Probing decoupled throats of AdS$_D$ black holes in $D = 6, 7$*

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Abstract: The Kerr/CFT correspondence establishes a correspondence between extremal black holes in higher dimensions and a chiral conformal field theory (CFT) in their near-horizon limit. A generalization of this framework, known as the EVH/CFT correspondence, has been developed for four- and five-dimensional AdS black holes. It was further proposed in [1] that, for AdS$_{D=6,7}$ black holes, a generalized duality between $(D-2)$-dimensional geometry and $(D-3)$-dimensional field theory may emerge in a suitably defined extremal vanishing horizon (EVH) limit. In this work, we show that, in the EVH limit, the near-EVH geometries of these AdS$_{D=6,7}$ black holes reduce to lower-dimensional black holes whose metrics are conformally related to solutions of Einstein-Maxwell-Maxwell-dilaton (EMMD) gravity. This structural resemblance suggests a potential route toward the microscopic counting of non-AdS black hole entropy via higher-dimensional AdS/CFT techniques.

Keywords: black holes, AdS/CFT, holography

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I. INTRODUCTION

To date, all gravitational-wave observations of black-hole mergers have been consistent with the Kerr description of black holes [2]. Understanding the quantum effects associated with Kerr black holes is therefore crucial for identifying potential signatures of quantum gravity. A major observational milestone is the confirmation of Hawking's area theorem [3], which states that the total event horizon area of black holes cannot decrease over time. In particular, recent observations demonstrate that the final horizon area after a merger exceeds the sum of the areas of the two initial black holes [4, 5].

Understanding the microscopic states of black holes is a cornerstone test for any candidate theory of quantum gravity. Two main approaches have been developed to study this problem across different frameworks.

- The AdS$_3$/CFT$_2$ correspondence provides a particularly transparent setting, since the powerful Virasoro symmetry completely determines the microscopic states of black holes in the near-horizon region [6]. This idea was later generalized to higher-dimensional extremal black holes through the Kerr/CFT correspondence [7] (and further to the extremal black hole/CFT correspondence [8–10]). These correspondences assert that the generic AdS$_2$ geometry emerging in the near-horizon limit of extremal black holes admits boundary conditions under which a Virasoro algebra governs the dynamics, independently of the details of the higher-dimensional black hole in the UV [1)].

- Within the AdS$_{d+1}$/CFT$_d$ ($d = D-1 > 2$) framework, the microscopic states of AdS black holes can be reproduced by counting gauge-invariant operators in the dual superconformal field theories. Remarkable progress has been made in understanding black holes in AdS$_4$ [12–16], AdS$_5$ [17–20], AdS$_6$ [14, 21], and AdS$_7$ [19, 22–24]. See [25] for a recent review. Most of these studies focus on BPS black holes, for which the counting of gauge-invariant states in the dual weakly coupled field theory remains valid. An exception is [26], which undertakes a field-theoretic computation to understand the entropy of near-BPS AdS$_5$ black holes.

The two approaches have been investigated in a unified framework for BPS black holes in AdS$_D$ spacetimes.

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1) There are also extensions of this idea to non-extremal black holes, where the Virasoro symmetry is realized as a geometric symmetry of the phase space of perturbations [11].

Two possible perspectives have been considered in the literature. One is to consider gravitational Cardy limits [27], in which the charges and angular momenta are rescaled so as to render the black hole ultra-spinning. The near-horizon geometry in these limits is generically of AdS_2 or AdS_3 type [28, 29], where the Virasoro algebra can determine the dynamics in the near-horizon limit. Another approach builds on earlier studies of the Extremal Vanishing Horizon (EVH) limit of black holes [30–41] and explores AdS_5 black holes in the BPS-EVH limit [1]. In the EVH limit, the black hole area $S_0 \sim A$ is taken to zero while the central charge c diverges, so that the product cA remains fixed. Given the expansion of the entropy S in terms of the temperature T:

$$S(T,Q_i,J_a) = S_0(Q_i,J_a) + S_1(Q_i,J_a)T + S_2(Q_i,J_a)T^2 + \cdots, \tag{1}$$

The dynamics of the extremal black hole can be activated in this limit by setting $S_0 = 0$, thereby circumventing the instability problem associated with AdS_2 geometry [42], and the geometry is enhanced to AdS_3 if $S_1 \neq 0$. In the supersymmetric AdS_5 black hole model, an extremal pinched BTZ black hole emerges in the near-EVH regime, consistent with the $S \sim T$ relation [1, 30–32, 34–37]. Moreover, since the microscopic states of the AdS_5 black hole can be computed via the superconformal indices of $\mathcal{N} = 4$ SYM, the authors of [1] demonstrated that a Cardy-like formula—used to count the degeneracies of states in an emergent CFT_2 [43]—can be derived from the inverse Laplace transform of the $\mathcal{N} = 4$ superconformal indices. This work represents a significant step toward proving the Kerr/CFT correspondence from the perspective of AdS/CFT, at least in the BPS limit. However, neither of these works can be considered to complete the proof of Kerr/CFT from AdS/CFT, since the mechanism underlying the emergent Virasoro algebra remains unclear.

An interesting generalization of EVH black holes proposed in [1, 30–41] involves emergent EVH configurations in the IR limit of AdS_6 [44] and AdS_7 black holes [45, 46]. These black holes can be appropriately embedded in ten-dimensional string theory and eleven-dimensional M-theory [47, 48]. The extremal limit of higher-dimensional AdS black holes requires more charges and angular momenta to be supported [8, 49, 50]. This is because gravitational interactions are stronger in higher dimensions, which requires more gauge forces or rotations to balance and achieve extremal black holes.

On the other hand, when more charges and spatial dimensions are present, we might expect that (1) can exhibit higher-order scalings, $S \sim T^2$ or $S \sim T^3$, by fine-tuning the charges and angular momenta, *i.e.*, setting $S_0(Q_i,J_a) = S_1(Q_i,J_a) = 0$ [31, 32, 35]. However, we will clarify that near-EVH black holes in rotating AdS_6 and AdS_7 can be defined without being restricted to the near-BPS limit, as previously proposed in [1], and can instead be realized through general near-extremal limits. We carefully examine the scaling relations between entropy S and temperature T for these black holes. Our analysis shows that their near-horizon geometries, in both $D = 6$ and $D = 7$, reduce to lower-dimensional black holes whose metrics are conformally equivalent to those of Einstein–Maxwell–Maxwell–Dilaton (EMMD) gravity, suggesting a close connection. These black holes emerging from EVH limits no longer have AdS asymptotics. However, as the microscopic states of supersymmetric $AdS_{6,7}$ black holes can be reproduced from the dual SCFT, we should, in principle, use the higher-dimensional SCFTs to understand the quantum states dual to these EMMD black holes. This may be an essential step toward understanding holographic duals of non-AdS black holes.

The paper is organized as follows. In Section II, we review the concept of near-EVH limits for Kerr–AdS_5 black holes based on [1, 32, 35]. In Sections III.A and III.B, we investigate the near-EVH phases in AdS_6 and AdS_7 black holes, respectively, within this geometric framework. Specifically, for AdS_6, the near-EVH geometry corresponds to a four-dimensional EMMD black hole with $S \sim T^2$. In AdS_7, we find both $S \sim T$ and $S \sim T^3$ configurations, which are realized as a BTZ black hole and a five-dimensional EMMD black hole, respectively. Finally, in Section IV, we discuss the holographic implications of these near-EVH limits, and conclude in Section V.

II. REVIEW OF AdS_5

Investigations of the (near-)EVH limits of AdS_5 black holes were initiated for static R-charged solutions in [30, 34] and further developed in [31]. Subsequent studies of the EVH limits of Kerr–RN–AdS_5 black holes and their dual field-theory interpretations began with [32], while an index-based account of the emergent IR CFT_2 and its entropy was later provided in [1]. Generic AdS_5 black holes in $U(1)^3$ gauged supergravity carry three charges and two angular momenta. The most general non-supersymmetric solution of this type was constructed in [51], building on earlier works [52–56]. However, the full generality of these solutions renders them rather complicated. For the purpose of capturing the essential physics of near-EVH limits and their holographic interpretation via superconformal indices [1], it suffices to consider the simpler special solutions presented in [32, 54]. We therefore restrict our analysis to these special cases.

The black hole solution under consideration, studied in [1, 10], carries two equal charges and is described by the following five-dimensional metric [54]:

$$
\begin{aligned}
\mathrm{d}s_5^2 =& H^{-\frac{4}{3}}\left[-\frac{X}{\rho^2}(\mathrm{d}t - a\sin^2\theta\frac{\mathrm{d}\phi}{\Xi_a} - b\cos^2\theta\frac{\mathrm{d}\psi}{\Xi_b})^2 + \frac{C}{\rho^2}\left(\frac{ab}{f_3}\mathrm{d}t - \frac{b}{f_2}\sin^2\theta\frac{\mathrm{d}\phi}{\Xi_a} - \frac{a}{f_1}\cos^2\theta\frac{\mathrm{d}\psi}{\Xi_b}\right)^2\right. \\
&\left.+\frac{Z\sin^2\theta}{\rho^2}\left(\frac{a}{f_3}\mathrm{d}t - \frac{1}{f_2}\frac{\mathrm{d}\phi}{\Xi_a}\right)^2 + \frac{W\cos^2\theta}{\rho^2}\left(\frac{b}{f_3}\mathrm{d}t - \frac{1}{f_1}\frac{\mathrm{d}\psi}{\Xi_b}\right)^2\right] + H^{\frac{2}{3}}\left[\frac{\rho^2}{X}\mathrm{d}r^2 + \frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2\right], \\
H =& 1 + \frac{q}{\rho^2}, \quad \rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta, \quad \Delta_\theta = 1 - a^2\cos^2\theta - b^2\sin^2\theta, \\
f_1 =& a^2 + r^2, \quad f_2 = b^2 + r^2, \quad f_3 = (a^2+r^2)(b^2+r^2) + qr^2, \\
X =& \frac{(a^2+r^2)(b^2+r^2)}{r^2} - 2m + (a^2+r^2+q)(b^2+r^2+q), \\
C =& f_1 f_2 (X + 2m - \frac{q^2}{\rho^2}), \qquad \Xi_a = 1 - a^2, \quad \Xi_b = 1 - b^2, \\
Z =& -b^2 C + \frac{f_2 f_3}{r^2}\left[f_3 - r^2(a^2-b^2)(a^2+r^2+q)\cos^2\theta\right], \\
W =& -a^2 C + \frac{f_1 f_3}{r^2}\left[f_3 + r^2(a^2-b^2)(b^2+r^2+q)\sin^2\theta\right].
\end{aligned} \tag{2}
$$

The corresponding thermodynamic quantities are as follows:

$$
\begin{aligned}
&\Omega_a = \frac{a(r_+^4 + r_+^2 b^2 + r_+^2 q + b^2 + r_+^2)}{(r_+^2+a^2)(r_+^2+b^2)+qr_+^2}, \qquad \Omega_b = \frac{b(r_+^4 + r_+^2 a^2 + r_+^2 q + a^2 + r_+^2)}{(r_+^2+a^2)(r_+^2+b^2)+qr_+^2}, \qquad \Phi_1 = \Phi_2 = \frac{\sqrt{q^2+2mq}r_+^2}{(r_+^2+a^2)(r_+^2+b^2)+qr_+^2}, \\
&\Phi_3 = \frac{qab}{(r_+^2+a^2)(r_+^2+b^2)+qr_+^2}, \qquad J_a = \frac{\pi a(2m+q\Xi_b)}{4G_N\Xi_b\Xi_a^2}, \qquad J_b = \frac{\pi b(2m+q\Xi_a)}{4G_N\Xi_a\Xi_b^2}, \qquad Q_1 = Q_2 = \frac{\pi\sqrt{q^2+2mq}}{4G_N\Xi_a\Xi_b}, \\
&Q_3 = -\frac{\pi abq}{4G_N\Xi_a\Xi_b}, \qquad S = \frac{\pi^2[(r_+^2+a^2)(r_+^2+b^2)+qr_+^2]}{2G_N\Xi_a\Xi_b r_+}, \qquad T = \frac{2r_+^6 + r_+^4(1+a^2+b^2+2q) - a^2b^2}{2\pi r_+[(r_+^2+a^2)(r_+^2+b^2)+qr_+^2]}, \\
&E = \frac{\pi}{8G_N\Xi_a^2\Xi_b^2}[2m(2\Xi_a + 2\Xi_b - \Xi_a\Xi_b) + q(2\Xi_a^2 + 2\Xi_b^2 + 2\Xi_a\Xi_b - \Xi_a^2\Xi_b - \Xi_b^2\Xi_a)].
\end{aligned} \tag{3}
$$

This solution is parametrized by four independent parameters, (a,b,m,q), and we set the cosmological constant to $g=1$. Here the Newton constant G_N is related to the rank of the $SU(N)$ gauge group in $\mathcal{N}=4$ SYM by $\frac{\pi}{2G_N}=N^2$. These expressions fully characterize the thermodynamic state of the black hole. The solution and its thermodynamic data play a central role in the subsequent analysis of the near-EVH limit and the emergence of an effective two-dimensional conformal description.

