NLO QCD sum rules analysis of 1^{-+} tetraquark states

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Abstract: We have performed a next-to-leading-order (NLO) QCD sum rules analysis of the 1^{-+} light tetraquark states. By investigating various compact and molecular tetraquark currents, we extracted the mass spectra of the corresponding states, all of which lie above 1.7 GeV. We have identified multiple 1^{-+} states around 2.0 GeV matching well with $\pi_1(2015)$, confirming that $\pi_1(2015)$ is an excellent tetraquark candidate. By contrast, our calculations exclude the possibility that the $\pi_1(1400)$ is a tetraquark or hybrid–tetraquark mixture. This result suggests that it may not exist, which is consistent with recent experimental results.

Keywords: QCD sum rules, NLO correction, tetraquark

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I. INTRODUCTION

The study of non-$q\bar{q}$ mesons beyond the conventional quark model remains a central topic in hadron physics [1, 2]. QCD predicts the existence of mesons with various structures, such as hybrid states containing excited gluons [3, 4], glueballs composed purely of gluons [5], and multiquark states [6–11]. Mesons with the exotic quantum numbers $J^{PC} = 1^{-+}$ have attracted extensive attention and are typically considered as candidates for hybrids, tetraquarks, or their mixtures. In particular, the ambiguity surrounding the isovector state $\pi_1(1400)$ has persisted for a long time. In a previous study, $\pi_1(1400)$ appeared in the $\pi^- p \to \pi^0 \eta n$ reaction [12] and was later observed in the $\pi^- p \to \eta\pi^- p$ channel [13, 14]. However, some recent evidence suggests that $\pi_1(1400)$ may be an artifact of the $\pi_1(1600)$ particle and might not exist [15]. Based on the partial-wave analysis data of the $\eta^{(\prime)}\pi$ system provided by the COMPASS Collaboration [16], a coupled-channel amplitude analysis [17] has been performed that enforces the unitarity and analyticity of the S matrix. The corresponding results demonstrate that a single pole is sufficient to fit the experimental data for both $\pi_1(1400)$ and $\pi_1(1600)$. In Ref. [18], $\pi_1(1400)$ was interpreted as a molecular tetraquark, whereas Ref. [19] suggests that a hybrid–tetraquark mixture might explain $\pi_1(1400)$—this conclusion is corroborated in a later work [20]. Studies on hybrids indicate that the mass of a 1^{-+} hybrid state is higher than 1.7 GeV [21–23], and consequently, matching the mass of $\pi_1(1400)$ is challenging. Therefore, within the framework of QCD sum rules, the existence of $\pi_1(1400)$ depends on the probability of existence of a tetraquark state with a mass approximately equal to or less than 1.4 GeV. In Ref. [24], a series of compact tetraquark currents were calculated, and several isospin-1 1^{-+} compact tetraquark states with masses around 1.6 GeV and 2.0 GeV were obtained. Therefore, $\pi_1(1600)$ and $\pi_1(2015)$ are considered as suitable candidates for four-quark states. However, all the aforementioned studies on tetraquark states focused on calcuations only at the leading order (LO) without considering next-to-leading-order (NLO) contributions, which can sometimes be significant [3]. This paper presents the first NLO analysis of the 1^{-+} tetraquark states. We have used the latest phenomenological parameters and a more precise running coupling to reanalyze the possibility of the existence of $\pi_1(1400)$ and to examine the feasibility of considering $\pi_1(1600)$ and $\pi_1(2015)$ as suitable tetraquark candidates.

II. TETRAQUARK CURRENTS AND RENORMALIZATION

Previous studies have shown that some currents with u, d, and s quarks (and their antiquarks) yield 1^{-+} states with masses of approximately 1.4–2.0 GeV at the LO order. [18, 24]. We expect that adding NLO corrections shifts their masses, making them a good choice for our study. We built the compact tetraquark currents using quark pairs (qq) and antiquark pairs ($\bar{q}\bar{q}$). Based on charge conjugation and flavor structure, we define four

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currents $\eta_1^\mu \sim \eta_4^\mu$:

$$\begin{aligned}\eta_1^\mu &= u_a^T C\gamma^\mu d_b(\bar{u}_a C\bar{d}_b^T + \bar{u}_b C\bar{d}_a^T)\\ &\quad + u_a^T C d_b(\bar{u}_a\gamma^\mu C\bar{d}_b^T + \bar{u}_b\gamma^\mu C\bar{d}_a^T),\\ \eta_2^\mu &= u_a^T C\sigma^{\mu\nu}\gamma^5 d_b(\bar{u}_a\gamma_\nu\gamma^5 C\bar{d}_b^T + \bar{u}_b\gamma_\nu\gamma^5 C\bar{d}_a^T)\\ &\quad + u_a^T C\gamma_\nu\gamma^5 d_b(\bar{u}_a\sigma^{\mu\nu}\gamma^5 C\bar{d}_b^T + \bar{u}_b\sigma^{\mu\nu}\gamma^5 C\bar{d}_a^T),\\ \eta_3^\mu &= u_a^T C\gamma^\mu d_b(\bar{u}_a C\bar{d}_b^T - \bar{u}_b C\bar{d}_a^T)\\ &\quad + u_a^T C d_b(\bar{u}_a\gamma^\mu C\bar{d}_b^T - \bar{u}_b\gamma^\mu C\bar{d}_a^T),\\ \eta_4^\mu &= u_a^T C\sigma^{\mu\nu}\gamma^5 d_b(\bar{u}_a\gamma_\nu\gamma^5 C\bar{d}_b^T - \bar{u}_b\gamma_\nu\gamma^5 C\bar{d}_a^T)\\ &\quad + u_a^T C\gamma_\nu\gamma^5 d_b(\bar{u}_a\sigma^{\mu\nu}\gamma^5 C\bar{d}_b^T - \bar{u}_b\sigma^{\mu\nu}\gamma^5 C\bar{d}_a^T),\end{aligned} \tag{1}$$

where C is the charge conjugation operator, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. By replacing the d quark with an s quark, we obtain

$$\begin{aligned}\eta_5^\mu &= u_a^T C\gamma^\mu s_b(\bar{u}_a C\bar{s}_b^T + \bar{u}_b C\bar{s}_a^T)\\ &\quad + u_a^T C s_b(\bar{u}_a\gamma^\mu C\bar{s}_b^T + \bar{u}_b\gamma^\mu C\bar{s}_a^T),\\ \eta_6^\mu &= u_a^T C\sigma^{\mu\nu}\gamma^5 s_b(\bar{u}_a\gamma_\nu\gamma^5 C\bar{s}_b^T + \bar{u}_b\gamma_\nu\gamma^5 C\bar{s}_a^T)\\ &\quad + u_a^T C\gamma_\nu\gamma^5 s_b(\bar{u}_a\sigma^{\mu\nu}\gamma^5 C\bar{s}_b^T + \bar{u}_b\sigma^{\mu\nu}\gamma^5 C\bar{s}_a^T),\\ \eta_7^\mu &= u_a^T C\gamma^\mu s_b(\bar{u}_a C\bar{s}_b^T - \bar{u}_b C\bar{s}_a^T)\\ &\quad + u_a^T C s_b(\bar{u}_a\gamma^\mu C\bar{s}_b^T - \bar{u}_b\gamma^\mu C\bar{s}_a^T),\\ \eta_8^\mu &= u_a^T C\sigma^{\mu\nu}\gamma^5 s_b(\bar{u}_a\gamma_\nu\gamma^5 C\bar{s}_b^T - \bar{u}_b\gamma_\nu\gamma^5 C\bar{s}_a^T)\\ &\quad + u_a^T C\gamma_\nu\gamma^5 s_b(\bar{u}_a\sigma^{\mu\nu}\gamma^5 C\bar{s}_b^T - \bar{u}_b\sigma^{\mu\nu}\gamma^5 C\bar{s}_a^T).\end{aligned} \tag{2}$$

Furthermore, for molecular tetraquark states, we also consider the following two currents [18]:

$$J_1^\mu = \frac{1}{2}\left(\bar{u}\gamma^5 u - \bar{d}\gamma^5 d\right)\left(\bar{u}\gamma^5\gamma^\mu u + \bar{d}\gamma^5\gamma^\mu d\right), \tag{3}$$

$$J_2^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\left(\bar{u}\gamma^5\gamma_\rho d\,\bar{d}\gamma_\sigma u - \bar{d}\gamma^5\gamma_\rho u\,\bar{u}\gamma_\sigma d\right), \tag{4}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric tensor with the convention $\epsilon^{0123} = +1$.

