Exploring toroidal α-cluster configurations in ^{28}Si within the 7α cluster model

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Abstract: Two 7α cluster configurations in ^{28}Si are investigated within the framework of the generator coordinate method (GCM) with Brink-Bloch 7α wave functions by taking the edge length as the generator coordinate, including a uniform toroidal configuration and a disk-like configuration. Within the framework of the 7α cluster model with imposed geometric symmetry, we observe that a uniform toroidal configuration may emerge at approximately 40 MeV above the 7α threshold. This provides a cluster-model perspective that complements some mean-field descriptions of toroidal structure in ^{28}Si.

Keywords: ^{28}Si, alpha cluster model, 7α, GCM Brink

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I. INTRODUCTION

Many light nuclei are known to exhibit exotic geometries and density distributions such as linear chains [1−3], toroidal configurations [4−6], and other novel states [7−11] at elevated excitation energies or under high-spin conditions. Investigations of such configurations provide crucial constraints on nuclear forces under extreme conditions and advance our understanding of the evolution of nuclear structure and dynamical processes [12−17].

Toroidal configurations in nuclei have been investigated for several decades. Wheeler [18] proposed that nuclei might assume toroidal configurations under certain conditions. Building on this idea, Wong applied the liquid-drop model to investigate the possible existence of toroidal nuclei and identified the ranges of mass numbers and angular momenta in which such configurations could emerge [19−21]. Subsequently, the existence of toroidal isomers in various light nuclei has been predicted using a range of theoretical approaches [4, 22−25].

As the lightest nucleus predicted by Wong to exhibit a stable toroidal configuration, ^{28}Si has been the subject of extensive theoretical and experimental investigations in recent years. For example, in terms of experimental investigations, both Cao *et al*. and Hannaman *et al*. have explored toroidal states in ^{28}Si by analyzing resonant peaks in the excitation energy spectrum of the inverse kinematics reaction ^{12}C(^{28}Si,7α)^{12}C at 35 MeV/u. Cao *et al*. reported potential toroidal resonances at 114,126, and 138 MeV [5]. By contrast, Hannaman *et al*. reported that no clearly statistically significant resonant peaks could be discerned above the background in the 7α excitation spectrum [26, 27]. The observed discrepancy between these experimental results is likely attributable to differences in the kinematic acceptances of the two detection systems [28]. In terms of theoretical analyses, Ren *et al*. conducted a systematic study of high-spin toroidal states in ^{28}Si [29] within the framework of cranking covariant density functional theory (CDFT) on a three-dimensional lattice. They reproduced the experimentally observed resonances and demonstrated pronounced α-cluster features through the α-localization function. Zheng *et al*. employed a hybrid α cluster model [30] to explain the resonant peaks observed in the 7α disassembly of ^{28}Si and linked them to toroidal high-spin configurations.

Although these experimental and theoretical studies suggest the possible existence of toroidal configurations in ^{28}Si, their actual presence still remains to be firmly established. Given the potential importance of α-clusters as indicated by previous investigations of toroidal configurations in ^{28}Si and in view of the significant role in various studies on nuclear structure [31−34], examining such an exotic nuclear structure from the perspective of microscopic α-clusters is meaningful.

Considering that our interest lies in the toroidal configurations of ^{28}Si, we assume that the relevant states consist of several α clusters arranged in prescribed geometrical patterns to allow reasonably relative predictions of

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certain physical properties as a practical approach [35, 36]. Building on this idea, we apply the generator coordinate method (GCM) constructed from fully antisymmetrized 7α-cluster wave functions and introduce generator coordinates tailored to toroidal geometries to explore the possible formation of toroidal configurations in ^{28}Si.

II. MODEL DESCRIPTION

We perform GCM calculations to investigate a toroidal configuration of 7α clusters in which the clusters are located at the vertices of a regular heptagon (referred to as a toroidal configuration). For comparison, we also consider a disk-like configuration in which 6α clusters are placed at the vertices of a regular hexagon and the seventh is located at the center (a disk-like configuration).

