative QCD calculations, in particular those involving FSIs.

CPC Accepted

References

[1] S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)
[2] F. L. Braghin, J. Phys. G **50**, 095101 (2023), arXiv: 2212.06616[hep-ph]
[3] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **121**, 081802 (2018), arXiv: 1803.02166[hep-ex]
[4] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **123**, 112001 (2019), arXiv: 1903.04118[hep-ex]
[5] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **104**, L071101 (2021), arXiv: 2106.13536[hep-ex]
[6] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **110**, L111102 (2024), arXiv: 2404.09219[hep-ex]
[7] Y.-K. Hsiao, S.-Q. Yang, W.-J. Wei, and B.-C. Ke, JHEP **12**, 226 (2025), arXiv: 2306.06091[hep-ph]
[8] Y.-K. Hsiao, Y. Yu, and B.-C. Ke, Eur. Phys. J. C **80**, 895 (2020), arXiv: 1909.07327[hep-ph]
[9] Y. Yu, Y.-K. Hsiao, and B.-C. Ke, Eur. Phys. J. C **81**, 1093 (2021), arXiv: 2108.02936[hep-ph]
[10] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977)
[11] T. V. Brito, F. S. Navarra, M. Nielsen, and M. E. Bracco, Phys. Lett. B **608**, 69 (2005), arXiv: hep-ph/0411233
[12] E. Klempt and A. Zaitsev, Phys. Rept. **454**, 1 (2007), arXiv: 0708.4016[hep-ph]
[13] C. Alexandrou, J. Berlin, M. Dalla Brida, J. Finkenrath, T. Leontiou, and M. Wagner, Phys. Rev. D **97**, 034506 (2018), arXiv: 1711.09815[hep-lat]
[14] T. Humanic *et al.* (ALICE Collaboration), Rev. Mex. Fis. Suppl. **3**, 0308039 (2022)
[15] J. D. Weinstein and N. Isgur, Phys. Rev. Lett. **48**, 659 (1982)
[16] L.-Y. Dai and M. R. Pennington, Phys. Lett. B **736**, 11 (2014), arXiv: 1403.7514[hep-ph]
[17] T. Sekihara and S. Kumano, Phys. Rev. D **92**, 034010 (2015), arXiv: 1409.2213[hep-ph]
[18] M.-Y. Duan, J.-Y. Wang, G.-Y. Wang, E. Wang, and D.-M. Li, Eur. Phys. J. C **80**, 1041 (2020), arXiv: 2008.10139[hep-ph]
[19] N. Ikeno, M. Bayar, and E. Oset, Eur. Phys. J. C **81**, 377 (2021), arXiv: 2102.01650[hep-ph]
[20] B.-C. Ke, J. Koponen, H.-B. Li, and Y. Zheng, Ann. Rev. Nucl. Part. Sci. **73**, 285 (2023), arXiv: 2310.05228[hep-ex]
[21] H.-Y. Cheng, C.-W. Chiang, and F. Xu, Phys. Rev. D **110**, 094052 (2024), arXiv: 2408.13942[hep-ph]
[22] D. R. Boito, J. P. Dedonder, B. El-Bennich, O. Leitner, and B. Loiseau, Phys. Rev. D **79**, 034020 (2009), arXiv: 0812.3843[hep-ph]
[23] H.-Y. Cheng, C.-W. Chiang, and Z.-Q. Zhang, Phys. Rev. D **105**, 033006 (2022), arXiv: 2201.00460[hep-ph]
[24] L. R. Dai and E. Oset, Phys. Rev. D **109**, 054008 (2024), arXiv: 2307.03014[hep-ph]
[25] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **132**, 131903 (2024), arXiv: 2309.05760[hep-ex]
[26] M. Ablikim *et al.* (BESIII Collaboration), Nucl. Instrum. Meth. A **614**, 345 (2010), arXiv: 0911.4960[physics.ins-det]
[27] M. Ablikim *et al.* (BESIII Collaboration), Chin. Phys. C **37**, 123001 (2013), arXiv: 1307.2022[hep-ex]
[28] M. Ablikim *et al.* (BESIII Collaboration), Chin. Phys. C **48**, 123001 (2024), arXiv: 2406.05827[hep-ex]
[29] S. Agostinelli *et al.* (GEANT4 Collaboration), Nucl. Instrum. Meth. A **506**, 250 (2003)
[30] K. X. Huang, Z. J. Li, Z. Qian, J. Zhu, H. Y. Li, Y. M. Zhang, S. S. Sun, and Z. Y. You, Nucl. Sci. Tech. **33**, 142 (2022), arXiv: 2206.10117[physics.ins-det]
[31] S. Jadach, B. F. L. Ward, and Z. Was, Phys. Rev. D **63**, 113009 (2001), arXiv: hep-ph/0006359
[32] D. J. Lange, Nucl. Instrum. Meth. A **462**, 152 (2001)
[33] R. G. Ping, Chin. Phys. C **32**, 599 (2008)
[34] J. C. Chen, G. S. Huang, X. R. Qi, D. H. Zhang, and Y. S. Zhu, Phys.Rev.D **62**, 034003 (2000)
[35] R. L. Yang, R. G. Ping, and H. Chen, Chin.Phys.Lett. **31**, 061301 (2014)
[36] E. Richter-Was, Phys. Lett. B **303**, 163 (1993)
[37] R. M. Baltrusaitis *et al.* (MARK-III Collaboration), Phys. Rev. Lett. **56**, 2140 (1986)
[38] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **111**, L091501 (2025), arXiv: 2411.07730[hep-ex]
[39] M. Ablikim *et al.* (BESIII), (2025), 10.1007/JHEP06(2025)220, arXiv: 2503.19542 [hep-ex].
[40] M. Ablikim *et al.* (BESIII Collaboration), Chin. Phys. C **49**, 083001 (2025), arXiv: 2503.16070[hep-ex]