When expanding the perturbative part of the correlation functions for these tetraquark currents to NLO, we need to calculate the Feynman diagrams as shown in Fig. 1. Because tetraquark currents are composite operators, the calculation results contain non-local divergence terms such as

$$\log(-q^2/\mu^2)/\varepsilon. \tag{5}$$

These non-local divergence terms cannot be completely removed using conventional Lagrangian renormalization methods; the operator currents must be renormalized. For the operator currents η_1^μ and η_3^μ, applying the tetraquark renormalization method [25] with the Feynman gauge yields

$$\begin{aligned}&\left(u_a^T C d_b\,\bar{u}_a\gamma^\mu C\bar{d}_b^T\right)_r\\ &= \left(Z_2^{-2} - \frac{2C_A^2+1}{32\pi^2\varepsilon C_A}g^2\right)u_a^T C d_b\,\bar{u}_a\gamma^\mu C\bar{d}_b^T\\ &\quad + \frac{3}{32\pi^2\varepsilon}g^2 u_a^T C d_b\,\bar{u}_b\gamma^\mu C\bar{d}_a^T\\ &\quad - \frac{C_A i g^2}{32\pi^2\varepsilon}u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_a\gamma_\nu\gamma^5 C\bar{d}_b^T\\ &\quad + \frac{i g^2}{32\pi^2\varepsilon}u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_b\gamma_\nu\gamma^5 C\bar{d}_a^T,\end{aligned} \tag{6}$$

$$\begin{aligned}&\left(u_a^T C d_b\,\bar{u}_b\gamma^\mu C\bar{d}_a^T\right)_r\\ &= \left(Z_2^{-2} - \frac{2C_A^2+1}{32\pi^2\varepsilon C_A}g^2\right)u_a^T C d_b\,\bar{u}_b\gamma^\mu C\bar{d}_a^T\\ &\quad + \frac{3}{32\pi^2\varepsilon}g^2 u_a^T C d_b\,\bar{u}_a\gamma^\mu C\bar{d}_b^T\\ &\quad + \frac{C_A i g^2}{32\pi^2\varepsilon}u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_b\gamma_\nu\gamma^5 C\bar{d}_a^T\\ &\quad - \frac{i g^2}{32\pi^2\varepsilon}u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_a\gamma_\nu\gamma^5 C\bar{d}_b^T\\ &\quad + \frac{1}{48\pi^2\varepsilon}\left(\bar{u}D^\alpha G_{\alpha\beta}\gamma^\beta\gamma^\mu\gamma^5 u - \bar{d}D^\alpha G_{\alpha\beta}\gamma^\beta\gamma^\mu\gamma^5 d\right),\end{aligned} \tag{7}$$

In the above equations,

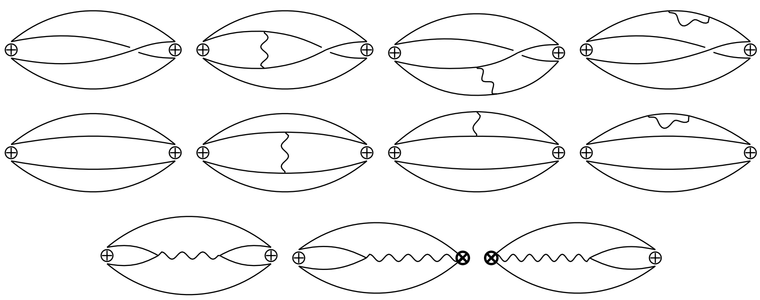

Fig. 1. Feynman diagrams for the perturbative term of tetraquark currents. Permutation diagrams are omitted. The bold cross vertex represents the hybrid-like counterterm. For compact tetraquark currents, the first four diagrams are unnecessary.

$$Z_2 = 1 - \frac{g^2 C_F}{16\pi^2 \epsilon}, \tag{8}$$

where $C_F = 4/3$ is the quadratic Casimir operator of the fundamental representation of the $SU(3)$ group, and $C_A = 3$ is the quadratic Casimir operator of the adjoint representation of the $SU(3)$ group; these parameters are also known as the quark and gluon color factors, respectively. For convenience, we suppress the generator matrix T of $SU(3)$ and the coupling constant g, *i.e.*,

$$D^\alpha G_{\alpha\beta} \equiv g D^\alpha G^n_{\alpha\beta} T^n. \tag{9}$$

The hybrid-like current $D^\alpha G_{\alpha\beta}$ only appears in operator currents where the summation is over different quark color indices. By replacing the d quark in Eqs. (6) and (7) with an s quark, we obtain the operator renormalization results for the tetraquark currents η_5^μ and η_7^μ. The molecular tetraquark currents can be obtained through linear combinations of the following renormalized operator currents:

$$\begin{aligned}\left(\bar{u}\gamma^5 u \bar{d}\gamma^5\gamma^\mu d\right)_r = &\left(Z_2^{-2} + \frac{5C_F}{16\pi^2\varepsilon} g^2\right) \bar{u}\gamma^5 u \bar{d}\gamma^5\gamma^\mu d \\ &+ \frac{\mathrm{i}g^2}{8\pi^2\varepsilon} \bar{u} T^n \sigma^{\mu\nu} u \bar{d}\gamma_\nu T^n d,\end{aligned} \tag{10}$$

$$\begin{aligned}\left(\bar{u}\gamma^5 u \bar{u}\gamma^5\gamma^\mu u\right)_r = &\left(Z_2^{-2} + \frac{5C_F}{16\pi^2\varepsilon} g^2\right) \bar{u}\gamma^5 u \bar{u}\gamma^5\gamma^\mu u \\ &+ \frac{\mathrm{i}g^2}{8\pi^2\varepsilon} \bar{u} T^n \sigma^{\mu\nu} u \bar{u}\gamma_\nu T^n u \\ &+ \frac{\mathrm{i}}{24\pi^2\varepsilon} \bar{u} D^\alpha G_{\alpha\beta} \sigma^{\beta\mu} u.\end{aligned} \tag{11}$$

The renormalization of other operator currents is explained in Appendix A.2.

III. QCD SUM RULES

QCD sum rules [2, 26–29] determine the corresponding hadronic spectral structure through the poles of operator current correlation functions in the complex plane. To calculate the correlation function, we first perform operator product expansion (OPE):

$$\begin{aligned}\Pi(q) &= \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx} \langle 0|T\{J^\mu(x)J^\nu(0)\}|0\rangle \\ &= C_0(q) + C_4(q)\langle O^4\rangle + C_6(q)\langle O^6\rangle + \cdots,\end{aligned} \tag{12}$$

where $C_i(q)$ denots Wilson coefficients and $\langle O^i\rangle$ vacuum condensates (i.e., the vacuum expectation values of various local operators). The first Wilson coefficient $C_0(q)$ corresponds to the perturbative diagrams in Fig. 1 and accounts for the high-energy contribution. Subsequent coefficients such as $C_4(q)$ and $C_6(q)$ can be calculated by cutting certain propagators in the perturbative diagrams or by introducing zero-momentum gluons from the vacuum; they represent the contribution from the low-energy region. For this part of the contribution, the calculation should be performed in coordinate space. We can use the techniques reported in Refs. [24, 30–32] to evaluate these non-perturbative contributions. The quark propagator including vacuum condensates is

$$\begin{aligned}\mathrm{i}S^{ab} &\equiv \langle 0|T[q^a(x)\bar{q}^b(0)]|0\rangle \\ &= \frac{\mathrm{i}\delta^{ab}}{2\pi^2 x^4}\not{x} - \frac{\mathrm{i}g}{32\pi^2} G^n_{\mu\nu} T^{nab} \frac{1}{x^2}(\sigma^{\mu\nu}\not{x} + \not{x}\sigma^{\mu\nu}) - \frac{\delta^{ab}}{12}\langle\bar{q}q\rangle \\ &\quad - \frac{\delta^{ab}x^2}{192}\langle g\bar{q}\sigma G q\rangle - \frac{m_q\delta^{ab}}{4\pi^2 x^2} + \frac{\mathrm{i}\delta^{ab} m_q\langle\bar{q}q\rangle}{48}\not{x} + \frac{\mathrm{i}\delta^{ab} m_q^2}{8\pi^2 x^2}\not{x}.\end{aligned} \tag{13}$$