We adopt Brink-Bloch 7α Slater determinants as intrinsic basis states with localized Gaussian packets [37],

$$\Phi^{\rm B}(\boldsymbol{R}_1,\boldsymbol{R}_2,\cdots,\boldsymbol{R}_7)=\mathcal{A}[\phi_1(\boldsymbol{R}_1)\cdots\phi_4(\boldsymbol{R}_1)\phi_5(\boldsymbol{R}_2)\cdots\phi_{28}(\boldsymbol{R}_7)], \tag{1}$$

where $\mathcal{A}$ represents the antisymmetrization operator, and the wave function for the k-th nucleon is defined as a Gaussian wave packet

$$\phi_k(\boldsymbol{R}_j)=\frac{1}{(\pi b^2)^{3/4}}\exp\left[-\frac{1}{2b^2}(\boldsymbol{r}_k-\boldsymbol{R}_j)^2\right]\chi_k\tau_k. \tag{2}$$

Here, $\boldsymbol{r}_k$ denotes the coordinate of the k-th nucleon, $\boldsymbol{R}_j$ is a generator coordinate specifying the center of the Gaussian wave packet for the j-th α cluster, and χ_k and τ_k respectively represent its spin and isospin. The oscillator parameter for the single-particle wave functions is set to b = 1.44 fm.

We directly use each edge length as the generator coordinate as shown in Fig. 1, and the distance between adjacent α clusters on the regular polygon (the edge length) is denoted as ρ. The 7α clusters are placed at positions $\boldsymbol{R}_j$ (j = 1,…,7), where $\boldsymbol{R}_j$ denotes the vectors defining the specific geometric distributions ($\sum_j \boldsymbol{R}_j=0$ to fix the center of mass). We vary the edge length ρ from 0.8 to 9.5 fm in steps of 0.3 fm, which yields 30 basis points for each configuration.

According to the GCM framework, the total wave function of ^{28}Si can be written as the superposition of angular momentum- and parity-projected Brink wave functions:

$$\Psi^{J\pi}_M=\sum_{i,K}c_{i,K}\hat{P}^J_{MK}\hat{P}^{\pi}\Phi^{\rm B}(\{\boldsymbol{R}\}_i), \tag{3}$$

where $\hat{P}^J_{MK}$ and $\hat{P}^{\pi}$ are the angular momentum and parity projectors, respectively. $\Phi^{\rm B}(\{\boldsymbol{R}\}_i)$ indicates a Brink wave function in Eq. (1) with a specified set of generator coordinates $\{\boldsymbol{R}\}_i$. The coefficients $c_{i,K}$ are determined by solving the Hill-Wheeler equation [38]. The Hamiltonian $\hat{H}$ of the system includes kinetic, effective nucleon-nucleon interaction, and Coulomb parts.

$$\hat{H}=-\frac{\hbar^2}{2m}\sum_i\nabla_i^2-T_{\rm c.m.}+\hat{V}_{NN}+\hat{V}_{\rm C}. \tag{4}$$

In the present work, we use the Tohsaki No.1 effective nucleon-nucleon interaction as the central N-N potential [39].

III. RESULTS AND DISCUSSION

The toroidal configuration arranged as a regular heptagon exhibits D_{7h} point group symmetry, whereas the disk-like configuration (a centered hexagon) is characterized by D_{6h} symmetry. These specific geometric symmetries impose strong constraints on the intrinsic quantum numbers, excitation energies, and resultant rotational bands. To investigate the structural stability of these distinct α-cluster configurations, we perform calculations for the energy as a function of the edge length (ρ) to search for local energy minima.