The EVH and near-EVH limits are defined as in [32][1)]

$$
\mathrm{EVH}: a = r_+ = 0; \qquad \text{near-EVH}: a = \lambda\epsilon^2, \quad r = \epsilon x, \tag{4}
$$

where the $\epsilon\to 0$ limit is taken alongside the near-horizon limit. The EVH point is treated as the ground state, while the near-EVH limit can be regarded as an excited sector of the theory, since it has a non-vanishing temperature of order ϵ. The corresponding geometries in the (near-)EVH limits are, respectively, a pinched AdS$_3$ and a BTZ black hole. The entropy of the BTZ black hole scales as $S\sim N^2\epsilon$, which remains finite if we keep $N^2\epsilon$ fixed as $N\to\infty$. For the classical description of gravity to be valid, $N^2\epsilon$ should be taken to be large.

We are especially interested in the near-EVH limit combined with the BPS condition, which incorporates supersymmetry

$$
E = J_a + J_b + Q_1 + Q_2 + Q_3, \tag{5}
$$

and the extremality condition, which requires the horizons to be degenerate. The chemical potentials that satisfy the supersymmetry condition are generically complex, and it is useful to define the following chemical potentials:

$$
\Delta_i = \beta(1-\Phi_i), \qquad \omega_I = \beta(1-\Omega_I), \qquad i=1,2,3; \quad I=1,2, \tag{6}
$$

1) The EVH limit is defined by $b=r_+=0$ in [1] and remains valid in the $q=0$ limit. Due to the symmetry between the a and b parameters, this does not change the essential physics.

and are subject to the linear constraint:

$$\Delta_1+\Delta_2+\Delta_3-\omega_a-\omega_b=2\pi\mathrm{i}\,. \tag{7}$$

This renders the parameters (q,m) generically complex unless the radius of the horizon is determined by a real constraint.

$$r_0^2=\frac{ab}{1+a+b}\,. \tag{8}$$

This condition eliminates closed timelike curves in spacetime. Solutions that satisfy both the supersymmetry condition (5) and the horizon-size condition (8) have real charges and entropy and are parametrized by (q,m) as

$$\begin{aligned}q&=\frac{(a+b)(1+a)(1+b)}{1+a+b}\,,\\ m&=\frac{(a+b)^2(1+a)(1+b)(2+a+b)}{2(1+a+b)}\,.\end{aligned} \tag{9}$$

Combining the BPS conditions (5) and (8) with the EVH conditions (4), we find that the entropy of the black hole is given by

$$S=\frac{\pi b}{1-b}\sqrt{\frac{b\lambda}{1+b}}N^2\epsilon\,. \tag{10}$$

The corresponding decoupling metric for AdS$_5$ takes the following form:

$$\begin{aligned}\mathrm{d}s^2&=\left(\frac{h}{\sin\theta}\right)^{-\frac{4}{3}}\left[h^2\mathrm{d}s_3^2+h^2\frac{b^2\mathrm{d}\theta^2}{1-b^2\sin^2\theta}\right.\\ &\left.\quad+\frac{1-b^2\sin^2\theta}{(1-b)^2}\frac{\cos^2\theta}{\sin^2\theta}\mathrm{d}\tilde{\psi}^2\right],\\ h&=\sin\theta+\frac{1}{b\sin\theta}\,,\qquad \tilde{\psi}=\psi-(1-b)t\,,\end{aligned} \tag{11}$$

where, in the EVH limit, $\mathrm{d}s_3^2$ is taken to be the metric of AdS$_3$

$$\mathrm{d}s_3^2=-\frac{x^2}{\ell_3^2}\mathrm{d}\tau^2+\frac{\ell_3^2}{x^2}\mathrm{d}x^2+x^2\mathrm{d}\tilde{\chi}^2,\quad \ell_3=\frac{b}{1+b},\quad t=\frac{1+b}{b}\tau, \tag{12}$$

In the near-EVH case, however, $\mathrm{d}s_3^2$ is replaced by the extremal BTZ geometry:

$$\mathrm{d}s_3^2=-\frac{\left(x^2-x_0^2\right)^2}{\ell_3^2x^2}\mathrm{d}\tau^2+\frac{\ell_3^2x^2\mathrm{d}x^2}{(x^2-x_0^2)^2}+x^2\left(\mathrm{d}\tilde{\chi}-\frac{x_0^2}{\ell_3x^2}\mathrm{d}\tau\right)^2. \tag{13}$$

This geometry is called pinched because the periodicity of the S^1 direction, $\tilde{\chi}=\epsilon\phi$, is $2\pi\epsilon$.

We are now ready to explain the entropy (10) of the BPS-EVH black hole from computations in the dual $\mathcal{N}=4$ SYM, using (13) and following [1]. It has been shown in various works [17–20, 57] that the entropy functional in the large-N limit of $\mathcal{N}=4$ SYM is

$$\begin{aligned}S&=\ln Z+(J_a+Q_3)\omega_a+(J_b+Q_3)\omega_b+(Q_1-Q_3)\Delta_1\\ &\quad+(Q_2-Q_3)\Delta_2+2\pi\mathrm{i}Q_3\\ \ln Z&=\frac{N^2}{2}\frac{\Delta_1\Delta_2\Delta_3}{\omega_a\omega_b},\quad \Delta_1+\Delta_2+\Delta_3-\omega_a-\omega_b=2\pi\mathrm{i}\,,\end{aligned} \tag{14}$$

where the chemical potentials are taken to be complex to avoid cancellations between the bosonic and fermionic degrees of freedom [12]. Upon performing a saddle-point approximation with respect to the chemical potentials $\Delta_i,\omega_a,\omega_b$, one finds that the entropy functional (14) reproduces the BPS black-hole entropy [56].

$$S=2\pi\sqrt{Q_1Q_2+Q_1Q_3+Q_2Q_3-\frac{N^2}{2}(J_a+J_b)}\,. \tag{15}$$

The entropy formula (15) is consistent with that for the black hole with two equal charges (3) in the BPS limit. Therefore, the BPS-EVH limit of the entropy (10) should also be encoded in the superconformal indices of $\mathcal{N}=4$ SYM and in the entropy functional (14).

As shown in [1], the near-EVH limit splits the extremization of the entropy functional within the saddle-point approximation into two steps. In the large-N limit, with the EVH condition $N^2\epsilon$ held fixed, the charges scale as $Q_3\sim J_a\sim N^2\epsilon^2$, while $Q_{1,2}\sim J_b\sim N^2$. Under this scaling, the saddle-point approximation of the functional in (14) is justified only for $\Delta_{1,2}$ and ω_b, but not for ω_a. Therefore, setting $\Delta_1=\Delta_2=\Delta$ in the solution (2) and performing the ω_b and Δ integrals first, we obtain

$$\mathrm{e}^S=\int\mathrm{d}\omega_a\exp\left[\frac{N^2\hat{\Delta}^2}{2\omega_a}\left(\frac{\hat{\Delta}}{\hat{\omega}_b}-1\right)+2\pi\mathrm{i}Q_3\right]\mathrm{e}^{\omega_a(J_a+Q_3)}, \tag{16}$$

where $\hat{\Delta}$ and $\hat{\omega}_b$ denote the values of the chemical potentials that satisfy the saddle-point equations. The expression (16) closely resembles the Cardy formula of a two-dimensional CFT. It may be viewed as a functional of the rescaled modulus $\tilde{\omega}_a=\epsilon\omega_a$. Since $N^2\epsilon$ is fixed and large, the extremization over $\tilde{\omega}_a$ is justified and precisely reproduces the near-EVH entropy given in (10).

This computation strongly supports the emergence of an effective two-dimensional conformal field theory in the near-horizon limit. The resulting EVH 2D CFT is closely related to the Kerr/CFT correspondence, although their central charges differ by a factor of $\sqrt{2}$, a discrepancy that may stem from different choices of time coordinate between AdS$_5$ and AdS$_3$ [1, 30, 32, 34]. This result should therefore be viewed as a key first step toward uncovering the microscopic mechanism underlying the Kerr/CFT correspondence within the framework of AdS/CFT.

However, this does not constitute sufficient evidence to claim that the EVH/CFT correspondence (a more special version of the Kerr/CFT correspondence) has been derived from AdS/CFT. First, although the central charge in the Cardy formula matches that of Kerr/CFT, its origin on the field-theory side remains obscure. It is unclear how this central charge, which is geometrically defined by the AdS$_2$ throat in the gravity picture, can be derived from the algebraic data of $\mathcal{N}=4$ SYM, such as operator conformal dimensions or the central charge of $\mathcal{N}=4$ SYM. Second, it is not understood how operators transforming under the 4D superconformal algebra PSU$(2,2|4)$ organize into representations of the Virasoro algebra[1]. This conceptual tension is reflected in the mismatch between the local symmetries of AdS$_5$ in the UV and AdS$_3$ in the IR. Moreover, it remains unclear whether the general SL$(3,\mathbb{Z})$ families of AdS black holes corresponding to root-of-unity configurations [58–67] admit such a near-EVH decoupling limit. These issues will not be discussed in this paper.

III. AdS BLACK HOLES IN $D = 6, 7$

AdS$_D$ black holes in $D = 6$ and 7 dimensions exhibit a considerably richer structure than their four- and five-dimensional counterparts. Of particular interest are solutions that either possess known holographic duals or admit consistent embeddings into string theory. Notable examples include AdS$_6$ black holes with two independent angular momenta and one R-charge [44], as well as AdS$_7$ black holes carrying up to three angular momenta and two distinct R-charges [46]. This classification closely mirrors the properties of superconformal field theories in $d = 5$ and 6 dimensions [68]. Moreover, these black hole solutions are of intrinsic interest owing to their realizations as specific brane configurations in string theory or M-theory [47, 48].

The metric of AdS$_D$ can be expressed in coordinates comprising the time t and radial coordinate r, together with $[\frac{D-1}{2}]$ azimuthal angles ϕ_i and $[\frac{D-2}{2}]$ latitude coordinates y_α on the sphere. To be precise, we set $D = 2n+1$ for odd dimensions and $D = 2n$ for even dimensions. The coordinates y_α are related to the direction cosines μ_i of the unit sphere $S^{[D/2]}$ via the Jacobi transformation:

$$\mu_i^2 = \frac{\prod_{\alpha=1}^{n-1}(a_i^2 - y_\alpha^2)}{\prod'^{n}_{k\neq i}(a_i^2 - a_k^2)}, \quad i = 1,\ldots,\left[\frac{D-1}{2}\right], \tag{17}$$

This automatically satisfies the constraint $\sum_{i=1}^{[D/2]}\mu_i^2 = 1$. Here, each a_i parametrizes a rotation in the corresponding ϕ_i direction. The symbol $\prod'$ indicates that the product omits any vanishing factor. In the case of even D, we set $a_n = 0$. This coordinate system, introduced in [50], provides a natural framework for generalizing Kerr–AdS black holes to include NUT charges. It also offers several structural advantages: for example, the metric on S^{D-2} becomes diagonal in these coordinates, and the coordinates y_α and r appear in a highly symmetric manner throughout the metric.

In this section, we describe the corresponding (near)-EVH limits of the AdS$_D$ black holes with $D = 6, 7$. An analysis of the thermodynamics of these black holes indicates that they exhibit $S \sim T^{D-4}$ scaling in the near-BPS and near-EVH limits [1]. These scaling relations between entropy and temperature were conjectured to hint at a possible AdS$_{D-2}$/CFT$_{D-3}$ duality emerging in the IR. In this section, we primarily provide two clarifications regarding these models.

- The near-BPS condition is not required to define the near-EVH limits.

- The near-EVH geometry is not that of AdS$_{D-2}$ black holes, but rather that of black holes which are solutions to EMMD theories [69] on a $(D-2)$-dimensional manifold.