The Lorentz structure of the operator currents indicates that a general vector current can couple to both vector and scalar particles. Therefore, we need to isolate the part corresponding to the 1^{-+} vector particle in the correlation function. In momentum space, the correlation function of the vector current can be decomposed into

$$\begin{aligned}&\mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx}\langle 0|T\{J^\mu(x)J^\nu(0)\}|0\rangle \\ &= \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi^V(q) + q^\mu q^\nu \Pi^S(q),\end{aligned} \tag{14}$$

where the term $\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)$ on the right-hand side of the equation is the transverse polarization factor, and $q^\mu q^\nu$ is the longitudinal polarization factor. The vector particle couples to the vector current J^μ through the matrix element

$$\langle 0|J^\mu|V(q)\rangle = \epsilon^\mu f(q^2). \tag{15}$$

The polarization vector ϵ^μ satisfies the completeness relation

$$\sum_{\lambda=-1}^{1} \epsilon_\lambda^{\mu *}\epsilon_\lambda^\nu = \frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}. \tag{16}$$

Therefore, the spectral density of the vector particle is related to the transverse polarization part of the correlation function. For the vector currents $\eta_1^\mu \sim \eta_8^\mu$ and J_1^μ, only the $\Pi^V(q)$ part is considered. For tensor currents, they can couple to scalar, vector, and tensor particles. Similar to the aforementioned analysis, considering that $J_2^{\mu\nu}$ is antisymmetric with respect to Lorentz indices, we decompose the tensor-current-operator correlation function in momentum space as

$$i\int d^4x e^{iqx}\langle 0|J_2^{\mu\nu}(x)J_2^{\dagger\alpha\beta}|0\rangle=(\eta^{\mu\alpha}\eta^{\nu\beta}-\eta^{\nu\alpha}\eta^{\mu\beta})\Pi'^{V}_{J_2}(q^2)$$
$$+(\eta^{\mu\alpha}q^\nu q^\beta-\eta^{\nu\alpha}q^\mu q^\beta-\eta^{\mu\beta}q^\nu q^\alpha+\eta^{\nu\beta}q^\mu q^\alpha)\frac{1}{q^2}\Pi^V_{J_2}(q^2),\quad(17)$$

where $\eta^{\mu\nu}\equiv\frac{q^\mu q^\nu}{q^2}-g^{\mu\nu}$. The coupling rule for vector particles and tensor currents is

$$\langle 0|J_2^{\mu\nu}|V(q)\rangle=(q^\mu\epsilon^\nu-q^\nu\epsilon^\mu)f(q^2).\quad(18)$$

Combined with Eq. (16), these relations indicate that the spectral density of the vector particle corresponds to the $\Pi^V_{J_2}(q^2)$ part.

We derive the OPE of the correlation function using dimensional regularization and the $\overline{\text{MS}}$ subtraction scheme while treating γ^5 in the BMHV scheme. Consequently, the correlation function Π^V for the current η_1^μ is given by

$$\begin{aligned}\Pi^V_{\eta_1}=\,&q^8\left[\left(-\frac{79g_s^2}{2654208\pi^8}-\frac{1}{18432\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right.\\&\left.-\frac{5g_s^2}{1327104\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)\right]\\&+q^4\frac{g_s^2}{18432\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)\langle GG\rangle\\&-q^2\frac{1}{18\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle^2\\&+\frac{1}{12\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\\&+\frac{1}{q^2}\left(\frac{5g_s^2}{864\pi^2}\langle GG\rangle\langle\bar{q}q\rangle^2-\frac{1}{48\pi^2}\langle\bar{q}Gq\rangle^2\right)\\&+\frac{1}{q^4}\left(-\frac{g_s^2}{576\pi^2}\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle-\frac{32}{81}g_s^2\langle\bar{q}q\rangle^4\right).\end{aligned}\quad(19)$$

The remaining correlation functions are given in Appendix A.3.

Using the dispersion relation and adopting the "δ + continuum" assumption, we have

$$\frac{1}{\pi}\text{Im}\Pi^V(q)=f^2\delta(s-m^2)+\theta(s-s_0)\rho_c(s),\quad(20)$$

where m is the mass of the lowest resonance state, f is the coupling strength, s_0 is the continuum threshold, and $\rho_c(s)$ is the spectral density of the continuum states. Applying the Borel transform to both sides of the above equation yields the n-th moment $\mathcal{M}^n(\tau,s_0)$:

$$\mathcal{M}^n(\tau,s_0)\equiv\frac{1}{\pi}\int_0^{s_0}ds\, s^n e^{-\tau s}\,\text{Im}\Pi^V(s)=f^2m^{2n}e^{-\tau m^2}.\quad(21)$$

The mass m of the lowest resonance state in the spectral density can be obtained from the ratio of the moment:

$$\mathcal{R}^n(\tau,s_0)\equiv\frac{\mathcal{M}^{n+1}(\tau,s_0)}{\mathcal{M}^n(\tau,s_0)}=m^2.\quad(22)$$

Upon taking the ratio of moments, the coupling constant f cancels out. In this study, $n=0$ is adopted.

IV. NUMERICAL ANALYSIS

In the numerical analysis, we adopt the following quark masses and vacuum condensate parameters at the scale $\mu=2$ GeV [33, 34]:

$$\begin{aligned}m_s&=93.5\pm0.8\ \text{MeV},\\\frac{g^2}{4\pi}\langle GG\rangle&=0.07\pm0.02\ \text{GeV}^4,\\\langle\bar{q}q\rangle&=-(0.276)^3\ \text{GeV}^3,\\\langle\bar{s}s\rangle&=0.74\langle\bar{q}q\rangle,\\\langle\bar{q}Gq\rangle&=M_0^2\langle\bar{q}q\rangle,\\\langle\bar{s}Gs\rangle&=M_0^2\langle\bar{s}s\rangle,\\M_0^2&=0.8\pm0.2\ \text{GeV}^2,\end{aligned}$$

where $q=u$ or d, $\langle GG\rangle=\langle G^n_{\mu\nu}G^{n\mu\nu}\rangle$, $\langle\bar{q}Gq\rangle=\langle\bar{q}gT^nG^n_{\mu\nu}\sigma^{\mu\nu}q\rangle$, and the masses of the light quarks u and d are neglected. The running coupling constant α_s is expressed in the one-loop approximation:

$$\alpha_s(\mu^2)=\frac{\alpha_s(m_\tau^2)}{1+\frac{\beta_0}{4\pi}\alpha_s(m_\tau^2)\log\left(\frac{\mu^2}{m_\tau^2}\right)},\quad(23)$$

where $m_\tau=1776.93\pm0.09$ MeV, $\alpha_s(m_\tau^2)=0.314\pm0.014$, with $n_f=3$, and $\beta_0=9$. Renormalization group improvement is achieved by setting $\mu^2=1/\tau$ [35].

The relative contributions of the perturbative and condensate terms to the moment $\mathcal{M}^0(\tau,s_0)$ of the current η_1^μ are calculated as shown in Fig. 2. Evidently, the OPE converges well, demonstrating the reliability of the calculation results.

