

Probing type II seesaw leptogenesis through lepton flavor violation*

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Abstract: Lepton flavor violation (LFV) offers a powerful probe of physics beyond the Standard Model, particularly in models addressing neutrino masses and the baryon asymmetry of the universe. In this study, we investigate LFV processes within the framework of type II seesaw leptogenesis, where the Standard Model is extended by an $SU(2)_L$ triplet Higgs field. We focus on key LFV processes, including $\mu^+ \rightarrow e^+ \gamma$, $\mu^+ \rightarrow e^+ e^- e^+$, and $\mu \rightarrow e$ conversion in nuclei, deriving stringent constraints on the parameter space from current experimental data. We scan the 3σ range of neutrino oscillation parameters and identify the most conservative bounds consistent with existing measurements. Our results reveal that the MEG experiment currently provides the most stringent constraints in the normal ordering (NO) scenario, while the SINDRUM experiment offers comparable sensitivity in the inverted ordering (IO) case. Future experiments, such as MEG II, Mu3e, Mu2e, and COMET, are predicted to significantly improve the sensitivity, testing larger regions of the parameter space.

Keywords: lepton flavor violation, Type II seesaw, leptogenesis

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I. INTRODUCTION

It is well-known that, in the Standard Model, the lepton flavors L_e, L_μ, L_τ are conserved, arising as accidental symmetries. However, with the discovery of neutrino oscillations [1–7], it is realized that these lepton flavors are not symmetric in nature. Although charged lepton flavor violation (CLFV) has not yet been observed, numerous neutrino mass models have provided such sources. Therefore, the CLFV experiments have become excellent probes of new physics, elucidating the origin of neutrino masses [8–12].

Among the various neutrino mass models, the seesaw models are the most popular [13–21]. Moreover, the lepton number violation and CP violation in seesaw models naturally lead to a mechanism called leptogenesis [22, 23], which is an appealing candidate for the explanation of baryon asymmetry of our universe. However, leptogenesis through type I or type III seesaws generally involves high energy scales [24], making it difficult to

probe. However, it is known that the type II seesaw mechanism cannot lead to successful thermal leptogenesis [25, 26]. Interestingly, recent works show that by considering the Affleck-Dine mechanism and non-minimal coupling of the scalar fields to gravity, it may be possible to realize successful non-thermal leptogenesis in the type II seesaw mechanism [27, 28]. Thus, one may explain the origin of neutrino mass, inflation, and baryon asymmetry simultaneously in the framework of type II seesaw leptogenesis.

An attractive feature of the type II seesaw model is that the relevant scale is relatively low among the numerous new physics models. This means that it can be tested by current and upcoming experiments, both in the high energy frontier with collider experiments [29–36] and in the intensity frontier with CLFV experiments [37–46]. Because both classes of experiments have obtained null result, constraints can be placed on the parameter space of the model.

In this paper, our discussions focus on the CLFV tests

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of type II seesaw leptogenesis; however, most of our methods and results are generic to any type II seesaw model. Compared to generic type II seesaw models, the requirement of successful leptogenesis leads to extra constraints on the parameter space, ruling out the upper left region of the $m_\Delta - \mu$ parameter space. Meanwhile, the remaining parameter regions may be probed by current and future CLFV experiments, providing an opportunity to test or rule out the type II seesaw leptogenesis mechanism. Analogous analyses have been conducted in [43]; however, they only obtained the constraints at several benchmark points of the neutrino oscillation parameters. In this paper, we scan the 3σ range of the neutrino data and obtain the most conservative constraints on the parameters of type II seesaw leptogenesis. In addition, comparing the analysis of $\mu \rightarrow e$ conversion in the titanium (Ti) nuclei in [43], we also include the $\mu \rightarrow e$ conversion in aluminum (Al) nuclei. In this work, we mainly focus on the constraints of CLFV experiments from the muon sector, including the search for $\mu^+ \rightarrow e^+ \gamma$, $\mu^+ \rightarrow e^+ e^- e^+$, and $\mu^- N \rightarrow e^- N$ processes. Current best limits on these processes are provided by the MEG, SINDRUM, and SINDRUM II experiments, respectively [47–50]. Moreover, several upcoming experiments will improve the sensitivity by up to four orders of magnitude [51–55], which will greatly extend the parameter region we are interested in.