A. AdS$_6$ black hole

The AdS$_6$ black hole is a solution of the $\mathcal{N}=4$ $SU(2)$ gauged supergravity theory in six dimensions, which includes a graviton, a two-form potential, a scalar, a one-form potential, and the $SU(2)$ Yang–Mills gauge potential. The bosonic part of the Lagrangian is given by [44, 70]

$$\begin{aligned}\mathcal{L}_6 = {}& R\star 1 - \frac{1}{2}\star d\varphi\wedge d\varphi - \frac{1}{2X^2}\left(\star F_{(2)}\wedge F_{(2)} + \star F^I_{(2)}\wedge F^I_{(2)}\right)\\ &-\frac{1}{2}X^4\star F_{(3)}\wedge F_{(3)} + \left(9X^2 + \frac{12}{X^2} - \frac{1}{X^6}\right)\star 1\\ &- A_{(2)}\wedge\left(\frac{1}{2}dA_{(1)}\wedge dA_{(1)} + \frac{1}{\sqrt{2}}A_{(2)}\wedge dA_{(1)}\right.\\ &\left.+\frac{1}{3}A_{(2)}\wedge A_{(2)} + \frac{1}{2}F^I_{(2)}\wedge F^I_{(2)}\right).\end{aligned} \tag{18}$$

For convenience, we adopt the following ansatz to de-

1) Evidence suggests that the emergence of the Virasoro algebra in the near-EVH limit is tied to the chiral algebra mechanism [1], since one of the components of the Virasoro generators $J_a + Q_3$ is a Schur operator. However, the set of letters contributing to the entropy and transforming under the Virasoro algebra also includes non-Schur operators. Therefore, a generalization of the chiral algebra mechanism appears necessary to account for the Virasoro symmetry emerging in near-EVH limits.

scribe the AdS_6 black hole solutions of this theory.

$$y_\alpha = (y,z), \quad (a_1,a_2,a_3) \equiv (a,b,0). \tag{19}$$

The charged, rotating AdS_6 solution in the asymptotic static frame is given by [1)] [10, 44]

$$\begin{aligned} ds^2 = H^{\frac{1}{2}} \Bigg[& \frac{(r^2+y^2)(r^2+z^2)}{X}dr^2 + \frac{(r^2+y^2)(y^2-z^2)}{Y}dy^2 \\ & + \frac{(r^2+z^2)(z^2-y^2)}{Z}dz^2 - \frac{X}{H^2(r^2+y^2)(r^2+z^2)}\mathcal{A}^2 \\ & + \frac{Y}{(r^2+y^2)(y^2-z^2)}\Big((1+r^2)(1-z^2)d\tilde{t} - (a^2+r^2)(a^2-z^2)d\tilde{\phi}_1 \\ & - (b^2+r^2)(b^2-z^2)d\tilde{\phi}_2 - \frac{qr\mathcal{A}}{H(r^2+y^2)(r^2+z^2)}\Big)^2 \\ & + \frac{Z}{(r^2+z^2)(z^2-y^2)}\Big((1+r^2)(1-y^2)d\tilde{t} - (a^2+r^2)(a^2-y^2)d\tilde{\phi}_1 \\ & - (b^2+r^2)(b^2-y^2)d\tilde{\phi}_2 - \frac{qr\mathcal{A}}{H(r^2+y^2)(r^2+z^2)}\Big)^2 \Bigg], \end{aligned} \tag{20}$$

where the functions in the metric are explicitly specified

$$\begin{aligned} X &= (r^2+a^2)(r^2+b^2) + [r(r^2+a^2)+q][r(r^2+b^2)+q] - 2mr, \\ Y &= -(1-y^2)(a^2-y^2)(b^2-y^2), \\ Z &= -(1-z^2)(a^2-z^2)(b^2-z^2), \\ H &= 1 + \frac{qr}{(r^2+y^2)(r^2+z^2)}, \\ \mathcal{A} &= (1-y^2)(1-z^2)d\tilde{t} - (a^2-y^2)(a^2-z^2)d\tilde{\phi}_1 \\ &\quad - (b^2-y^2)(b^2-z^2)d\tilde{\phi}_2. \end{aligned} \tag{21}$$

Without loss of generality, following [44], we adopt the convention that the coordinates y and z are restricted to the region

$$-a \le y \le a \le z \le b. \tag{22}$$

The coordinates $\tilde{\phi}_i$ and $\tilde{t}$ are related to the standard Boyer–Lindquist coordinates, in which the S^1 directions ϕ_i are 2π-periodic, via

$$\tilde{t} = \frac{t}{\Xi_a\Xi_b}, \quad \tilde{\phi}_1 = \frac{\phi_1}{a\Xi_a(a^2-b^2)}, \quad \tilde{\phi}_2 = \frac{\phi_2}{b\Xi_b(b^2-a^2)}. \tag{23}$$

This redefinition is introduced to simplify notation, following [50]. Thermodynamic quantities can be computed directly from the metric (20), yielding the following expressions [14, 44, 71, 72].

$$\begin{aligned} E &= \frac{2\pi m}{3G_N\Xi_a\Xi_b}\left[\frac{1}{\Xi_a} + \frac{1}{\Xi_b} + \frac{q}{2m}\left(1 + \frac{\Xi_b}{\Xi_a} + \frac{\Xi_a}{\Xi_b}\right)\right], \\ S &= \frac{2\pi^2[(r_+^2+a^2)(r_+^2+b^2)+qr_+]}{3G_N\Xi_a\Xi_b}, \\ J_a &= \frac{2\pi ma}{3G_N\Xi_a^2\Xi_b}\left(1 + \frac{\Xi_b q}{2m}\right), \qquad J_b = \frac{2\pi mb}{3G_N\Xi_a\Xi_b^2}\left(1 + \frac{\Xi_a q}{2m}\right), \\ T &= \frac{2r_+^2(1+r_+^2)(2r_+^2+a^2+b^2) - (1-r_+^2)(r_+^2+a^2)(r_+^2+b^2) + 4qr_+^3 - q^2}{4\pi r_+[(r_+^2+a^2)(r_+^2+b^2)+qr_+]} \\ \Omega_a &= \frac{a[(r_+^2+1)(r_+^2+b^2)+qr_+]}{(r_+^2+a^2)(r_+^2+b^2)+qr_+}, \qquad \Omega_b = \frac{b[(r_+^2+1)(r_+^2+a^2)+qr_+]}{(r_+^2+a^2)(r_+^2+b^2)+qr_+}, \\ \Phi &= \frac{\sqrt{q^2+2mqr_+}}{(r_+^2+a^2)(r_+^2+b^2)+qr_+}, \qquad Q = \frac{\pi\sqrt{q^2+2mq}}{G_N\Xi_a\Xi_b}. \end{aligned} \tag{24}$$

The Newton constant G_N is related to the field-theory constant by $\frac{27\sqrt{2}}{5\pi}\frac{N^{\frac{5}{2}}}{\sqrt{8-N_f}} = G_N^{-1}$ [21], where N_f denotes the number of flavors. These quantities obey the first law of black-hole thermodynamics [44].

We are particularly interested in the (near-)EVH limit, where the extremal black-hole horizon scales as ϵ and $N^{\frac{5}{2}}\epsilon^2$ is held fixed so as to retain nontrivial dynamics. In this framework, the exact EVH limit corresponds to the ground state of the near-EVH geometry, characterized by a vanishing horizon. Achieving such a configuration requires careful parameter tuning, particularly because in spacetime dimensions $D \ge 6$, the existence of extremal AdS black holes typically requires additional charges and angular momenta to balance the gravitational potential.

The EVH limit is generically defined as:

$$a \equiv 0, \quad r = \epsilon x, \quad r_+ = \epsilon x_+, \quad t = \epsilon\tau, \quad q = 0. \tag{25}$$

The blackening factor exhibits a vanishing horizon for the following choice of parameters:

$$X(r) = (r^2+b^2)(r^2+1)r^2, \tag{26}$$

This indicates that the limit yields the ground-state geometry. However, the function Y and the coordinate y in the metric (20) vanish within the definition domain (22), while $\tilde{\phi}_1$ becomes singular in the EVH limit (25). They therefore need to be appropriately normalized. The solution that is compatible with both the definition domain (22) and the Jacobi transformation (17) in this EVH limit is:

1) We use the static frame rather than the other ψ_i coordinates, as it is appropriate for computing thermodynamic quantities and is also suitable for taking the near-EVH limits later.

$$y = a\cos\theta_1, \quad z = b\cos\theta_2, \quad a\tilde{\phi}_1 = -\frac{\phi_1}{b^2},$$
$$0 \leq \theta_1 \leq \pi, \qquad 0 \leq \theta_2 \leq \frac{\pi}{2}. \tag{27}$$

The geometry yields

$$\begin{aligned} \mathrm{d}s^2 = &-\frac{(1+r^2)(1-b^2\cos^2\theta_2)}{1-b^2}\mathrm{d}t^2 + \frac{r^2+b^2\cos^2\theta_2}{(r^2+1)(r^2+b^2)}\mathrm{d}r^2 \\ &+ r^2\cos^2\theta_2(\mathrm{d}\theta_1^2 + \sin^2\theta_1\mathrm{d}\phi_1^2) \\ &+ \frac{r^2+b^2}{1-b^2}\sin^2\theta_2\mathrm{d}\phi_2^2 + \frac{r^2+b^2\cos^2\theta_2}{1-b^2\cos^2\theta_2}\mathrm{d}\theta_2^2. \end{aligned} \tag{28}$$

A special limit is $z \to b$, where the geometry manifestly decouples into global AdS$_4$, spanned by (t, r, θ_1, ϕ_1), and an S^2 compact manifold spanned by (θ_2, ϕ_2).

We are now ready to explore the near-EVH limit. This limit is implemented by the following scalings:

$$a = \lambda\epsilon^2, \qquad q = bx_+\epsilon + q^{(3)}\epsilon^3, \qquad r_+ = \epsilon x_+ + y_3\epsilon^3, \tag{29}$$

Together with the coordinate transformation (27), the range (22) is preserved. The scaling between a and ϵ was proposed as a possible generalization of the near-EVH limits found in the AdS$_5$ black hole counterparts [1]. The parameter $q^{(3)}$ must be chosen to guarantee the existence of the extremal horizon, where the temperature vanishes. To see this, we introduce an ϵ^3-order perturbation of the radial coordinate in (29). The temperature T does not vanish at order ϵ for generic values of $q^{(3)}$, which results in the scaling relation $S \sim T^2$:

$$\begin{aligned} S &= \frac{2\pi^2 b x_+^2}{3G_N(1-b)}\epsilon^2, \\ T &= \frac{-2bx_+q^{(3)} + (3+4b+3b^2)x_+^4 - b^2\lambda^2 + 2y_3b^2x_+}{4\pi b(1+b)x_+^3}\epsilon + O(\epsilon^3). \end{aligned} \tag{30}$$

The temperature vanishes at order ϵ if $q^{(3)}$ satisfies

$$q^{(3)} = \left(2 + \frac{3}{2b} + \frac{3b}{2}\right)x_+^3 - \frac{b\lambda^2}{2x_+} + y_3 b. \tag{31}$$

Higher-order terms in ϵ do not affect the expansion (30) or the extremality constraint (31). By iterating this procedure to fix the coefficients in the ϵ expansion, the black hole becomes a decoupled near-EVH extremal black hole. If the decoupling conditions do not satisfy (31), the near-EVH black holes are generically non-extremal.

Therefore, the near-EVH scaling relations are possible even without imposing the near-BPS condition proposed in [1]. In the near-EVH limit, the dynamics of interest is restricted to

$$\frac{\epsilon^2}{G} \sim N^{\frac{5}{2}}\epsilon^2 \quad \text{fixed}, \qquad \epsilon \to 0. \tag{32}$$

Although the $S \sim T^2$ scaling relation is not unique to the EVH limit, the particular scaling of a with ϵ was motivated by potential generalizations of near-EVH limits in the context of AdS$_5$ black holes [1]. Meanwhile, the scaling of q with ϵ is chosen to ensure the persistence of an extremal horizon in the limit. Moreover, this ansatz naturally incorporates near-BPS EVH limits without further adjustment.