Using the ratio of moments in Eq. (22), we extract the mass of the lowest resonance by $m=\sqrt{\mathcal{R}^0(\tau,s_0)}$, which depends on the Borel parameter τ and the continuum threshold s_0. Since the Borel parameter τ is auxiliary, the extracted mass should exhibit a stable plateau with respect to τ. We determine the mass using stability criteria [34]: the mass is extracted from the extremum of the $m(\tau)$

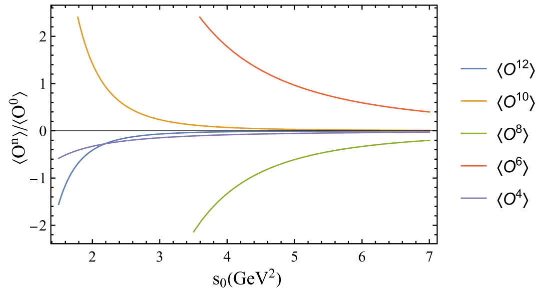

Fig. 2. (color online) Ratio of condensate terms $\langle O^n\rangle$ to the perturbative term $\langle O^0\rangle$ for η_1^μ, at $\tau = 0.3\,\mathrm{GeV}^{-2}$.

curve or its plateau region[1]. The general procedure is to first determine an approximate value of s_0 such that a plateau appears in the $m(\tau)$ curve. Subsequently, the range of τ corresponding to this plateau is identified as the working region. Finally, the mass of the resonance state is determined by the value of m at the plateau. It is worth noting that the moment $\mathcal{M}^0$ corresponds to the spectral density and thus is always positive. If a negative value appears, it indicates that the QCD sum rules analysis breaks down, and the corresponding mass is non-physical. In Fig. 3, we list the $\mathcal{M}^0 \sim s_0$ and $m \sim s_0$ curves for the tensor current $J_2^{\mu\nu}$. In the left figure, the region $s_0 \lesssim 4.5\,\mathrm{GeV}^2$ is non-physical, so the corresponding mass $\lesssim 1.5\,\mathrm{GeV}$ in the right figure is not credible; the true mass should be $\gtrsim 2.3\,\mathrm{GeV}$. In the subsequent analysis, we discuss within the physical interval where $\mathcal{M}^0$ is positive.

Figure 4 shows that for $J_2^{\mu\nu}$, the NLO perturbative correction is approximately 30% of the LO contribution and thus cannot be neglected. For η_2^μ, the NLO perturbative correction is even larger than the LO term, i.e., approximately 130% of the LO contribution. Furthermore, in Fig. 5, the left-hand side shows the estimated mass of the operator current $J_2^{\mu\nu}$ when expanded to the LO, and the right-hand side shows the estimated mass after adding the NLO perturbation correction. Incorporating the NLO correction significantly affects the mass determination. In particular, the mass curve is lowered by approximately 0.2 GeV and becomes flatter, which leads to improved stability.

In Appendix A.1, we present the resonance mass m as a function of the Borel parameter τ for all currents, with s_0 fixed at various values. Based on the stability criterion, we select three s_0 values that yield the flattest curves in each figure. The corresponding optimal mass estimates and s_0 values are listed in Table 1. The masses for the compact tetraquark currents $\eta_1^\mu \sim \eta_4^\mu$ (with u and d quarks) are 2.05, 1.88, 1.70, and 1.99 GeV. For $\eta_5^\mu \sim \eta_8^\mu$ (with u and s quarks), the corresponding masses are 2.36, 2.06, 2.26, and 2.34 GeV. For the molecular currents, we obtain 1.71 GeV for J_1^μ and 2.44 GeV for $J_2^{\mu\nu}$. In previous LO studies [24], the currents $\eta_1^\mu \sim \eta_4^\mu$ yielded the lowest resonance masses of approximately 1.6–1.7 GeV and are considered to couple to the $\pi_1(1600)$. However, after NLO corrections, the lowest resonance masses increased, shifting away from the peak position of the $\pi_1(1600)$. Compared with the results obtained in previous LO QCD

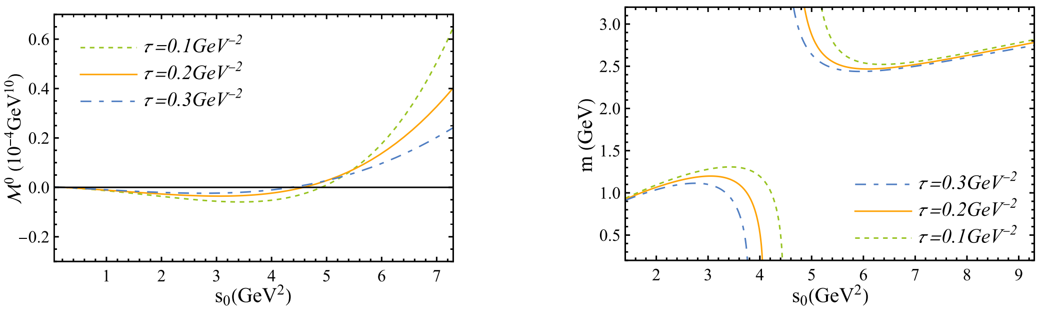

Fig. 3. (color online) NLO results for the moment $\mathcal{M}^0$ and mass m versus s_0 for the current $J_2^{\mu\nu}$.

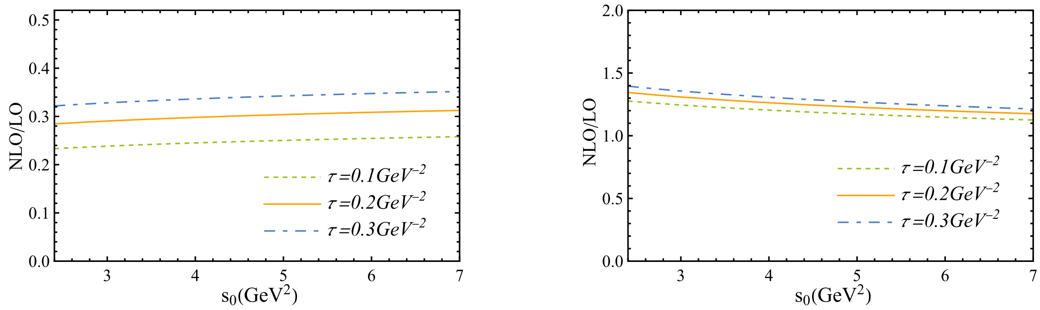

Fig. 4. (color online) Ratio of NLO contribution to LO contribution in moment $\mathcal{M}^0$ for the currents $J_2^{\mu\nu}$ (left) and η_2^μ (right).

1) When applying the sum rule to the harmonic oscillator in quantum mechanics, the minimum of $R(\tau)$ corresponds to the exact value of the ground state energy E_0.

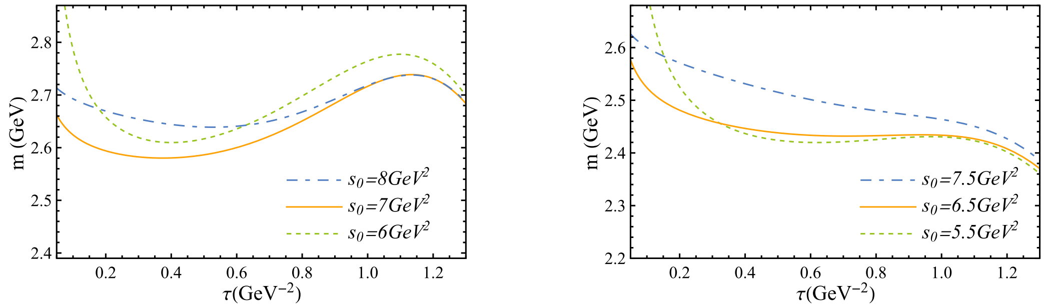

Fig. 5. (color online) Mass predictions for the current $J_2^{\mu\nu}$ at the LO (left) and NLO (right) levels.

Table 1. Resonance masses corresponding to operator currents.