[41] M. Ablikim *et al*. (BESIII Collaboration), Phys. Rev. Lett. **124**, 241803 (2020), arXiv: 2004.13910[hep-ex]
[42] See Supplemental Material at[URL] for the 2D fit, background PDF modeling and symmetry considerations in amplitude construction, as well as systematic uncertainties of amplitude analysis and BF measurements.
[43] B. S. Zou and D. V. Bugg, Eur. Phys. J. A **16**, 537 (2003), arXiv: hep-ph/0211457
[44] J. D. Jackson, Nuovo Cim. **34**, 1644 (1964)
[45] G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. **21**, 244 (1968)
[46] A. Abele *et al*., Phys. Rev. D **57**, 3860 (1998)
[47] D. V. Bugg, B. S. Zou, and A. V. Sarantsev, Nucl. Phys. B **471**, 59 (1996)
[48] A. Rogozhnikov, J. Phys. Conf. Ser. **762**, 012036 (2016), arXiv: 1608.05806[physics.data-an]
[49] B. Liu, X. Xiong, G. Hou, S. Song, and L. Shen, EPJ Web Conf. **214**, 06033 (2019)
[50] M. Artuso *et al*. (CLEO Collaboration), Phys. Rev. D **85**, 122002 (2012), arXiv: 1201.5716[hep-ex]
[51] M. Williams, JINST **5**, P09004 (2010), arXiv: 1006.3019[hep-ex]
[52] M. Ablikim *et al*. (BESIII Collaboration), Phys. Rev. D **99**, 092008 (2019), arXiv: 1903.06316[hep-ex]
[53] M. Ablikim *et al*. (BES Collaboration), Phys. Lett. B **598**, 149 (2004), arXiv: hep-ex/0406038
[54] M. Ablikim *et al*. (BESIII Collaboration), Phys. Rev. D **104**, 012016 (2021), arXiv: 2011.08041[hep-ex]
[55] L. L. Chau and H. Y. Cheng, Phys. Rev. Lett. **56**, 1655 (1986)
[56] H.-Y. Cheng and C.-W. Chiang, Phys. Rev. D **81**, 074021 (2010), arXiv: 1001.0987[hep-ph]
[57] Y.-L. Wang, S.-T. Cai, and Y.-K. Hsiao, Eur. Phys. J. C **86**, 89 (2026), arXiv: 2509.16879[hep-ph]

CPC Accepted