The BPS limit of AdS$_6$ black holes and the corresponding field theory interpretation have been discussed in [14]. The supersymmetry condition yields

$$E = J_a + J_b + Q. \tag{33}$$

As in AdS$_5$ [1, 73], the parameter q that satisfies the supersymmetry condition is generically complex unless the BPS value of the horizon size r_0 is chosen:

$$q = (a+b+ab)r_+ - r_+^3 + \mathrm{i}(1+a+b)\left(r_+^2 - r_0^2\right), \quad r_0^2 = \frac{ab}{1+a+b}. \tag{34}$$

The chemical potentials at the BPS points are defined as follows.

$$\Delta = \lim_{r_+\to r_0}\beta(1-\Phi), \;\; \omega_a = \lim_{r_+\to r_0}\beta(1-\Omega_a), \;\; \omega_b = \lim_{r_+\to r_0}\beta(1-\Omega_b), \tag{35}$$

which are generically complex and subject to the condition

$$\omega_a + \omega_b - 3\Delta = 2\pi\mathrm{i}. \tag{36}$$

This condition will be crucial for reproducing the black hole entropy from the superconformal indices, as will be discussed in Section 4. As a generalization of the EVH-BPS limit of AdS$_5$, Ref. [1] considered the combined near-BPS and near-EVH limits of an AdS$_6$ black hole, where the horizon size reduces to

$$x_+ = \sqrt{\frac{b\lambda}{1+b}}. \tag{37}$$

Besides, in the BPS limit, the entropy of the black hole is given by:

$$S = \frac{2\pi^2 b^2\lambda}{3G_N(1-b^2)}\epsilon^2, \tag{38}$$

which remains finite in the near-EVH limit. This is precisely a special case of the near-extremal black hole described by (30).

The near-EVH metric, with extremality determined by the condition (29), is [1]:

$$\begin{aligned}
\mathrm{d}\tilde{s}^2 = H^{\frac{1}{2}} \Bigg[& \cos^2\theta_2 \epsilon^2 x^2 \frac{\mathrm{d}x^2}{(x-x_+)^2} + \epsilon^2 \cos^2\theta_2 x^2 \mathrm{d}\theta_1^2 \\
& + \epsilon^2 x^2 \cos^2\theta_2 \sin^2\theta_1 \mathrm{d}\phi_1^2 \\
& - \frac{(x-x_+)^2[b^2 \sin^2\theta_2 \mathrm{d}\phi_2 - \epsilon b(1-b^2\cos^2\theta_2)\mathrm{d}\tau]^2}{(xb\cos\theta_2 + x_+\sec\theta_2)^2(1-b^2)^2} \\
& + \frac{b^2\cos^2\theta_2}{1-b^2\cos^2\theta_2}\mathrm{d}\theta_2^2 \\
& + \frac{b^2\sin^2\theta_2\cos^2\theta_2(1-b^2\cos^2\theta_2)(bx+x_+)^2}{(1-b^2)^2(x_+ + xb\cos^2\theta_2)^2}\mathrm{d}\phi_2^2 \Bigg], \\
H = 1 + & \frac{x_+}{xb\cos^2\theta_2}.
\end{aligned} \tag{39}$$

The decoupled geometry behaves differently for different values of θ_2. We therefore consider two limits. In the $\theta_2 \to 0$ limit (28), the four-dimensional black hole, decoupled from the IR limit of the AdS$_6$ black hole, is parametrized by the coordinates $(\tau, x, \theta_1, \phi_1)$, and its metric takes the following form:

$$\mathrm{d}\tilde{s}^2 = H^{\frac{1}{2}}\left[\epsilon^2 \mathrm{d}s_4^2 + \frac{b^2}{1-b^2}\mathrm{d}\Omega_2^2\right], \quad \mathrm{d}\Omega_2^2 = \mathrm{d}\theta_2^2 + \sin^2\theta_2 \mathrm{d}\phi_2^2. \tag{40}$$

The decoupled four-dimensional black hole can be written as follows:

$$\mathrm{d}s_4^2 = -\frac{(x-x_+)^2}{(x+\frac{x_+}{b})^2}\mathrm{d}\tau^2 + \frac{x^2\mathrm{d}x^2}{(x-x_+)^2} + x^2\mathrm{d}\theta_1^2 + x^2\sin^2\theta_1\mathrm{d}\phi_1^2. \tag{41}$$

Typically, the metric (41) is conformally related to the extremal EMMD black hole solutions [69, 74], which are also reviewed in Appendix A. For instance, the EMMD solution with $N_1 = N_2 = 2$ reads:

$$\begin{aligned}
\mathrm{d}s_{\mathrm{EMMD}}^2 = \frac{\tilde{r}+q_2}{\tilde{r}+q_1}\Bigg[& -\frac{(\tilde{r}-\mu)\tilde{r}}{(\tilde{r}+q_2)^2}\mathrm{d}t^2 + \frac{(\tilde{r}+q_1)^2}{\tilde{r}(\tilde{r}-\mu)}\mathrm{d}\tilde{r}^2 \\
& + (\tilde{r}+q_1)^2(\mathrm{d}\theta_1^2 + \sin^2\theta_1\mathrm{d}\phi_1^2)\Bigg].
\end{aligned} \tag{42}$$

Without loss of generality, we assume that $q_2 > q_1$. The metric within the square brackets in (42) is precisely the decoupled near-EVH geometry (41) upon identifying

$$\mu = 0, \quad x_+ = q_1, \quad \frac{x_+}{b} = q_2 - q_1, \quad x = \tilde{r} + q_1. \tag{43}$$

If the BPS condition is imposed [1], the horizon size x_+ and charge q are both fixed by the rotation parameters, which correspond to special values of the two $U(1)$ charges in the EMMD theory.

Recall that in AdS$_5$ black hole models, the EVH limit yields a pinched AdS$_3$ geometry in Poincaré coordinates, whereas the near-EVH limit leads to a pinched BTZ geometry [1, 30–32]. This can be understood holographically: the BTZ geometry is dual to thermal excitations above the AdS$_3$ vacuum. The situation in the AdS$_6$ black hole model studied here is qualitatively different. In the exact EVH limit, the geometry described by (28) contains a decoupled global AdS$_4$ submanifold, whose $r \to 0$ limit reduces to four-dimensional Minkowski spacetime. The decoupled black hole geometry (41) can therefore be regarded as an excitation above this vacuum, since setting $x_+ = 0$ indeed recovers Minkowski spacetime. Although the decoupled black hole solution (41) is asymptotically flat at leading order in the near-EVH decoupling limit, higher-order corrections — following the framework of EMMD theory with a cosmological constant [69] — could, in principle, introduce an effective cosmological constant. However, such contributions do not affect the leading infrared physics of the near-EVH limit, and we will not pursue them further in this work.

This near-EVH limit of black holes can be further generalized to non-extremal models, whose decoupled geometries can be conformally related to non-extremal EMMD black holes. The corresponding near-EVH limit (29) should be replaced by

$$q = b\sqrt{x_+x_-}\epsilon + q^{(3)}\epsilon^3, \tag{44}$$

which yields the four-dimensional decoupled black hole geometry

$$\begin{aligned}
\mathrm{d}s_4^2 = & -\frac{(x-x_+)(x-x_-)\mathrm{d}\tau^2}{(x+\frac{\sqrt{x_+x_-}}{b})^2} + \frac{x^2\mathrm{d}x^2}{(x-x_+)(x-x_-)} \\
& + x^2(\mathrm{d}\theta_1^2 + \sin^2\theta_1\mathrm{d}\phi_1^2).
\end{aligned} \tag{45}$$

This geometry is conformally related to the non-extremal EMMD black hole (42) under the identification

$$q_1 + \mu = x_+, \quad x_- = q_1, \quad \frac{\sqrt{x_+x_-}}{b} = q_2 - q_1, \quad x = \tilde{r} + q_1, \tag{46}$$

In this context, the conformal factor can be absorbed into the dilaton, reflecting a change of frame.

1) The temperature of an extremal black hole must be zero. We choose $q^{(3)}$ in (29) appropriately to make T identically vanish. However, any such choice does not affect the leading order in the near-EVH limit of AdS$_6$ black holes.

B. AdS$_7$ black hole

The AdS$_7$ black hole with two R-charges and three angular momenta possesses a richer parameter space. Such solutions can be obtained via a consistent reduction from $D=11$ supergravity on S^4 [75–77]. The bosonic part of the Lagrangian of the $D=7$ $SO(5)$ supergravity theory is given by

$$\mathcal{L}_7 = R\star 1+2\left(8X_1X_2+\frac{4(X_1+X_2)}{X_1^2X_2^2}-\frac{1}{X_1^4X_2^4}\right)\star 1-\frac{1}{2}\sum_{I=1}^{2}d\varphi_I\wedge\star d\varphi_I$$
$$-\frac{1}{2}\sum_{I=1}^{2}\frac{1}{X_I^2}F^I_{(2)}\wedge\star F^I_{(2)}-\frac{1}{2}X_1^2X_2^2F_{(4)}\wedge\star F_{(4)}+F_{(4)}\wedge A_{(3)}+F^1_{(2)}\wedge F^2_{(2)}\wedge A_{(3)}\,. \tag{47}$$

The most general AdS$_7$ black hole solution of this kind was constructed analytically in [46]; it includes all the known special solutions with one rotation turned off [78, 79], equal charges [45], or equal angular momenta [80, 81]. The most general solution to 7d gauged supergravity in the asymptotically static frame (t,r,y,z,ϕ_i), with $i=1,2,3$, is given explicitly in [46].

$$\begin{aligned}\mathrm{d}s^2 = (H_1H_2)^{\frac{1}{5}}\Bigg[&-\frac{(1+r^2)(1-y^2)(1-z^2)}{\Xi_1\Xi_2\Xi_3}\mathrm{d}t^2+\frac{(r^2+y^2)(r^2+z^2)}{U}\mathrm{d}r^2+\frac{(r^2+y^2)(y^2-z^2)y^2}{(1-y^2)(a_1^2-y^2)(a_2^2-y^2)(a_3^2-y^2)}\mathrm{d}y^2\\ &+\frac{(r^2+z^2)(z^2-y^2)z^2}{(1-z^2)(a_1^2-z^2)(a_2^2-z^2)(a_3^2-z^2)}\mathrm{d}z^2+\frac{(r^2+a_1^2)(a_1^2-y^2)(a_1^2-z^2)}{\Xi_1(a_1^2-a_2^2)(a_1^2-a_3^2)}\mathrm{d}\phi_1^2+\frac{(r^2+a_2^2)(a_2^2-y^2)(a_2^2-z^2)}{\Xi_2(a_2^2-a_1^2)(a_2^2-a_3^2)}\mathrm{d}\phi_2^2\\ &+\frac{(r^2+a_3^2)(a_3^2-y^2)(a_3^2-z^2)}{\Xi_3(a_3^2-a_1^2)(a_3^2-a_2^2)}\mathrm{d}\phi_3^2+\frac{1-H_1^{-1}}{1-(s_2/s_1)^2}K_1^2+\frac{1-H_2^{-1}}{1-(s_1/s_2)^2}K_2^2\Bigg]\,.\end{aligned} \tag{48}$$

where, for simplicity, we also set the cosmological constant to $g^2=1$. The factors in the metric are defined as follows (with $i,j,k=1,2,3$ denoting distinct indices).

$$\begin{aligned}&s_I=\sinh\delta_I,\quad c_I=\cosh\delta_I,\quad \Xi_i=1-a_i^2,\quad I=1,2,\quad H_I=1+\frac{2ms_I^2}{(r^2+y^2)(r^2+z^2)},\\ &U(r)=\frac{(1+r^2)\prod_{i=1}^3(r^2+a_i^2)}{r^2}-2m+m(s_1^2+s_2^2)\left(2r^2+\sum_{i=1}^{3}a_i^2\right)+\frac{4m^2s_1^2s_2^2}{r^2}\\ &\quad-\frac{2m(s_1^2+s_2^2)a_1a_2a_3}{r^2}+\frac{2m(c_1-c_2)^2}{r^2}(a_1+a_2a_3)(a_2+a_1a_3)(a_3+a_1a_2)+\frac{m(c_1-c_2)^2}{2}\Big[2\sum_{i=1}^{3}a_i^2+8a_1a_2a_3\\ &\quad+(a_1+a_2+a_3)(a_2+a_3-a_1)(a_1+a_3-a_2)(a_1+a_3-a_2)\Big],\\ &K_1=\frac{c_1+c_2}{2s_1}\mathcal{A}[y^2,z^2,0]+\frac{c_1-c_2}{2s_1}\mathcal{Y},\quad K_2=\frac{c_1+c_2}{2s_2}\mathcal{A}[y^2,z^2,0]-\frac{c_1-c_2}{2s_2}\mathcal{Y},\\ &\mathcal{A}[y^2,z^2,0]=\frac{(1-y^2)(1-z^2)}{\Xi_1\Xi_2\Xi_3}\mathrm{d}t-\sum_{i=1}^{3}\frac{a_i(a_i^2-y^2)(a_i^2-z^2)}{\Xi_i(a_i^2-a_j^2)(a_i^2-a_k^2)}\mathrm{d}\phi_i,\\ &\mathcal{Y}=\frac{(1-y^2)(1-z^2)[1-(a_1^2+a_2^2+a_3^2)-2a_1a_2a_3]}{\Xi_1\Xi_2\Xi_3}\mathrm{d}t+\sum_{i=1}^{3}\frac{a_i(a_i^2-y^2)(a_i^2-z^2)[1-(a_i^2-a_j^2-a_k^2)+\frac{2a_ja_k}{a_i}]}{\Xi_i(a_i^2-a_j^2)(a_i^2-a_k^2)}\mathrm{d}\phi_i\,,\end{aligned} \tag{49}$$

and, without loss of generality, the y- and z-coordinates are taken to lie in the region

$$0\le a_1\le y\le a_2\le z\le a_3\le 1\,. \tag{50}$$

The parameterization of this metric differs slightly from that used in the AdS$_5$ and AdS$_6$ black hole solutions, for which redefining the charge parameter as $q_I=2ms_I^2$ is particularly convenient. However, this simplification does not apply to the AdS$_7$ solution given in (48), because of the (c_1-c_2) term in the blackening factor. The resulting expression becomes tractable only in the equal-charge case. The event horizon is located at r_+, defined as the largest root of $U(r)=0$. In the AdS_5 and AdS_6 cases, one can invert this relation to express m explicitly in terms of the horizon radius, yielding a convenient form of the blackening factor. In contrast, for AdS_7, the function $U(r)$ is quadratic in m, which significantly complicates such an inversion. This increased complexity enriches the structure of the parameter space and opens new possibilities for exploring distinct EVH decoupling limits.