	η_1	η_2	η_3	η_4	η_5
m	2.05 ± 0.05	1.88 ± 0.06	1.70 ± 0.06	1.99 ± 0.04	2.36 ± 0.06
s_0	5.1 ± 0.3	4.3 ± 0.3	3.5 ± 0.3	4.8 ± 0.2	6.5 ± 0.4
	η_6	η_7	η_8	J_1	J_2
m	2.06 ± 0.04	2.24 ± 0.05	2.234 ± 0.03	1.71 ± 0.07	2.44 ± 0.02
s_0	5.0 ± 0.2	6.0 ± 0.3	6.5 ± 0.2	3.5 ± 0.3	6.5 ± 0.3

sum rules studies, our mass predictions are shifted by $0.1-0.3$ GeV for each current, as shown in Appendix A.4. These corrections are primarily attributed to the NLO contributions, the updated QCD parameters, a more precise running coupling, and renormalization group improvements.

Our corrected results do not yield a 1^{-+} resonance mass corresponding to tetraquark currents around or below 1.4 GeV. Consequently, $\pi_1(1400)$ cannot be interpreted as a tetraquark or hybrid–tetraquark mixture candidate. Recent studies suggest that previous analyses of $\pi_1(1400)$ experimental data may be flawed, and a single resonance peak centered at 1.6 GeV is sufficient to fit all experimental data [17]. Therefore, we can conclude that $\pi_1(1400)$ may not exist and that its signal is merely an artifact of the $\pi_1(1600)$. Particle Data Group (PDG) has included the $\pi_1(1400)$ data in the $\pi_1(1600)$ entry. Our calculations provide further theoretical support for this perspective within the QCD sum rules framework. In addition, our results show that the tetraquark mass is greater than 1.7 GeV, indicating that it exceeds previous estimates and is difficult to reconcile with the $\pi_1(1600)$. This result indicates potential inconsistencies in interpreting $\pi_1(1600)$ as a tetraquark, and our findings suggest that the non-tetraquark interpretation is preferable. However, we have identified multiple resonance states around 2.0 GeV; these states match well with $\pi_1(2015)$, confirming its potential as a tetraquark candidate.

V. CONCLUSION

We performed an NLO QCD sum rules analysis of the 1^{-+} light tetraquark states. The results show that NLO corrections are essential for mass predictions and help improve the stability. Sometimes the significant NLO corrections pose a challenge to achieving convergence, which might be clarified by further NNLO calculations. Nevertheless, the results demonstrate that the LO result alone is insufficient. All the obtained states have masses greater than 1.7 GeV, with the masses of most of them being equal to or greater than 2.0 GeV. These values are 0.1–0.3 GeV higher than those reported in previous studies, implying that the masses of tetraquark states should be higher than their previously anticipated values. Our results exclude the possibility of $\pi_1(1400)$ being a tetraquark or a hybrid–tetraquark mixture. This result implies that the particle may not exist, in agreement with recent experimental data. By contrast, we obtained multiple 1^{-+} states around 2.0 GeV, matching well with $\pi_1(2015)$. This result reinforces our view that—$\pi_1(2015)$ is an excellent tetraquark candidate.

APPENDIX A

A.1. $m(\tau)$ curves of resonance states

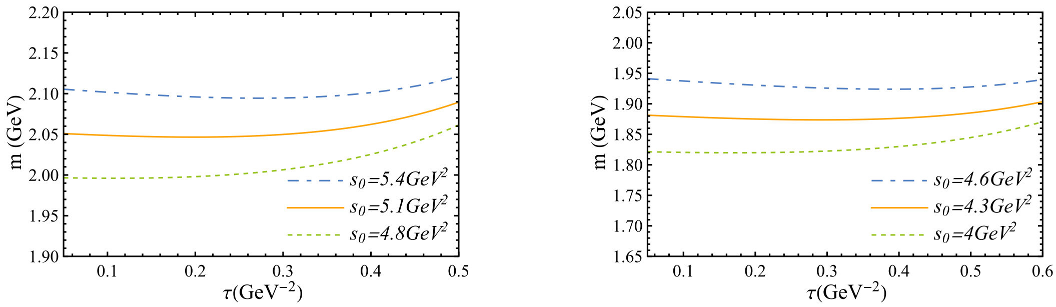

Fig. A1. (color online) Mass predictions for the currents η_1^{μ} (left) and η_2^{μ} (right).

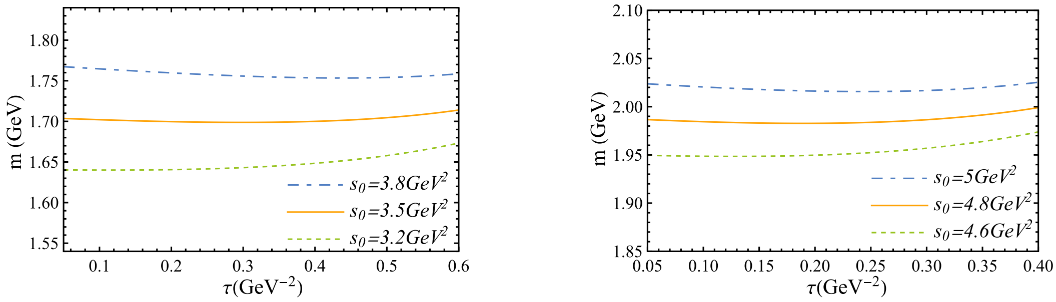

Fig. A2. (color online) Mass predictions for the currents η_3^μ (left) and η_4^μ (right).

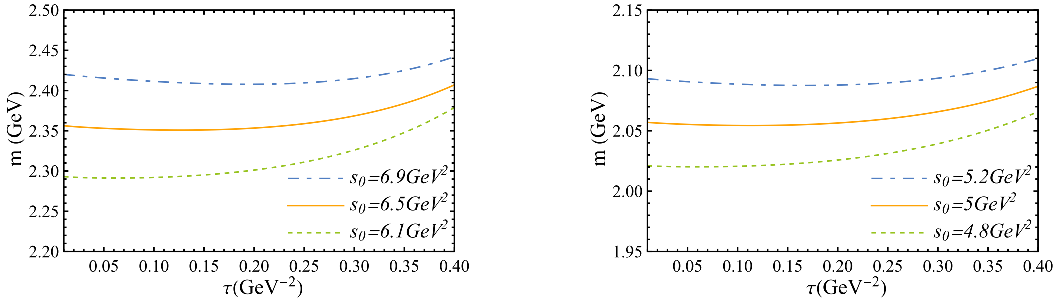

Fig. A3. (color online) Mass predictions for the currents η_5^μ (left) and η_6^μ (right).

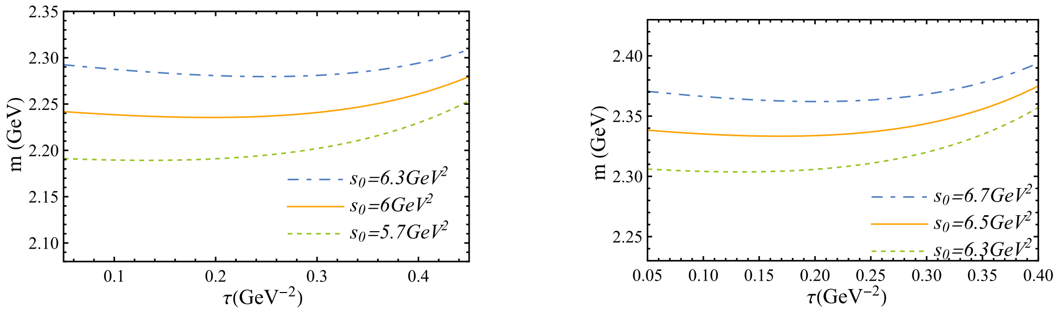

Fig. A4. (color online) Mass predictions for the currents η_7^μ (left) and η_8^μ (right).