Figure 2 displays the resultant potential energy curves

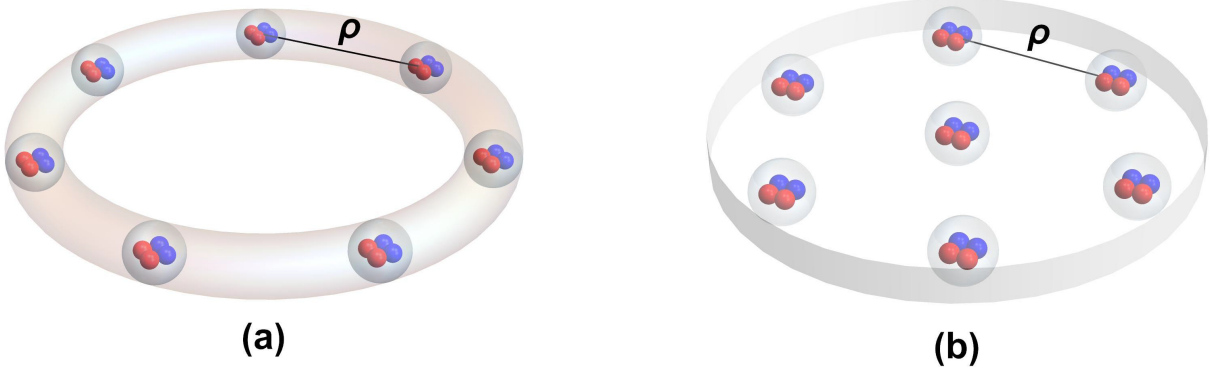

Fig. 1. (color online) Geometric arrangements of the 7α clusters: (a) toroidal configuration; (b) disk-like configuration.

as functions of the edge length ρ for the two distinct α-cluster geometric arrangements. Here, the reference of the energy is taken as the 7α breakup energy. In both cases, the surfaces develop a clear potential pocket at intermediate ρ and an outer barrier at larger ρ, which indicates partial shape stabilization along this one-dimensional coordinate. The existence of such a local minimum (pocket) is a necessary condition for the formation of a metastable cluster structure and its corresponding resonance; this approach to configuration analysis is systematically discussed in works on exotic states by Tohsaki *et al.* [35, 36].

The stability of these exotic structures is further enhanced by the centrifugal potential, particularly at high-spin states. Since the moment of inertia $\mathcal{I}$ for the toroidal configuration scales with the square of the edge length ($\mathcal{I} \propto \rho^2$), the centrifugal term $\hbar^2 J(J+1)/2\mathcal{I}$ scales as ρ^{-2}. This dependence introduces a strong repulsive potential at short distances, which significantly elevates the inner barrier of the potential pocket. Consequently, higher angular momenta effectively suppress the breathing-mode decay toward the compact limit. This dynamical stabilization mechanism corroborates the self-consistent mean-field predictions reported in Ref. [24], where toroidal isomers were found to be stabilized specifically at high spin.

Compared with the toroidal configuration, the disk-like configuration develops a substantially deeper pocket that emerges at a smaller value of ρ (indicative of a more compact arrangement) with an excitation energy of approximately 14 MeV above the experimental 7α breakup threshold. This energy lowering can be attributed primarily to the increased coordination number of the central α, which maximizes the short-range nuclear attraction and overrides the increased Coulomb repulsion.

It should be noted that the presence of a pocket alone does not establish a physical resonance; confirmation requires enlarged configuration spaces and some other quantities. Nevertheless, the calculated pockets delineate an experimentally relevant energy window for toroidal or disk-like α-cluster candidates.

To obtain more refined results, we further perform angular momentum- and parity-projected GCM mixing along the generator coordinate ρ (and across both geometric configurations). This procedure removes the explicit ρ-dependence of the wave functions and enables the extraction of correlated cluster candidates for $J^\pi = 0^+, 2^+, 4^+$.

For the toroidal configuration, the obtained energies of the 0^+, 2^+, and 4^+ states are –151.22, –150.66, and –149.38 MeV, respectively, while for the disk-like configuration, the corresponding energies of the 0^+, 2^+, and 4^+ states are –179.28, –178.62, and –177.06 MeV, respectively. The calculated cluster states are compared with the available experimental data and other theoretical predictions in Fig. 3.