The thermodynamic quantities of the AdS_7 black hole (48) are computed using the Komar integral [46]:

$$\begin{aligned}
S &= \frac{\pi^3}{4G_N\Xi_1\Xi_2\Xi_3}\frac{\sqrt{\mathcal{S}(r_+)}}{r_+}, \qquad T=\frac{r_+^2U'(r_+)}{4\pi\sqrt{\mathcal{S}(r_+)}},\\
J_1 &= \frac{\pi^2 m}{16G_N\Xi_1\prod_{j=1}^3\Pi_j}\Big[4a_1c_1c_2+4(1-c_1c_2)(a_2+a_1a_3)(a_3+a_1a_2)\\
&\quad+(c_1-c_2)^2\left(2a_2a_3+a_1(1+2\Xi_1-\sum_{j=1}^3\Xi_j)\right)\left(1+2a_1a_2a_3-\sum_{j=1}^3\Xi_j\right)\Big],\\
Q_1 &= \frac{\pi^2 m s_1}{4G_N\Xi_1\Xi_2\Xi_3}\left[2c_1-(c_1-c_2)(a_1^2+a_2^2+a_3^2+2a_1a_2a_3)\right],
\end{aligned}\tag{51}$$

with the angular velocity given by

$$\begin{aligned}
\Omega_1 &= \frac{1}{\mathcal{S}(r_+)}\Bigg[\frac12\left(\prod_{i=1}^3(r_+^2+a_i^2)+2ms_1^2(r_+^2-a_1a_2a_3)\right)\times\left(a_1(1+r_+^2)(r_+^2+a_2^2)(r_+^2+a_3^2)+2ms_2^2(a_1r_+^2-a_2a_3)\right)\\
&\quad+\frac12\left(\prod_{i=1}^3(r_+^2+a_i^2)+2ms_2^2(r_+^2-a_1a_2a_3)\right)\times\left(a_1(1+r_+^2)(r_+^2+a_2^2)(r_+^2+a_3^2)+2ms_1^2(a_1r_+^2-a_2a_3)\right)\\
&\quad-m(c_1-c_2)^2(r_+^2+a_2^2)(r_+^2+a_3^2)\Big\{2(r_+^2-a_1)(a_1+a_2a_3)(a_2+a_1a_3)(a_3+a_1a_2)\\
&\quad+(1+a_1)^2(a_1+a_2a_3)(1-a_1-a_2-a_3)(1-a_1+a_2+a_3)\Big\}\Bigg],
\end{aligned}\tag{52}$$

together with chemical potentials as

$$\begin{aligned}
\Phi_1 &= \frac{2mr_+^2s_1c_1[\prod_i(r_+^2+a_i^2)+2ms_2^2(r_+^2-a_1a_2a_3)]}{\mathcal{S}(r_+)}-\frac{mr_+^2s_1(c_1-c_2)}{\mathcal{S}(r_+)}\Big[(a_1^2+a_2^2+a_3^2+2a_1a_2a_3)\\
&\quad\times\left(\prod_i(r_+^2+a_i^2)+2ms_2^2(r_+^2-2a_1a_2a_3)\right)+4ms_2^2(a_1+a_2a_3)(a_2+a_1a_3)(a_3+a_1a_2)\Big],
\end{aligned}\tag{53}$$

where the function $\mathcal{S}$ is defined as:

$$\mathcal{S}(r)=\prod_{I=1}^2\left[(r^2+a_1^2)(r^2+a_2^2)(r^2+a_3^2)+2ms_I^2(r^2-a_1a_2a_3)\right]+2m(c_1-c_2)^2(r^2+a_1^2)(r^2+a_2^2)(r^2+a_3^2)(a_1+a_2a_3)(a_2+a_1a_3)(a_3+a_1a_2).\tag{54}$$

The charges, angular momenta, and chemical potentials for other indices can be obtained by permuting the indices. The energy is given by

$$\begin{aligned}
E &= \frac{m\pi^2}{8G_N\Xi_1\Xi_2\Xi_3}\left[\sum_{i=1}^3\frac{2}{\Xi_i}-1+\frac{5(s_1^2+s_2^2)}{2}+\frac{s_1^2+s_2^2}{2}\sum_{i=1}^3\left(\frac{2(1+a_i^2-\Sigma_2-2\Pi_1)}{\Xi_i}-\Xi_i\right)\right]+\frac{m\pi(c_1-c_2)^2}{32G_N\Xi_1\Xi_2\Xi_3}\Big[-10\Sigma_2-16\Pi_1+11\Sigma_4\\
&\quad+13\Pi_{22}+32\Pi_1\Sigma_2-3(\Sigma_6+5\Pi_{42}+4\Pi_1^2)-16\Pi_1\Sigma_2^2+\Pi_{62}+3\Pi_{44}-5\Pi_1^2\Sigma_2+8\Pi_1(2\Pi_2+\Pi_{42})+\Pi_1^2(\Sigma_4+3\Pi_{22})\Big],
\end{aligned}\tag{55}$$

where the factors are defined in [46].

$$\Sigma_n = a_1^n + a_2^n + a_3^n\,, \qquad \Pi_n = a_1^n a_2^n a_3^n\,,$$
$$\Pi_{nm} = a_1^n(a_2^m + a_3^m) + a_2^n(a_1^m + a_3^m) + a_3^n(a_1^m + a_2^m)\,. \tag{56}$$

These quantities were checked in [46] to satisfy the first law of thermodynamics, as a consistency check of the solution's validity. The thermodynamics of more specialized solutions were also studied in [73, 82]. The supersymmetry condition for this solution[1)] is given by

$$E = Q_1 + Q_2 - J_1 - J_2 - J_3\,, \tag{57}$$

This imposes a relation among the charges:

$$a_1 + a_2 + a_3 = \frac{2}{1 - \mathrm{e}^{\delta_1 + \delta_2}}\,. \tag{58}$$

This is supplemented by a constraint on the horizon size that satisfies

$$r_0^2 = \frac{a_1 a_2 + a_2 a_3 + a_1 a_3 - a_1 a_2 a_3}{1 - a_1 - a_2 - a_3}\,. \tag{59}$$

To fully determine the parameters describing the BPS black hole, we define the chemical potentials in the BPS limit as

$$\Delta_I = \beta(\Phi_I + 1), \qquad \omega_i = \beta(\Omega_i + 1)\,, \tag{60}$$

which is subject to the constraint

$$\sum_{i=1}^{3} \omega_i - 2\sum_{I=1}^{2} \Delta_I = 2\pi \mathrm{i}\,. \tag{61}$$

We are also interested in identifying potential (near-)EVH limits of these black hole solutions. To systematically explore such limits, we adopt the following guidelines:

● The near-horizon limit is defined by taking $\epsilon \to 0$, with the radial coordinate rescaled as $r = \epsilon x$ so that ϵ is dimensionless.

● The entropy S and the temperature T are assumed to scale with ϵ as follows:

$$S \sim \epsilon^n, \qquad T \sim \epsilon^{m-n+1}, \qquad m,n \in \mathbb{Z}\,, \tag{62}$$

This, in turn, fixes the scaling of the entropy function and the blackening factor:

$$\mathcal{S}(r_+) \sim \epsilon^{2n+2}, \qquad U'(r_+) \sim \epsilon^m\,. \tag{63}$$

For simplicity, we restrict our analysis to integer-valued m and n.

Our task is to determine the scalings of the parameters a_i and q_I that match (63), in order to achieve the near-EVH scalings of the entropy and temperature (62). In this paper, we restrict to cases with $S \sim T^k$ for some integer k, which can be interpreted within field theories exhibiting scaling symmetry, while excluding Lifshitz scaling [83] and hyperscaling-violating scalings [84].

Therefore, the possible solutions are:

$$\begin{aligned} (m,n) &= (1,1): & S &\sim T, \\ (m,n) &= (2,2): & S &\sim T^2, \\ (m,n) &= (3,3): & S &\sim T^3\,. \end{aligned} \tag{64}$$

Within these choices, the $S \sim T$ near-EVH limits are well known to produce pinched BTZ-like geometries in the IR. Just as the near-EVH BTZ black holes emerging from static R-charged AdS$_5$ black holes are supported by two charges of order G_N^{-1} and one perturbative R-charge [30], the BTZ geometry in the near-EVH limit of AdS$_7$ black holes can be supported by two angular momenta of order G_N^{-1} and one perturbative angular momentum.

More nontrivial are the scalings $S \sim T^2$ and $S \sim T^3$. Previous analyses of the near-EVH limit in AdS$_6$ identify $S \sim T^2$ as signaling the emergence of a $D=4$ EMMD black hole in the infrared. In AdS$_7$, however, no suitable parameter regime exists that realizes a consistent near-EVH limit with $S \sim T^2$. On the other hand, the case $S \sim T^3$ has previously been identified in the near-EVH limit of AdS$_7$ black holes [1]. In the following, we present the specific parameter choices that define each of these limits and describe the corresponding infrared geometries.

Example: $D=3$ decoupled geometry

It has been observed [27, 50] that AdS$_3$ and the BTZ geometry can appear in the near-horizon limit of AdS$_7$ black holes. Such limits can be achieved by taking two nonvanishing angular momenta and treating the third as perturbative.