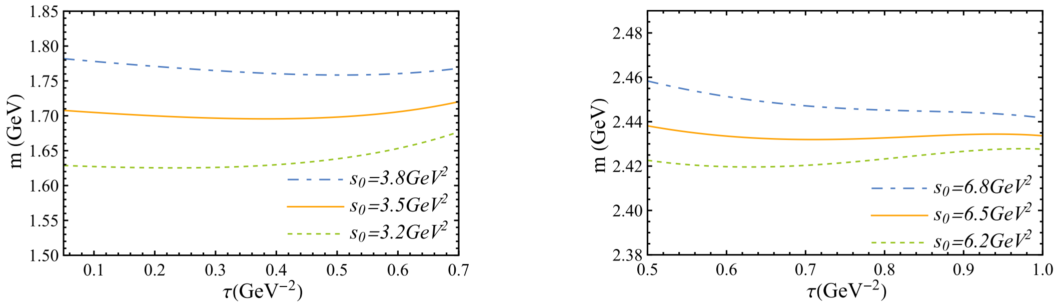

Fig. A5. (color online) Mass predictions for the currents J_1^μ (left) and $J_2^{\mu\nu}$ (right).

A.2. Renormalized operator currents

Renormalized operator current $J_2^{\mu\nu}$:

$$\left(\epsilon^{\mu\nu\rho\sigma}\bar{u}\gamma^5\gamma_\rho d\bar{d}\gamma_\sigma u\right)_r = \epsilon^{\mu\nu\rho\sigma}\left[\left(Z_2^{-2}+\frac{5C_F}{8\pi^2\varepsilon}g^2\right)\bar{u}\gamma_\rho\gamma^5 d\bar{d}\gamma_\sigma u - \frac{g^2}{8\pi^2\varepsilon}\bar{u}\gamma^5\gamma_\rho T^n d\bar{d}\gamma_\sigma\gamma^5 T^n u - \frac{\mathrm{i}}{48\pi^2\varepsilon}\epsilon_{\rho\sigma\beta\eta}\left(\bar{u}D_\alpha G^{\alpha\beta}\gamma^\eta u - \bar{d}D_\alpha G^{\alpha\beta}\gamma^\eta d\right)\right]. \quad \text{(A1)}$$

η_2^μ and η_4^μ can be renormalized through linear combinations of the following renormalization currents:

$$\begin{aligned}\left(u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_a\gamma_\nu\gamma^5 C\bar{d}_b^T\right)_r =& \left(Z_2^{-2}-\frac{4C_A^2-7}{32\pi^2\varepsilon C_A}g^2\right)u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_a\gamma_\nu\gamma^5 C\bar{d}_b^T-\frac{3g^2}{32\pi^2\varepsilon}u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_b\gamma_\nu\gamma^5 C\bar{d}_a^T\\&-\frac{3\mathrm{i}C_A g^2}{32\pi^2\varepsilon}u_a^T C d_b\,\bar{u}_a\gamma^\mu C\bar{d}_b^T+\frac{3\mathrm{i}g^2}{32\pi^2\varepsilon}u_a^T C d_b\,\bar{u}_b\gamma^\mu C\bar{d}_a^T,\end{aligned}\tag{A2}$$

$$\begin{aligned}\left(u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_b\gamma_\nu\gamma^5 C\bar{d}_a^T\right)_r =& \left(Z_2^{-2}-\frac{4C_A^2-7}{32\pi^2\varepsilon C_A}g^2\right)u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_b\gamma_\nu\gamma^5 C\bar{d}_a^T-\frac{3g^2}{32\pi^2\varepsilon}u_a^T C\sigma^{\mu\nu}\gamma^5 d_b\,\bar{u}_a\gamma_\nu\gamma^5 C\bar{d}_b^T\\&+\frac{3\mathrm{i}C_A g^2}{32\pi^2\varepsilon}u_a^T C d_b\,\bar{u}_b\gamma^\mu C\bar{d}_a^T-\frac{3\mathrm{i}g^2}{32\pi^2\varepsilon}u_a^T C d_b\,\bar{u}_a\gamma^\mu C\bar{d}_b^T-\frac{1}{48\pi^2\varepsilon}\left(\bar{u}D^\alpha G_{\alpha\beta}\sigma^{\beta\mu}u-\bar{d}D^\alpha G_{\alpha\beta}\sigma^{\beta\mu}d\right)\\&-\frac{\mathrm{i}}{16\pi^2\varepsilon}\left(\bar{u}D_\alpha G^{\alpha\mu}u-\bar{d}D_\alpha G^{\alpha\mu}d\right).\end{aligned}\tag{A3}$$

By replacing the d quark with an s quark, the operator currents η_6^μ and η_8^μ can be renormalized.

A.3. Correlation functions

$$\begin{aligned}\Pi_{\eta_1}^V =& q^8\left[\left(-\frac{79g_s^2}{2654208\pi^8}-\frac{1}{18432\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)-\frac{5g_s^2}{1327104\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)\right]+q^4\frac{g_s^2}{18432\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)\langle GG\rangle\\&-q^2\frac{1}{18\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle^2+\frac{1}{12\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle+\frac{1}{q^2}\left(\frac{5g_s^2}{864\pi^2}\langle GG\rangle\langle\bar{q}q\rangle^2-\frac{1}{48\pi^2}\langle\bar{q}Gq\rangle^2\right)\\&+\frac{1}{q^4}\left(-\frac{g_s^2}{576\pi^2}\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle-\frac{32}{81}g_s^2\langle\bar{q}q\rangle^4\right),\end{aligned}$$

$$\begin{aligned}\Pi_{\eta_2}^V =& q^8\left[\frac{65g_s^2}{1769472\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)+\left(-\frac{297929g_s^2}{557383680\pi^8}-\frac{1}{6144\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right]-q^4\frac{11g_s}{18432\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)\langle GG\rangle\\&-q^2\frac{1}{6\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle^2+\frac{1}{4\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle+\frac{1}{q^2}\left(-\frac{5g_s^2}{864\pi^2}\langle GG\rangle\langle\bar{q}q\rangle^2-\frac{1}{16\pi^2}\langle\bar{q}Gq\rangle^2\right)\\&+\frac{1}{q^4}\left(\frac{g_s^2}{576\pi^2}\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle-\frac{32g_s^2}{27}\langle\bar{q}q\rangle^4\right),\end{aligned}$$

$$\begin{aligned}\Pi_{\eta_3}^V =& q^8\left[\frac{5g_s^2}{884736\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)+q^8\left(-\frac{3503g_s^2}{39813120\pi^8}-\frac{1}{36864\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right]-q^4\frac{g_s^2}{18432\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)\langle GG\rangle\\&-q^2\frac{1}{36\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle^2+\frac{1}{24\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle+\frac{1}{q^2}\left(-\frac{5g_s^2}{864\pi^2}\langle GG\rangle\langle\bar{q}q\rangle^2-\frac{1}{96\pi^2}\langle\bar{q}Gq\rangle^2\right)\\&+\frac{1}{q^4}\left(\frac{g_s^2}{576\pi^2}\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle-\frac{16}{81}g_s^2\langle\bar{q}q\rangle^4\right),\end{aligned}$$

$$\begin{aligned}\Pi_{\eta_4}^V =& q^8\left[\frac{5g_s^2}{1769472\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)+\left(-\frac{26021g_s^2}{557383680\pi^8}-\frac{1}{12288\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right]-q^4\frac{g_s^2}{18432\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)\langle GG\rangle\\&-q^2\frac{1}{12\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle^2+\frac{1}{8\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle+\frac{1}{q^2}\left(\frac{5g_s^2}{864\pi^2}\langle GG\rangle\langle\bar{q}q\rangle^2-\frac{1}{32\pi^2}\langle\bar{q}Gq\rangle^2\right)\\&+\frac{1}{q^4}\left(-\frac{g_s^2}{576\pi^2}\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle-\frac{16}{27}g_s^2\langle\bar{q}q\rangle^4\right),\end{aligned}$$