In terms of experimental investigations, both Cao *et al.* [5] and Hannaman *et al.* [26, 27] faced limitations that prevented direct observation of the initial geometric configuration or determination of its angular momentum, and their identification of possible resonance peaks necessarily involved a degree of dependence on prior theoretical assumptions. Nevertheless, the energy intervals associated with the inferred toroidal configurations remain broadly consistent across various studies based on these two datasets [5, 28, 40]. Cao *et al.* [5] reported three resonance peaks extracted from their analysis; these values are listed in the first category. Hannaman *et al.* [26, 27] did not identify resonance peaks in their original analysis; however, subsequent studies have suggested that several peaks may indeed be present in their dataset [40]. More recently, Depastas *et al.* [28] reanalyzed both experiments using deep learning-based analysis trained on data from the rotating silicon HαC system and extracted additional resonance peaks from each dataset. The peaks obtained by Depastas *et al.* from the dataset of Cao *et al.* are listed in the second category, whereas those from the dataset of Hannaman *et al.* are shown in the third category. The fourth and fifth categories present our theoretical predictions: the results for the toroidal configurations are listed in the fourth category, whereas those for the disk-like configurations are shown in the fifth. Categories six through seven summarize some additional mean-field model theoretical calculations reported in the literature, including the result of the HFB calculations by Staszczak *et al.* [24] (sixth category) and the result of CDFT calculations by Ren *et al.* [29] (seventh category).

We first examine the numerical stability of the resulting levels with respect to enlarging the basis set. We found that the spectra shown in Fig. 3 remain robust within the tested ranges. Since the present 7α model space is constructed under specific geometrical constraints rather than to reproduce the exact ground state of ^{28}Si, the theor-

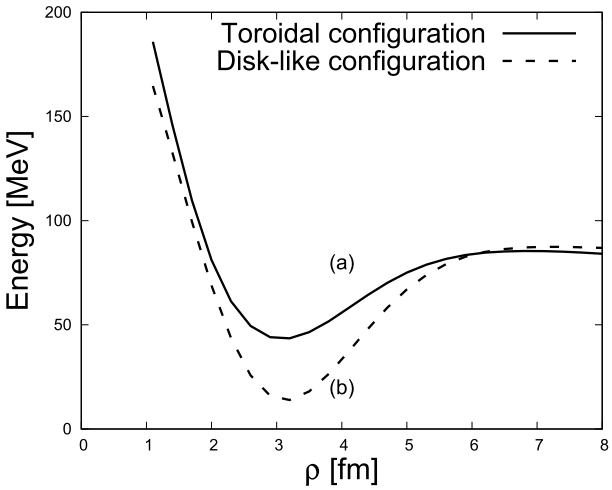

Fig. 2. Energy as a function of the edge length ρ, (a) toroidal configuration; (b) disk-like configuration.

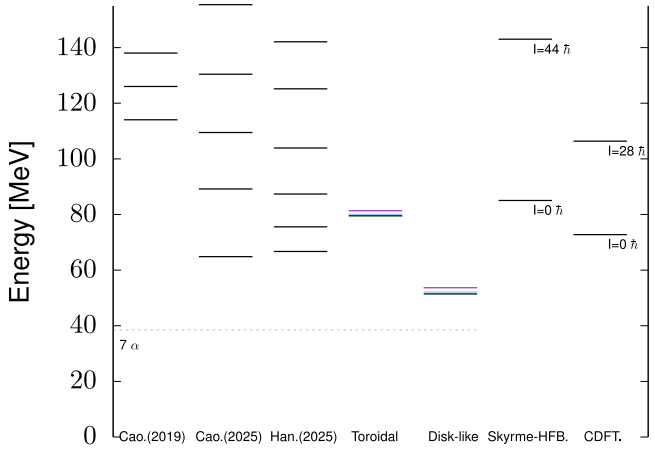

Fig. 3. (color online) The energy spectra of ^{28}Si obtained from the experiment by Cao *et al.* [5], analysis with artificial intelligence [28], 7α GCM calculation after superposition including 0^+ (black line), 2^+ (blue line), and 4^+ (purple line) states, Skyrme-Hartree-Fock-Bogoliubov (Skyrme-HFB) [24] and covariant density functional theory (CDFT) [29]. All energies are quoted relative to the 7α breakup threshold, which is marked by the dashed lines.

etical 7α threshold obtained from the GCM calculation is shifted to coincide with the experimental value. The ground states for the other theoretical calculations (which do not incorporate a 7α threshold) are aligned with the experimental ground state. This alignment is employed to ensure a consistent comparison of relative excitation energies between various theoretical models and the experimental data.