We can simply take the near-EVH limit as

$$s_I \equiv 0, \qquad a_3 = r_+ = 0, \qquad r = \epsilon x, \tag{65}$$

while the near-EVH limit is defined as

1) Our conventions for the BPS condition follow [46] and differ from those for AdS$_{5,6}$ BPS black holes by the replacement $a_i \to -a_i$.

$$a_3 = \lambda\epsilon^2, \qquad r_+ = \epsilon x_+ . \tag{66}$$

Note that these solutions reduce exactly to the Kerr-AdS$_7$ black holes [49, 50] and are incompatible with the supersymmetry condition (58). Their thermodynamic quantities are given by

$$\begin{aligned}
E &= \frac{m\pi^2}{4G_N\Xi_1\Xi_2\Xi_3}\left(\sum_{i=1}^{3}\frac{1}{\Xi_i}-\frac{1}{2}\right),\\
J_i &= \frac{ma_i\pi^2}{4G_N\Xi_i(\prod_{j=1}^{3}\Xi_j)},\\
S &= \frac{\pi^2}{4G_Nr_+}\prod_{i=1}^{3}\frac{r_+^2+a_i^2}{\Xi_i}, \qquad \Omega_i = \frac{a_i\Xi_i}{r_+^2+a_i^2},\\
T &= \frac{1}{2\pi}\left[r_+(1+r_+^2)\sum_{i=1}^{3}\frac{1}{r_+^2+a_i^2}-\frac{1}{r_+}\right].
\end{aligned} \tag{67}$$

The near-EVH limits defined in (66) thus result in

$$\begin{aligned}
&S\sim T\sim\epsilon,\\
&\lim_{\epsilon\to 0}\frac{S}{T} = \frac{\pi^3a_1^2a_2^2}{2G_N(1-a_1^2)(1-a_2^2)\left(1+\dfrac{1}{a_1^2}+\dfrac{1}{a_2^2}-\dfrac{\lambda}{r_+^4}\right)},
\end{aligned} \tag{68}$$

This confirms that the near-EVH limit is reasonable. The black hole geometry in the limit (66) takes the following form:

$$\begin{aligned}
\mathrm{d}s_7^2 = {}& \epsilon^2\frac{y^2z^2}{a_1^2a_2^2}\mathrm{d}s_3^2+\frac{y^2(y^2-z^2)}{(1-y^2)(a_1^2-y^2)(a_2^2-y^2)}\mathrm{d}y^2\\
&+\frac{z^2(z^2-y^2)}{(1-z^2)(a_1^2-z^2)(a_2^2-z^2)}\mathrm{d}z^2\\
&+\frac{(1-y^2)(a_1^2-y^2)(a_2^2-y^2)}{y^2(y^2-z^2)}\\
&\times\left(-\frac{a_1(a_1^2-z^2)}{\Xi_1(a_1^2-a_2^2)}\mathrm{d}\phi_1-\frac{a_2(a_2^2-z^2)}{\Xi_2(a_2^2-a_1^2)}\mathrm{d}\phi_2\right)^2\\
&+\frac{(1-z^2)(a_1^2-z^2)(a_2^2-z^2)}{z^2(z^2-y^2)}\\
&\times\left(-\frac{a_1(a_1^2-y^2)}{\Xi_1(a_1^2-a_2^2)}\mathrm{d}\phi_1-\frac{a_2(a_2^2-y^2)}{\Xi_2(a_2^2-a_1^2)}\mathrm{d}\phi_2\right)^2,
\end{aligned} \tag{69}$$

where the three-dimensional part is a pinched AdS$_3$ geometry in the EVH limit.

$$\mathrm{d}s_3^2 = -\frac{x^2}{l_3^2}\mathrm{d}\tau^2+\frac{l_3^2}{x^2}\mathrm{d}x^2+x^2\mathrm{d}\tilde{\phi}_3^2, \quad l_3^2 = \frac{a_1^2a_2^2}{a_1^2+a_2^2+a_1^2a_2^2}, \tag{70}$$

and a pinched extremal BTZ in the near-EVH limits, with the extremality condition $r_+ = r_-$:

$$\mathrm{d}s_3^2 = -\frac{(x^2-x_+^2)^2}{l_3^2x^2}\mathrm{d}\tau^2+\frac{l_3^2x^2}{(x^2-x_+^2)^2}\mathrm{d}x^2+x^2\left(\frac{x_+^2}{l_3x^2}\mathrm{d}\tau-\mathrm{d}\tilde{\phi}_3\right)^2. \tag{71}$$

In string theory, a local AdS$_3$ geometry arises as the near-horizon limit of a bound state consisting of n_1 D1-branes, n_5 D5-branes, and a momentum charge P along a common compact direction. The D1-branes, viewed as instantons inside the D5-branes, together with the momentum charge, give rise to a two-dimensional conformal field theory known as the D1–D5 CFT. This theory has central charge $c = 6n_1n_5$, which counts the number of degrees of freedom in the bound state. We refer the reader to [85] for a comprehensive review. In the context of R-charged AdS$_5$ black holes, a local AdS$_3$ geometry similarly emerges in the near-EVH limit when two R-charges are present. The three R-charges, corresponding to the Cartan generators of the $SO(6)$ R-symmetry, represent different species of giant gravitons. The effective theory arising from two species of giant gravitons, localized at their intersection, has degrees of freedom proportional to the product of the two macroscopic R-charges, yielding a central charge $c\sim Q_1Q_2$ [30, 34]. Similarly, for Kerr–AdS$_7$ black holes in the near-EVH limit, the central charge of the dual CFT$_2$ associated with the emergent AdS$_3$ is proportional to the product of two angular momenta:

$$c\sim l_3\sim a_1a_2\sim J_1J_2 . \tag{72}$$

This suggests that the CFT$_2$ dual to the emergent AdS$_3$ resides on the intersection of dual-giant-like objects in AdS$_7$ [86].

Example: $D = 5$ decoupled geometry

Since the $S\sim T^3$ near-EVH limit exists in equal-charge AdS black holes with three distinct angular momenta, we take the equal-charge solution [45] to examine its near-EVH geometry for simplicity. Therefore, we take $q = 2ms_1^2 = 2ms_2^2$. In particular, in the BPS limit, the horizon size is completely fixed by the rotation parameters (59). Therefore, the EVH-BPS limit associated with taking $r_+\sim\epsilon$ is completely fixed by the scalings of the rotation parameters, independent of whether the charges are taken to be equal or not.

The EVH limit that yields $S\sim T^3$ is defined as

$$a_1 = a_2 = 0, \quad q = 0, \quad r = \epsilon x, \quad x_+ = 0, \quad t = \epsilon\tau . \tag{73}$$

To ensure a well-defined geometry in this limit, consistent with (50), we choose the parametrization

$$y^2 = a_1^2\cos^2\theta_1+a_2^2\sin^2\theta_1, \qquad z = a_3\cos\theta_2 , \tag{74}$$

We can automatically translate from the Jacobi coordinates (17) to the spherical coordinates (θ_1, θ_2). In these coordinates, the EVH geometry of the AdS$_7$ black hole (48) is parameterized by $(t, r, \theta_1, \theta_2, \phi_i)$ with $i = 1,2,3$.

$$\begin{aligned} \mathrm{d}s^2 = &-\frac{(1-a_3^2\cos^2\theta_2)}{1-a_3^2}(1+r^2)\mathrm{d}t^2 + \frac{r^2+a_3^2\cos^2\theta_2}{(r^2+1)(r^2+a_3^2)}\mathrm{d}r^2 \\ &+ r^2\cos^2\theta_2(\mathrm{d}\theta_1^2+\sin^2\theta_1\mathrm{d}\phi_1^2+\cos^2\theta_1\mathrm{d}\phi_2^2) \\ &+\frac{r^2+a_3^2\cos^2\theta_2}{1-a_3^2\cos^2\theta_2}\mathrm{d}\theta_2^2+\frac{r^2+a_3^2}{1-a_3^2}\sin^2\theta_2\mathrm{d}\phi_3^2 . \end{aligned} \tag{75}$$

In the limit $\theta_2 \to 0$, the geometry decouples into global AdS$_5$, spanned by $(t, r, \theta_1, \phi_{1,2})$, and an S^2, spanned by (θ_2, ϕ_3). We adopt the metric ansatz of [45] and implement the following near-EVH limits

$$a_1 = \lambda_1\epsilon^2, \quad a_2 = \lambda_2\epsilon^2, \quad q = a_3x_+^2\epsilon^2 + q^{(4)}\epsilon^4, \quad r = \epsilon x + y_4\epsilon^3 . \tag{76}$$

It is convenient to define the new coordinate $y = w\epsilon^2$.

$$w^2 = \lambda_1^2\cos^2\theta_1 + \lambda_2^2\sin^2\theta_1 . \tag{77}$$

The entropy and temperature in the near-EVH limit (76) are then determined by the perturbative ansatz (76).

$$\begin{aligned} S &= \frac{\pi^3 a_3 x_+^3\epsilon^3}{4G_N(1-a_3)} + O(\epsilon^4) \\ T &= \frac{(1+a_3+a_3^2)x_+^4 + a_3^2\lambda_1\lambda_2 - q^{(4)}a_3 + 2y_4a_3^2x_+}{a_3(1+a_3)\pi x_+^3}\epsilon + O(\epsilon^2), \end{aligned} \tag{78}$$

confirming the $S \sim T^3$ scaling in this limit for generic values of $q^{(4)}$. The special value of $q^{(4)}$ for which the temperature vanishes is determined to be

$$q^{(4)} = \left(1+\frac{1}{a_3}+a_3\right)x_+^4 + a_3\lambda_1\lambda_2 + 2y_4a_3x_+ . \tag{79}$$

In this paper, we do not consider such fine-tuned values, as we are interested in the general near-EVH decoupling limits. The relations (78) hold even without imposing the BPS condition, thereby generalizing the EVH-BPS limit of AdS$_7$ proposed in [1]. If the supersymmetry condition is imposed, the horizon size satisfies $x_+ = x_0$, where

$$x_0^2 = \frac{a_3(\lambda_1+\lambda_2)}{1-a_3}, \tag{80}$$

and the entropy of the EVH-BPS black hole is given by

$$S = \frac{\pi^3}{4G_N}\left(\frac{a_3}{1-a_3}\right)^{\frac{5}{2}}(\lambda_1+\lambda_2)^{\frac{3}{2}}\epsilon^3 . \tag{81}$$

We now apply the near-EVH limit given in (76) to the AdS$_7$ black hole metric. As in the AdS$_6$ case, the angular coordinate θ_2, which ranges from 0 to $\pi/2$, governs the emergence of distinct, decoupled geometries in the near-horizon region. In the limit $\theta_2 \to 0$, the metric reduces to the following five-dimensional form:

$$\mathrm{d}s_7^2 = H^{\frac{2}{5}}\left[\epsilon^2\mathrm{d}s_5^2 + \frac{a_3^2}{(1-a_3^2)}\mathrm{d}\Omega_2^2\right], \quad \mathrm{d}\Omega_2^2 = \mathrm{d}\theta_2^2 + \sin^2\theta_2\mathrm{d}\phi_3^2, \tag{82}$$

where the five-dimensional geometry is given by:

$$\begin{aligned} \mathrm{d}s_5^2 &= -\frac{(x^2-x_+^2)^2}{(x^2+\frac{x_+^2}{a_3})^2}\mathrm{d}\tau^2 + \frac{x^4\mathrm{d}x^2}{(x^2-x_+^2)^2} + x^2\mathrm{d}\Omega_3^2, \\ \mathrm{d}\Omega_3^2 &= \mathrm{d}\theta_1^2 + \sin^2\theta_1\mathrm{d}\phi_1^2 + \cos^2\theta_1\mathrm{d}\phi_2^2 . \end{aligned} \tag{83}$$

Moreover, the geometry described by (83) is conformally related to a $D = 5$ EMMD black hole with exponents $(N_1, N_2) = (1,2)$. In the extremal limit, the latter is given by (cf. (A15)):

$$\mathrm{d}\tilde{s}_5^2 = \frac{(x^2+q_2-q_1)^{\frac{2}{3}}}{x^{\frac{4}{3}}}\left[-\frac{(x^2-q_1)^2\mathrm{d}t^2}{(x^2+q_2-q_1)^2} + \frac{x^4\mathrm{d}x^2}{(x^2-q_1)^2} + x^2\mathrm{d}\Omega_3^2\right] . \tag{84}$$

By identifying the EMMD charges q_1 and q_2 (assuming, without loss of generality, that $q_1 < q_2$) with the parameters of the AdS$_7$ solution, we obtain:

$$x_+^2 = q_1, \qquad \frac{x_+^2}{a_3} = q_2 - q_1, \qquad q_2 > q_1 . \tag{85}$$

The near-EVH limit of AdS$_7$ black holes exhibits several structural similarities with the AdS$_6$ case analyzed in Section III.A. First, in both cases, the emergent, decoupled near-EVH geometry of an AdS$_D$ black hole is an asymptotically flat EMMD-type solution in $(D-2)$ dimensions. In the formal limit in which the Boyer–Lindquist rotation parameter tends to infinity, these geometries reduce to Reissner–Nordström black holes, a regime that lies outside the domain of validity of the near-EVH approximation. Second, as in the AdS$_6$ case, the correspondence between the near-EVH limit of the AdS$_7$ black hole and the five-dimensional EMMD description also extends to non-extremal configurations. This generalization is achieved by modifying the near-EVH scaling in (76) to incorporate a subleading correction:

$$q = a_3 x_+ x_- \epsilon^2 + q^{(4)}\epsilon^4, \tag{86}$$

This yields the following decoupled five-dimensional metric, conformal to the EMMD black hole metric in $D=5$:

$$\mathrm{d}s_5^2 = -\frac{(x^2-x_+^2)(x^2-x_-^2)}{(x^2+\frac{x_+x_-}{a_3})^2}\mathrm{d}\tau^2 + \frac{x^4\mathrm{d}x^2}{(x^2-x_+^2)(x^2-x_-^2)} + x^2\mathrm{d}\Omega_3^2. \tag{87}$$

Finally, in both AdS_6 and AdS_7 settings, the EMMD decoupling limit becomes fully explicit only when the direction-cosine coordinate is fixed at the special value $\theta_2 \to 0$. This choice forces z to approach its upper bound, thereby collapsing S^2 (spanned by μ_i^2) to S^1. The physical implications of this parameter restriction will be addressed in Section III.C.