$$
\begin{aligned}
\Pi^V_{\eta_5} = {} & q^8\left[\left(-\frac{2243g_s^2}{79626240\pi^8}-\frac{1}{18432\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)-\frac{7g_s^2}{1769472\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)\right]+q^6\frac{17m_s^2}{7680\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)+q^4\log\left(-\frac{q^2}{\mu^2}\right)\\
& \times\left(-\frac{m_s\langle\bar{s}s\rangle}{48\pi^4}+\frac{m_s\langle\bar{q}q\rangle}{96\pi^4}+\frac{g_s^2\langle GG\rangle}{18432\pi^6}\right)+q^2\log\left(-\frac{q^2}{\mu^2}\right)\left(\frac{m_s\langle\bar{s}Gs\rangle}{96\pi^4}-\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{9\pi^2}+\frac{\langle\bar{s}s\rangle^2}{36\pi^2}-\frac{m_s\langle\bar{q}Gq\rangle}{48\pi^4}+\frac{\langle\bar{q}q\rangle^2}{36\pi^2}-\frac{g_s^2m_s^2\langle GG\rangle}{4608\pi^6}\right)\\
& +\log\left(-\frac{q^2}{\mu^2}\right)\left(\frac{\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{12\pi^2}-\frac{\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle}{24\pi^2}+\frac{\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{12\pi^2}+\frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{2\pi^2}-\frac{m_s^2\langle\bar{s}s\rangle^2}{24\pi^2}-\frac{g_s^2m_s\langle GG\rangle\langle\bar{q}q\rangle}{256\pi^4}-\frac{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{24\pi^2}+\frac{m_s^2\langle\bar{q}q\rangle^2}{6\pi^2}\right)\\
& +\frac{1}{q^2}\left(-\frac{\langle\bar{q}Gq\rangle\langle\bar{s}Gs\rangle}{24\pi^2}-\frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{6\pi^2}+\frac{\langle\bar{s}Gs\rangle^2}{96\pi^2}+\frac{5g_s^2\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{864\pi^2}-\frac{m_s^2\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{4\pi^2}-\frac{4}{9}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle^2-\frac{2}{3}m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle\right.\\
& \left.+\frac{5g_s^2m_s\langle GG\rangle\langle\bar{q}Gq\rangle}{4608\pi^4}+\frac{\langle\bar{q}Gq\rangle^2}{96\pi^2}\right)+\frac{1}{q^4}\left(-\frac{g_s^2\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{1152\pi^2}-\frac{m_s^2\langle\bar{q}Gq\rangle\langle\bar{s}Gs\rangle}{24\pi^2}+\frac{2}{9}m_s\langle\bar{q}q\rangle^2\langle\bar{s}Gs\rangle-\frac{1}{9}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle\right.\\
& \left.-\frac{g_s^2\langle GG\rangle\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{1152\pi^2}-\frac{32}{81}g_s^2\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle^2-\frac{1}{9}m_s\langle\bar{s}s\rangle^2\langle\bar{q}Gq\rangle+\frac{5}{9}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle+\frac{m_s^2\langle\bar{q}Gq\rangle^2}{24\pi^2}\right),
\end{aligned}
$$

$$
\begin{aligned}
\Pi^V_{\eta_6} = {} & q^8\left[\frac{65g_s^2}{1769472\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)+\left(-\frac{297929g_s^2}{557383680\pi^8}-\frac{1}{6144\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right]+q^6\frac{17m_s^2}{2560\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)+q^4\log\left(-\frac{q^2}{\mu^2}\right)\\
& \times\left(-\frac{m_s\langle\bar{s}s\rangle}{16\pi^4}+\frac{m_s\langle\bar{q}q\rangle}{32\pi^4}-\frac{11g_s^2\langle GG\rangle}{18432\pi^6}\right)+q^2\log\left(-\frac{q^2}{\mu^2}\right)\left(\frac{m_s\langle\bar{s}Gs\rangle}{32\pi^4}-\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{3\pi^2}+\frac{\langle\bar{s}s\rangle^2}{12\pi^2}-\frac{m_s\langle\bar{q}Gq\rangle}{16\pi^4}+\frac{\langle\bar{q}q\rangle^2}{12\pi^2}+\frac{109g_s^2m_s^2\langle GG\rangle}{18432\pi^6}\right)\\
& +\log\left(-\frac{q^2}{\mu^2}\right)\left(\frac{\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{4\pi^2}-\frac{\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle}{8\pi^2}-\frac{5g_s^2m_s\langle GG\rangle\langle\bar{s}s\rangle}{256\pi^4}+\frac{\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{4\pi^2}+\frac{3m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{2\pi^2}-\frac{m_s^2\langle\bar{s}s\rangle^2}{8\pi^2}+\frac{3g_s^2m_s\langle GG\rangle\langle\bar{q}q\rangle}{128\pi^4}\right.\\
& \left.-\frac{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{8\pi^2}+\frac{m_s^2\langle\bar{q}q\rangle^2}{2\pi^2}\right)+\frac{1}{q^2}\left(\frac{25g_s^2m_s\langle GG\rangle\langle\bar{s}Gs\rangle}{4608\pi^4}-\frac{\langle\bar{q}Gq\rangle\langle\bar{s}Gs\rangle}{8\pi^2}-\frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{2\pi^2}+\frac{\langle\bar{s}Gs\rangle^2}{32\pi^2}-\frac{5g_s^2\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{144\pi^2}\right.\\
& \left.+\frac{25g_s^2\langle GG\rangle\langle\bar{s}s\rangle^2}{1728\pi^2}-\frac{3m_s^2\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{4\pi^2}-\frac{4}{3}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle^2-2m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle-\frac{5g_s^2m_s\langle GG\rangle\langle\bar{q}Gq\rangle}{768\pi^4}+\frac{25g_s^2\langle GG\rangle\langle\bar{q}q\rangle^2}{1728\pi^2}+\frac{\langle\bar{q}Gq\rangle^2}{32\pi^2}\right)\\
& +\frac{1}{q^4}\left(\frac{g_s^2\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{192\pi^2}-\frac{5g_s^2\langle GG\rangle\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle}{1152\pi^2}-\frac{m_s^2\langle\bar{q}Gq\rangle\langle\bar{s}Gs\rangle}{8\pi^2}+\frac{2}{3}m_s\langle\bar{q}q\rangle^2\langle\bar{s}Gs\rangle-\frac{1}{3}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle\right.\\
& -\frac{5g_s^2m_s^2\langle GG\rangle\langle\bar{s}s\rangle^2}{1152\pi^2}+\frac{g_s^2\langle GG\rangle\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{192\pi^2}-\frac{32}{27}g_s^2\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle^2-\frac{1}{3}m_s\langle\bar{s}s\rangle^2\langle\bar{q}Gq\rangle+\frac{5}{3}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle\\
& \left.-\frac{5g_s^2\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{1152\pi^2}+\frac{m_s^2\langle\bar{q}Gq\rangle^2}{8\pi^2}\right),
\end{aligned}
$$