The lowest 0^+ candidate for the uniform toroidal configuration is found to appear at approximately 40 MeV above the 7α threshold. This energy position is notably close to the $I = 0$ extrapolation of the toroidal bands predicted by both Skyrme-HFB [24] and CDFT [29] calculations. While mean-field models typically stabilize toroidal isomers only at high spin due to the centrifugal barrier, our GCM result suggests that the observed 0^+ state may serve as the bandhead of this exotic toroidal rotational sequence. The consistency between our microscopic cluster result and the mean-field predictions lends support to the idea that a toroidal shape minimum exists, possessing intrinsic stability against the breathing mode. Furthermore, this energy region seems to be generally consistent with recent experimental signals observed in the 7α breakup channel of ^{28}Si [28, 40]. The rotational band structures, 2^+ and 4^+, are extracted from our projected GCM calculation. Due to the computational complexity of the current model space, we are limited to discussing only these low-spin states and cannot explore the high-spin excited states seen in the mean-field predictions.

Comparing the two geometric configurations, the toroidal configuration is significantly higher in energy (by approximately 28 MeV) than the disk-like configuration (14 MeV above the threshold). This large energy separation can be understood as the substantial energy cost required to form the central hole of the torus: it primarily results from the loss of significant nuclear attraction that would otherwise be provided by a central α cluster, an effect that dominates over any reduction in Coulomb repulsion achieved by distributing the clusters on a larger periphery. While the disk-like configuration is energetically lower and asymmetric or non-planar cluster structures may generally be more favorable, our present investigation is specifically directed toward geometric cluster isomers in ^{28}Si with high D_{7h} or D_{6h} symmetry, most notably the D_{7h} toroidal configuration. For this reason, other asymmetric or non-planar low-energy configurations are not discussed in detail here.

Despite these findings, which suggest the possible excitation energy region for the toroidal structure, the present model involves some inherent limitations that pose some notable challenges for future research. (1) Limited configuration space: We rely on a very simple symmetry assumption for the geometric arrangements (uniform heptagon and centered hexagon). This approach ignores the potential influence of other complex configurations (such as nonuniform toroidal or fully three-dimensional cluster arrangements) and their mixing effects. (2) Absence of spin-orbit interaction: The nα cluster assumption inherently neglects nucleon-level spin-orbit interaction. This simplification makes it relatively difficult to describe the ground- and higher-J states accurately. For example, the calculated 2^+ and 4^+ energy spacings and rotational band structures are primarily determined by the purely geometric moment of inertia of the clusters, which could potentially overlook the crucial effects of spin coupling among nucleons within or near the cluster boundaries. (3) Continuum effects: Since the calculated resonant states appear relatively high above the 7α breakup threshold, they are embedded in the continuum. The current constrained GCM method does not treat the continuum effects appropriately and thus cannot provide reliable estimates for the decay widths of these resonances, which are essential for a precise comparison with experimental observation. Thus, resonance methods should be adopted to address the coupling to the continuum.

IV. SUMMARY

In this study, we explore the possible existence of a toroidal α-cluster structure in ^{28}Si using a microscopic 7α cluster GCM framework. By employing constrained toroidal and disk-like edge lengths as generator coordinates, we extract correlated cluster candidate states with low angular momenta ($J^\pi = 0^+, 2^+, 4^+$). The results indicate that the lowest 0^+ candidate of the uniform toroidal configuration is located around 40 MeV above the 7α breakup threshold. This energy placement shows reasonable con-

sistency with the intrinsic energy of toroidal structures predicted by mean-field calculations and is generally comparable to possible resonant signals observed in recent experiments. It should be emphasized, however, that the present GCM approach relies on highly simplified 7α geometric constraints, which limit its predictive power for a complete description of the continuum and structural mixing. Future research should expand the configuration space, incorporate continuum effects to determine decay widths, and investigate spin-orbit coupling effects for a more complete description of these highly excited states.

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