C. Discussion

The AdS_D black holes in $D=6$ and $D=7$ analyzed in this section exhibit an infrared scaling relation $S \sim T^{D-4}$, signaling the emergence of an effective $(D-2)$-dimensional geometry in the decoupling limit. These on-shell geometries are conformally related to EMMD black holes [69]. Furthermore, the rich parameter space of AdS_7 also allows three-dimensional BTZ geometries to emerge in the decoupling limit. The decoupled geometries discussed in this work can be viewed as ground states—analogous to AdS_2—that encode the ground-state degeneracy of the higher-dimensional black holes. A key distinction, however, is that the near-EVH limit permits excitations along additional directions, giving rise to a lower-dimensional effective field theory. In this picture, extremal EMMD black holes—characterized by vanishing temperature—correspond to the ground states of these lower-dimensional effective theories. There is also a structural difference between the AdS_3-type near-EVH geometry and the EMMD-type geometry. The former corresponds to a pinched, locally AdS_3 spacetime, in which the compact S^1 direction has period $2\pi\epsilon$ and the $\mathrm{d}r^2$ component remains of order ϵ^0 [1, 32, 87]. In contrast, the near-EVH limits of $\mathrm{AdS}_{D=6,7}$ exhibit a decoupling structure of the form:

$$\mathrm{d}s_D^2 = \epsilon^2 \mathrm{d}s_{\mathrm{EMMD}}^2 + \mathrm{d}\Omega_2^2, \tag{88}$$

This resembles a class of non-relativistic geometries known as MpT spacetimes [88]. We will revisit these observations in Section V.

In both the near-EVH limits of AdS_6 and AdS_7 black holes, the lower-dimensional EMMD geometry becomes explicit in the limit $\theta_2 \to 0$, which effectively reduces the S^2 parameterized by the direction cosines to an S^1. Given the analogous structures in the two models, we shall use AdS_6 to illustrate the underlying physical picture. From a string-theory viewpoint, the AdS_6 background is supported by a D4–D8 bound state in massive type-IIA supergravity [70]. The limit $\theta_2 \to 0$ selects a sector in which excitations are confined to modes that do not probe the shrinking direction — *i.e.*, collective low-energy modes of the bound state that are effectively four-dimensional. The resulting EMMD theory—with its metric, two gauge fields, and a dilaton—thus provides a consistent infrared description of this non-conformal, decoupled sector. Its microscopic origin should be traceable to an appropriate subsector of the dual five-dimensional SCFT. Because a more detailed understanding of this system in terms of giant gravitons is still lacking, we leave a more complete analysis for future work.

Several limitations in our study deserve mention. First, we have restricted our analysis to (near-)EVH limits in which the rotation parameters scale with ϵ as integer powers, following the approach of [1, 32]. While most generic scaling choices do not yield a well-defined decoupling limit with a smooth horizon geometry, it may be worthwhile to explore more exotic scalings—such as those involving non-integer powers—that could be dual to other decoupled sectors in the holographic field theory. Second, our discussion has been confined to on-shell solutions of supergravity in $D=6,7$ [44, 45]. An interesting open question is whether these supergravity Lagrangians (18) and (47) can be related off-shell to the EMMD theory Lagrangian, for example, (A1). Such a connection would help clarify how EMMD theories emerge from fine-tuned decoupled vacua of ten-dimensional string theory or 11D supergravity via specific brane configurations.

As reviewed in Appendix A, the supergravity framework admits three distinct classes of EMMD black hole geometries, labeled by a pair of integers (N_1, N_2) satisfying:

$$N_1 + N_2 = \frac{2(D-2)}{D-3}. \tag{89}$$

For $D>5$, this constraint cannot be satisfied by any pair of integers—a fact that may be related to the observation that the AdS_7 black hole is the highest-dimensional AdS black hole that is embeddable in string theory and admits a known superconformal field theory dual. For lower dimensions, the possible integer pairs (taking $N_1 < N_2$ without loss of generality by symmetry) are:

$$\begin{aligned} D=4: &\quad (N_1,N_2)=(2,2), \quad (1,3), \\ D=5: &\quad (N_1,N_2)=(1,2). \end{aligned} \tag{90}$$

It remains unclear why the near-EVH limits of $\mathrm{AdS}_{6,7}$ black holes yield EMMD black holes of types $(2,2)$ and

$(1,2)$, respectively, whereas the $(1,3)$ type—to the best of our knowledge—does not appear in the near-EVH limits studied here. It is also possible that such a solution could emerge from the near-EVH limit of as-yet-unknown supergravity solutions in $D=6$ or 7, which would be an interesting direction for future work.

IV. HOLOGRAPHY WITH $S \sim T^{D-4}$

The entropy of BPS AdS$_D$ black holes is known via microstate counting in the dual superconformal field theory [14, 21–24, 89]. These results can be used to study the entropy of the emergent $(D-2)$-dimensional geometries that arise in the near-extremal vanishing horizon (EVH) limit. Although the extremization of the entropy functional in $D=6$ and 7 shares certain similarities, it differs from that in the near-EVH limit for AdS$_5$. This further confirms that the near-EVH decoupling geometries in $D=6$ and 7, whose entropy scales as $S \sim T^{D-4}$, are not of BTZ type.

As an example, consider the AdS$_6$ black hole. Its entropy is given by extremizing the functional [14, 21]

$$S = \mathrm{i}\frac{\pi}{3G_N}\frac{\Delta^3}{\omega_a\omega_b}+\Delta Q+\omega_a J_a+\omega_b J_b+\Lambda(3\Delta-\omega_a-\omega_b+2\pi\mathrm{i}), \tag{91}$$

where Λ is a Lagrange multiplier that enforces the constraint among the chemical potentials. Extremizing this functional via the saddle-point approximation yields

$$\frac{\partial S}{\partial \Delta}=\frac{\partial S}{\partial \omega_a}=\frac{\partial S}{\partial \omega_b}=0, \tag{92}$$

simplifies the entropy expression to

$$S = 2\pi\mathrm{i}\Lambda. \tag{93}$$

To ensure that the entropy is real for real physical charges, Λ must be purely imaginary. This, in turn, leads to two constraint equations involving S [14]:

$$\begin{aligned} &Q\frac{S^2}{4\pi^2}+\frac{J_a+J_b}{6G_N}S-\frac{Q^3}{27}=0\\ &\left(\frac{S}{2\pi}\right)^3-\frac{\pi}{3G_N}\left(\frac{S}{2\pi}\right)^2-\frac{Q^2}{6\pi}S+\frac{\pi}{3G_N}J_aJ_b=0. \end{aligned} \tag{94}$$

Solving these equations yields the BPS entropy in terms of the charges and angular momenta:

$$S = \frac{\pi}{Q}\left[\sqrt{(J_a+J_b)^2\frac{\pi^2}{9G_N^2}+\frac{4}{27}Q^3}-(J_a+J_b)\frac{\pi}{3G_N}\right], \tag{95}$$

subject to the charge constraint:

$$\begin{aligned} &\frac{1}{Q}\left[\sqrt{(J_a+J_b)^2\frac{\pi^2}{9G_N^2}+\frac{4}{27}Q^3}-(J_a+J_b)\frac{\pi}{3G_N}\right]\\ &=\frac{2\pi}{3G_N}Q^2\frac{Q^2+(J_a+J_b)Q-27J_aJ_b}{3(J_a+J_b)^2\frac{\pi^2}{G_N^2}+3\frac{\pi^2}{G_N^2}(J_a+J_b)Q-8Q^4}. \end{aligned} \tag{96}$$

In the EVH limit $Q=J_a=0$, the right-hand side vanishes identically, while the left-hand side remains proportional to the entropy S. Hence, the entropy vanishes in this limit.

In the near-EVH limit specified by (29), together with the BPS condition, the entropy expression (38) can be reproduced by extremizing the functional in (91). This mechanism differs significantly from the near-EVH case in AdS$_5$ (see (14), where the entropy scales as $N^2\epsilon$ due to an extremization between terms of order N^2 and $N^2\epsilon^2$, leading to the emergence of a 2D Cardy formula [1]). In the AdS$_6$ case, the thermodynamic quantities scale as follows:

$$\begin{aligned} &S\sim \mathrm{i}\frac{\pi}{3G_N}\frac{\Delta^3}{\omega_a\omega_b}\sim N^{\frac{5}{2}}\epsilon^2, \qquad \Delta\sim\omega_b\sim\epsilon, \quad Q\sim J_b\sim N^{\frac{5}{2}}\epsilon\\ &J_a\sim N^{\frac{5}{2}}\epsilon^3, \qquad \omega_a\sim 2\pi\mathrm{i}+O(\epsilon). \end{aligned} \tag{97}$$

Since the angular momentum J_a is subleading compared to J_b and Q, it can be neglected at leading order. Incorporating the near-EVH scaling relations (97) into the entropy functional, together with the chemical potential constraint, correctly reproduces the black hole entropy in the near-EVH limit (38).

The entropy of the AdS$_7$ black hole is reproduced by the following functional [19, 25, 46, 73]

$$=-\frac{\pi^2}{8G_N}\frac{\Delta^4}{\omega_1\omega_2\omega_3}+Q\Delta-\sum_{i=1}^{3}J_i\omega_i-\Lambda\left(\sum_{i=1}^{3}\omega_i-4\Delta-2\pi\mathrm{i}\right). \tag{98}$$

This can be derived from either a 6d SCFT or the 5d $\mathcal{N}=2$ Yang-Mills theory on S^5 [90]. The entropy, evaluated in the saddle-point approximation, is still captured by the Lagrange multiplier (93) due to the homogeneity of this functional. To obtain a real-valued physical entropy for real charges and angular momenta, the entropy is determined by two equations

$$\begin{aligned} &S^2=\frac{4\pi^4(J_1J_2+J_2J_3+J_1J_3)-2G_N\pi^2Q^3}{\pi^2-8G_NQ},\\ &\frac{2S^4}{\pi^4}+\left(\frac{J_1+J_2+J_3}{G_N}-\frac{3Q^2}{\pi^2}\right)S^2+\frac{Q^4}{8}-\frac{4\pi^2}{G_N}J_1J_2J_3=0. \end{aligned} \tag{99}$$

In the exact EVH limit where $J_{1,2}=Q=0$, the constraint

(99) implies that the entropy vanishes, consistent with the computation on the gravity side. In the near-EVH limit (76), the thermodynamic quantities scale with ϵ as:

$$\omega_1 \sim \omega_2 \sim \mathrm{i}\pi, \quad \Delta \sim \omega_3 \sim \epsilon, \quad J_1 \sim J_2 \sim N^3\epsilon^4, \quad J_3 \sim Q \sim N^3\epsilon^2. \tag{100}$$

Therefore, the contributions from the angular momenta $J_{1,2}$ are entirely subleading in the near-EVH limit, where $N^3\epsilon^3$ is held fixed. This phenomenon is reminiscent of the case in which one extremizes the AdS_6 entropy functional rather than that of AdS_5, in which the subleading angular momenta are crucial for deriving the Cardy formula.

A few remarks are in order.

- First, the contribution from the nearly vanishing angular momenta is subleading, yielding an inhomogeneous entropy functional that does not decompose into a two-step extremization procedure, which is consistent with the fact that the near-EVH geometry is not of the BTZ type.

- Second, we propose interpreting the entropy functional in the limit (97) as that of an EMMD black hole. It is important to note that EMMD black holes carry no angular momentum and exhibit no angular velocities [69]. To establish this correspondence, the quantities in (97) must be mapped to the thermodynamic variables of EMMD black holes in the decoupled limit. Furthermore, the explicit EMMD metric depends mildly on the coordinate θ_2, which is integrated out in the near-EVH black hole entropy expressions (38) and (81). Consequently, the near-EVH black hole entropy effectively averages over a family of EMMD geometries, rendering the matching between the EMMD black hole entropy and the near-EVH limit more subtle. This additional degree of freedom can be absorbed into a redefinition of the lower-dimensional Newton constant.

- Finally, the EVH limits are defined by setting a certain subset of charges and angular momenta to zero, leading to enhanced supersymmetry. This is equivalent to defining appropriate subsectors of the superconformal field theories dual to the $AdS_{6,7}$ black holes.

However, all of these holographic aspects lie beyond the scope of the present work.