$$
\begin{aligned}
\Pi^V_{\eta_7} = {} & q^8\left[\frac{5g_s^2}{1769472\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)+\left(-\frac{3503g_s^2}{79626240\pi^8}-\frac{1}{36864\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right]+q^6\frac{17m_s^2}{15360\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)+q^4\log\left(-\frac{q^2}{\mu^2}\right)\\
& \times\left(-\frac{m_s\langle\bar{s}s\rangle}{96\pi^4}+\frac{m_s\langle\bar{q}q\rangle}{192\pi^4}-\frac{g_s^2\langle GG\rangle}{18432\pi^6}\right)+q^2\log\left(-\frac{q^2}{\mu^2}\right)\left(\frac{m_s\langle\bar{s}Gs\rangle}{192\pi^4}-\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{18\pi^2}+\frac{\langle\bar{s}s\rangle^2}{72\pi^2}-\frac{m_s\langle\bar{q}Gq\rangle}{96\pi^4}+\frac{\langle\bar{q}q\rangle^2}{72\pi^2}+\frac{g_s^2m_s^2\langle GG\rangle}{4608\pi^6}\right)\\
& +\log\left(-\frac{q^2}{\mu^2}\right)\left(\frac{\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{24\pi^2}-\frac{\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle}{48\pi^2}+\frac{\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{24\pi^2}+\frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{4\pi^2}-\frac{m_s^2\langle\bar{s}s\rangle^2}{48\pi^2}+\frac{g_s^2m_s\langle GG\rangle\langle\bar{q}q\rangle}{256\pi^4}-\frac{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{48\pi^2}+\frac{m_s^2\langle\bar{q}q\rangle^2}{12\pi^2}\right)\\
& +\frac{1}{q^2}\left(-\frac{\langle\bar{q}Gq\rangle\langle\bar{s}Gs\rangle}{48\pi^2}-\frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{12\pi^2}+\frac{\langle\bar{s}Gs\rangle^2}{192\pi^2}+\frac{5g_s^2\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{864\pi^2}-\frac{m_s^2\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{8\pi^2}-\frac{2}{9}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle^2-\frac{1}{3}m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle\right.\\
& \left.-\frac{5g_s^2m_s\langle GG\rangle\langle\bar{q}Gq\rangle}{4608\pi^4}+\frac{\langle\bar{q}Gq\rangle^2}{192\pi^2}\right)+\frac{1}{q^4}\left(-\frac{m_s^2\langle\bar{q}Gq\rangle\langle\bar{s}Gs\rangle}{48\pi^2}+\frac{1}{9}m_s\langle\bar{q}q\rangle^2\langle\bar{s}Gs\rangle-\frac{1}{18}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle\right.\\
& \left.+\frac{g_s^2\langle GG\rangle\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{1152\pi^2}-\frac{16}{81}g_s^2\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle^2-\frac{1}{18}m_s\langle\bar{s}s\rangle^2\langle\bar{q}Gq\rangle+\frac{5}{18}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle+\frac{g_s^2\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{1152\pi^2}+\frac{m_s^2\langle\bar{q}Gq\rangle^2}{48\pi^2}\right),
\end{aligned}
$$

$$
\begin{aligned}
\Pi^V_{\eta_8} =\; & q^8\left[\frac{5g_s^2}{1769472\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)+\left(-\frac{26021g_s^2}{557383680\pi^8}-\frac{1}{12288\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right]+q^6\frac{17m_s^2}{5120\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)+q^4\log\left(-\frac{q^2}{\mu^2}\right)\\
& \times\left(-\frac{m_s\langle\bar{s}s\rangle}{32\pi^4}+\frac{m_s\langle\bar{q}q\rangle}{64\pi^4}-\frac{g_s^2\langle GG\rangle}{18432\pi^6}\right)+q^2\log\left(-\frac{q^2}{\mu^2}\right)\left(\frac{m_s\langle\bar{s}Gs\rangle}{64\pi^4}-\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{6\pi^2}+\frac{\langle\bar{s}s\rangle^2}{24\pi^2}-\frac{m_s\langle\bar{q}Gq\rangle}{32\pi^4}+\frac{\langle\bar{q}q\rangle^2}{24\pi^2}+\frac{17g_s^2m_s^2\langle GG\rangle}{18432\pi^6}\right)\\
& +\log\left(-\frac{q^2}{\mu^2}\right)\left(\frac{\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{8\pi^2}-\frac{\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle}{16\pi^2}-\frac{g_s^2m_s\langle GG\rangle\langle\bar{s}s\rangle}{256\pi^4}+\frac{\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{8\pi^2}+\frac{3m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{4\pi^2}-\frac{m_s^2\langle\bar{s}s\rangle^2}{16\pi^2}-\frac{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{16\pi^2}+\frac{m_s^2\langle\bar{q}q\rangle^2}{4\pi^2}\right)\\
& +\frac{1}{q^2}\left(\frac{5g_s^2m_s\langle GG\rangle\langle\bar{s}Gs\rangle}{4608\pi^4}-\frac{\langle\bar{q}Gq\rangle\langle\bar{s}Gs\rangle}{16\pi^2}-\frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}Gs\rangle}{4\pi^2}+\frac{\langle\bar{s}Gs\rangle^2}{64\pi^2}+\frac{5g_s^2\langle GG\rangle\langle\bar{s}s\rangle^2}{1728\pi^2}-\frac{3m_s^2\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle}{8\pi^2}-\frac{2}{3}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle^2\right.\\
& \left.-m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle+\frac{5g_s^2\langle GG\rangle\langle\bar{q}q\rangle^2}{1728\pi^2}+\frac{\langle\bar{q}Gq\rangle^2}{64\pi^2}\right)+\frac{1}{q^4}\left(-\frac{g_s^2\langle GG\rangle\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle}{1152\pi^2}-\frac{m_s^2\langle\bar{q}Gq\rangle\langle\bar{s}Gs\rangle}{16\pi^2}+\frac{1}{3}m_s\langle\bar{q}q\rangle^2\langle\bar{s}Gs\rangle\right.\\
& -\frac{1}{6}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{s}Gs\rangle-\frac{g_s^2m_s^2\langle GG\rangle\langle\bar{s}s\rangle^2}{1152\pi^2}-\frac{16}{27}g_s^2\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle^2-\frac{1}{6}m_s\langle\bar{s}s\rangle^2\langle\bar{q}Gq\rangle+\frac{5}{6}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{q}Gq\rangle\\
& \left.-\frac{g_s^2\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{1152\pi^2}+\frac{m_s^2\langle\bar{q}Gq\rangle^2}{16\pi^2}\right),
\end{aligned}
$$

$$
\begin{aligned}
\Pi^V_{J_1} =\; & q^8\left[\frac{25g_s^2}{14155776\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)+\left(-\frac{41221g_s^2}{1114767360\pi^8}-\frac{11}{1179648\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right]-q^4\frac{g_s^2}{32768\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)\langle GG\rangle\\
& +q^2\frac{1}{128\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle^2-\frac{3}{256\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle+\frac{1}{q^2}\left(\frac{3}{1024\pi^2}\langle\bar{q}Gq\rangle^2+\frac{5g_s^2}{1536\pi^2}\langle GG\rangle\langle\bar{q}q\rangle^2\right)\\
& +\frac{1}{q^4}\frac{g_s^2}{3072\pi^2}\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle,
\end{aligned}
$$

$$
\begin{aligned}
\Pi^V_{J_2} =\; & q^8\left[\left(-\frac{77g_s^2}{2764800\pi^8}-\frac{1}{30720\pi^6}\right)\log\left(-\frac{q^2}{\mu^2}\right)-\frac{g_s^2}{829440\pi^8}\log^2\left(-\frac{q^2}{\mu^2}\right)\right]+q^4\frac{g_s^2}{3072\pi^6}\log\left(-\frac{q^2}{\mu^2}\right)\langle GG\rangle\\
& +q^2\left[\left(-\frac{5\gamma g_s^2}{216\pi^4}-\frac{5g_s^2}{1296\pi^4}\right)\log\left(-\frac{q^2}{\mu^2}\right)-\frac{5g_s^2}{216\pi^4}\log^2\left(-\frac{q^2}{\mu^2}\right)\right]\langle\bar{q}q\rangle^2\\
& +\frac{1}{48\pi^2}\log\left(-\frac{q^2}{\mu^2}\right)\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle+\frac{1}{q^2}\left(-\frac{g_s^2}{864\pi^2}\langle GG\rangle\langle\bar{q}q\rangle^2-\frac{1}{192\pi^2}\langle\bar{q}Gq\rangle^2\right)+\frac{1}{q^4}\frac{g_s^2}{1728\pi^2}\langle GG\rangle\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle.
\end{aligned}
$$

A.4. Mass correction

The figure compares the masses determined at the NLO and LO. For j_1^μ and $j_2^{\mu\nu}$, the previous study also considered the zero mode contribution; therefore, they are not compared here.

Table A1. Comparison between the NLO-corrected and LO masses. m' and m denote the LO and NLO masses, respectively, and Δm represents the mass correction.

	η_1	η_2	η_3	η_4	η_5	η_6	η_8	η_8
m'	1.70	1.60	1.60	1.70	2.10	2.00	1.90	2.00
m	2.05	1.88	1.70	1.99	2.36	2.06	2.24	2.34
Δm	0.35	0.28	0.1	0.29	0.26	0.06	0.34	0.34

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