V. CONCLUSION AND FUTURE WORK

In this work, we systematically explore possible near-horizon decoupled geometries in higher-dimensional AdS black holes embedded in string theory and M-theory [48]. Thanks to their high dimensionality and rich parameter space, the $D=6,7$ black holes admit model-dependent decoupled geometries of dimension $D\geq 4$ that are conformal to black hole solutions of EMMD gravity. These generalize the universal AdS_2 throats that appear in the near-horizon limits of extremal black holes. (Our analysis is restricted to leading order in the near-EVH limit, which may be insensitive to the presence of a cosmological constant in the decoupled EMMD geometries.)

As summarized in Table 1, the emergence of these distinct decoupled geometries has several crucial holographic implications. Most notably, the near-EVH limits in $D=6,7$ yield EMMD spacetimes rather than pure AdS_{D-2} factors, directly indicating that the decoupled low-energy sectors do not exhibit standard conformal invariance. Instead, the presence of a running dilaton in the EMMD solutions suggests that the dual infrared effective field theories may be governed by generalized scaling symmetries or hyperscaling-violating properties [84, 91, 92], rather than by traditional CFTs.

The first open question is to characterize the microscopic states dual to the classes of decoupled EMMD black holes (41) and (83), which are asymptotically flat [69, 74]. By rigorously embedding these non-AdS geometries into the well-defined near-EVH limits of AdS_6 and AdS_7, we establish a controlled, top-down holographic framework. This provides a clear pathway to tackle the microstate-counting problem for non-AdS black holes. Specifically, the distinctive thermodynamic scaling $S\sim T^{D-4}$ observed in these limits offers a macroscopic prediction for the density of states, setting a precise target for future field-theoretic computations and partition-function evaluations in the dual IR theory. The field theories dual to these black holes should be viewed as decoupled subsectors of the higher-dimensional dual SCFTs in $D=5,6$ [21–23]. However, these subsectors are not as well studied as the well-understood counterpart in $\mathcal{N}=4$ SYM, namely Spin Matrix theory [93–99]. A major challenge is that the SCFTs in $D=5,6$, and especially the 6d $\mathcal{N}=(2,0)$ theory, remain far from fully understood, as they lack weakly coupled descriptions [100], though efforts to construct Lagrangian descriptions exist [101]. Despite these challenges, our analysis indicates that the decoupled subsectors of SCFTs in $D=5,6$ provide a natural framework for describing the holographic duals to asymptotically flat EMMD black holes. Ultimately, these models suggest a novel strategy: utilizing higher-dimen-

Table 1. A summary of the decoupled geometries that appear in the near-EVH limit is provided.

Black hole	Allowed near horizon decoupled geometry
AdS_4	AdS_2, BTZ
AdS_5	AdS_2, BTZ
AdS_6	AdS_2, EMMD in $D=4$
AdS_7	AdS_2, BTZ or EMMD in $D=5$

sional AdS/CFT embeddings to decode the microscopic states of flat or non-AdS black holes, thereby offering a concrete example of holography beyond the standard AdS/CFT correspondence.

Analytic solutions for black objects in higher dimensions are more readily available in asymptotically flat spacetimes. Notable examples include the Myers–Perry black holes with the maximal number of independent rotations [102], the Cvetic–Youm solution incorporating $U(1)$ charges in arbitrary dimensions [103], and solutions with nontrivial horizon topologies such as black rings [104–106] and black saturns [107]. For comprehensive reviews, see [108]. These configurations may give rise to even richer geometric structures in the near-EVH regime [109, 110].

Unlike AdS_5 black holes, whose near-horizon limits are pinched AdS_3/BTZ geometries [31, 35, 87], the near-EVH limits of $\mathrm{AdS}_{D=6,7}$ black holes are of the following forms

$$\mathrm{d}s_D^2 = \epsilon^2\Omega\times \mathrm{d}s^2_{\mathrm{EMMD}_{D-2}} + \mathrm{d}s^2_{S^2}\,, \tag{101}$$

where Ω is a conformal factor. These geometries are similar to the recently introduced MpT geometries, which generalize non-relativistic Newton–Cartan geometry so that Dp-brane actions admit well-defined non-relativistic limits [88, 111]. The form of the decoupled geometry (101) in the near-EVH limits of $\mathrm{AdS}_{6,7}$ suggests that these backgrounds can be studied from the perspective of non-relativistic strings and branes. They might provide non-perturbative configurations in non-relativistic string theory at finite temperature.

The EMMD geometries emerging in the near-EVH limits of AdS_D black holes are specific to certain gauged supergravity models, whereas AdS_2 throats universally appear in the near-horizon region of near-extremal black holes. Although finite-energy excitations break the strict AdS_2 boundary conditions, their dynamics are universally captured by Jackiw–Teitelboim (JT) gravity — a nearly-AdS_2 theory proposed to be dual to the Sachdev–Ye–Kitaev (SYK) model [112–116]. This framework accounts for the linear term S_1T in the low-temperature entropy expansion $S=S_0+S_1T$; see [117, 118] for pedagogical treatments. Extensions to three dimensions have also been explored, providing a higher-dimensional analogue of the near-AdS_2/SYK correspondence [119]. These developments have been further applied to study the dynamics of near-BPS AdS_5 black holes and to identify mass gaps [120]. In our context, the EMMD black holes serve as novel low-energy effective geometries in the infrared, characterized by the higher power-law scaling $S\sim T^{D-4}$. They may similarly be understood as gravitational duals to strongly coupled low-energy sectors, much like higher-dimensional generalizations of the SYK model capturing nontrivial infrared dynamics.

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APPENDIX A: REVIEW OF EMMD GRAVITY

In this section, we briefly review the Einstein–Maxwell–Maxwell–Dilaton gravity model studied in [69, 74]. The Lagrangian of the D-dimensional supergravity under consideration is given by

$$\frac{1}{e}\mathcal{L} = R-\frac{1}{2}(\partial\phi)^2-\frac{1}{4}\mathrm{e}^{a_1\phi}F_1^2-\frac{1}{4}\mathrm{e}^{a_2\phi}F_2^2-V(\phi), \tag{A1}$$

where a_1 and a_2 are the dilaton coupling constants. To obtain analytic solutions within this supergravity theory, it is convenient to parameterize the couplings as

$$a_1^2=\frac{4}{N_1}-\frac{2(D-3)}{D-2},\qquad a_2^2=\frac{4}{N_2}-\frac{2(D-3)}{D-2}, \tag{A2}$$

Here, N_i are positive integers in the context of supergravity. We focus in particular on black hole solutions that satisfy

$$a_1a_2=-\frac{2(D-3)}{D-2}. \tag{A3}$$

Under this condition, analytic solutions with two independent charges can be constructed, as shown in [69].

The general solutions to the field equations derived from the action (A1) take the following form:

$$\begin{aligned}
&\mathrm{d}s^2 = -(H_1^{N_1}H_2^{N_2})^{-\frac{D-3}{D-2}}f\mathrm{d}t^2+(H_1^{N_1}H_2^{N_2})^{\frac{1}{D-3}}\left(\frac{\mathrm{d}r^2}{f}+r^2\mathrm{d}\Omega_{D-2}^2\right),\\
&A_1=\frac{\sqrt{N_1}c_1}{s_1}\frac{\mathrm{d}t}{H_1},\qquad A_2=\frac{\sqrt{N_2}c_2}{s_2}\frac{\mathrm{d}t}{H_2},\\
&\phi=\frac{1}{2}N_1a_1\ln H_1+\frac{1}{2}N_2a_2\ln H_2,\qquad f=1-\frac{\mu}{r^{D-3}},\\
&H_1=1+\frac{\mu s_1^2}{r^{D-3}},\qquad H_2=1+\frac{\mu s_2^2}{r^{D-3}},
\end{aligned} \tag{A4}$$

subject to the constraints

$$N_1a_1+N_2a_2=0,\qquad N_1+N_2=\frac{2(D-2)}{D-3}. \tag{A5}$$

In four dimensions ($D=4$), the possible solutions include $(N_1,N_2)=(2,2)$, $(1,3)$, $(3,1)$, while in five dimensions ($D=5$) the only admissible pairs are $(N_1,N_2)=(2,1)$, $(1,2)$. We now examine each of these cases in turn.

$D=4$ **geometry with** $(N_1,N_2)=(2,2)$

This symmetric choice of parameters, with $a_1=-a_2=1$, yields a particularly simple and elegant solution. We define the charge parameters as

$$q_1=\mu s_1^2,\qquad q_2=\mu s_2^2, \tag{A6}$$

The metric in (A4) simplifies to

$$\mathrm{d}s^2=-\frac{r(r-\mu)}{(r+q_1)(r+q_2)}\mathrm{d}t^2+\frac{(r+q_1)(r+q_2)}{r(r-\mu)}\mathrm{d}r^2 +(r+q_1)(r+q_2)\mathrm{d}\Omega_2^2. \tag{A7}$$

To make its underlying conformal structure explicit, we factor the metric. Assuming, without loss of generality, that $q_2-q_1=\mathfrak{q}>0$, we can recast it in the form

$$\mathrm{d}s^2=\frac{r+q_2}{r+q_1}\left[-\frac{(r-\mu)r}{(r+q_2)^2}\mathrm{d}t^2+\frac{(r+q_1)^2}{r(r-\mu)}\mathrm{d}r^2+(r+q_1)^2\mathrm{d}\Omega_2^2\right]. \tag{A8}$$

This conformal representation shows that the solution interpolates between different geometries while preserving the same asymptotics; the prefactor plays a crucial role.

$D=4$ **geometry with** $(N_1,N_2)=(1,3)$

Owing to the symmetry between N_1 and N_2, it suffices to analyze the $(1,3)$ case, since the $(3,1)$ solution is its direct counterpart. The corresponding dilaton couplings are fixed as follows:

$$a_1=\sqrt{3},\qquad a_2=-\frac{1}{\sqrt{3}}. \tag{A9}$$

In this asymmetric configuration, the metric (A4) reduces to:

$$\mathrm{d}s^2=-\frac{r(r-\mu)}{\sqrt{(r+q_1)(r+q_2)^3}}\mathrm{d}t^2+\frac{\sqrt{(r+q_1)(r+q_2)^3}}{r(r-\mu)}\mathrm{d}r^2 +\sqrt{(r+q_1)(r+q_2)^3}\mathrm{d}\Omega_2^2. \tag{A10}$$

Expressing this metric in a conformally rescaled frame yields a clearer geometric interpretation:

$$\mathrm{d}s^2=\sqrt{\frac{r+q_1}{r+q_2}}\left[-\frac{r(r-\mu)}{(r+q_1)(r+q_2)}\mathrm{d}t^2 +\frac{(r+q_2)^2\mathrm{d}r^2}{r(r-\mu)}+(r+q_2)^2\mathrm{d}\Omega_2^2\right]. \tag{A11}$$

$D=5$ **geometry with** $(N_1,N_2)=(1,2)$

We now consider a five-dimensional example with $(N_1,N_2)=(1,2)$. The corresponding dilaton couplings are given by:

$$a_1=\sqrt{\frac{8}{3}},\qquad a_2=-\sqrt{\frac{2}{3}}. \tag{A12}$$

The full metric in this case reads

$$\mathrm{d}s^2=-\frac{r^2(r^2-\mu)\mathrm{d}t^2}{(r^2+q_1)^{\frac{2}{3}}(r^2+q_2)^{\frac{4}{3}}} +\frac{(r^2+q_1)^{\frac{1}{3}}(r^2+q_2)^{\frac{2}{3}}}{r^2-\mu}\mathrm{d}r^2 +(r^2+q_1)^{\frac{1}{3}}(r^2+q_2)^{\frac{2}{3}}\mathrm{d}\Omega_3^2. \tag{A13}$$

A clearer picture of the geometry emerges upon applying the coordinate transformation.

$$x^2=r^2+q_1,\qquad \mathfrak{q}=q_2-q_1>0. \tag{A14}$$

In these new coordinates, the metric takes the following conformally flat form:

$$\mathrm{d}s^2=\frac{(x^2+\mathfrak{q})^{\frac{2}{3}}}{x^{\frac{4}{3}}}\left[-\frac{(x^2-q_1)(x^2-q_1-\mu)\mathrm{d}t^2}{(x^2+\mathfrak{q})^2} +\frac{x^4\mathrm{d}x^2}{(x^2-q_1)(x^2-q_1-\mu)}+x^2\mathrm{d}\Omega_3^2\right]. \tag{A15}$$

This final form reduces to the standard five-dimensional RN black hole upon setting $\mathfrak{q}=0$.

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