The remainder of this paper is organized as follows. In Sec. II, we briefly review the framework of type II seesaw leptogenesis. In Sec. III, we calculate the CLFV processes in the muon sector and obtain the current most stringent constraints on the parameter space of type II seesaw leptogenesis. Then, we analyze the sensitivity of future CLFV experiments to the model. Finally, in Sec. IV, we provide a summary and draw conclusions.

II. TYPE II SEESAW LEPTOGENESIS

A. Type II seesaw mechanism

In the type II seesaw mechanism, the SM is minimally extended by an $SU(2)_L$ triplet Higgs field Δ with hypercharge $Y = 1$ and lepton number $L = -2$:

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \quad (1)$$

and the SM Higgs is

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (2)$$

The Lagrangian of the type-II seesaw scenario is given by

$$\mathcal{L}_{\text{seesaw}}^{\text{II}} = \mathcal{L}_{\text{SM}} + \text{Tr}[(D_\mu \Delta)^\dagger (D^\mu \Delta)] - V(H, \Delta) - \mathcal{L}_{\text{Yukawa}}, \quad (3)$$

where

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{Yukawa}}^{\text{SM}} - h_{ij} \bar{L}_i^c (i\sigma^2) \Delta L_j + \text{h.c.} \quad (4)$$

The Higgs potential $V(H, \Delta)$ has the form

$$\begin{aligned} V(H, \Delta) = & -m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) \\ & + \lambda_1 (H^\dagger H) \text{Tr}(\Delta^\dagger \Delta) + \lambda_2 (\text{Tr}(\Delta^\dagger \Delta))^2 \\ & + \lambda_3 \text{Tr}(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H + [\mu (H^T i\sigma^2 \Delta^\dagger H) \\ & + \frac{\lambda_5}{M_p} (H^T i\sigma^2 \Delta^\dagger H)(H^\dagger H) \\ & + \frac{\lambda'_5}{M_p} (H^T i\sigma^2 \Delta^\dagger H)(\Delta^\dagger \Delta) + \text{h.c.}], \end{aligned} \quad (5)$$

where the terms in the square brackets violate the lepton number, which is necessary for leptogenesis during inflation. At the low energy scales we are interested in, only the μ -term contributes.

After electroweak symmetry breaking (EWSB), the neutral components of H and Δ take nonzero vacuum expectation value (vev). In the limit $m_\Delta \gg v_{\text{EW}}$, which is consistent with the constraints from LHC [56], we get the tiny triplet vev:

$$v_\Delta \equiv \langle \Delta^0 \rangle \simeq \frac{\mu v_{\text{EW}}^2}{2m_\Delta^2} \quad (6)$$

where $v_{\text{EW}} \simeq 174$ GeV. Once the neutral component of the triplet Higgs Δ^0 reaches non-zero vev, a tiny neutrino mass is generated from the Yukawa term. The neutrino mass matrix can be computed as

$$(m_\nu)_{ll'} \equiv m_{ll'} \simeq 2h_{ll'} v_\Delta. \quad (7)$$

Thus, the matrix of Yukawa couplings $h_{ll'}$ is directly related to the PMNS neutrino mixing matrix $U_{\text{PMNS}} \equiv U$:

$$h_{ll'} \equiv \frac{1}{2v_\Delta} (U^* \text{diag}(m_1, m_2, m_3) U^\dagger)_{ll'}. \quad (8)$$

where m_i are the masses of the neutrino mass eigenstates, and U is the PMNS matrix, which takes the form

$$U = V(\theta_{12}, \theta_{23}, \theta_{13}, \delta) Q(\alpha_{21}, \alpha_{31}), \quad (9)$$

where

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}, \quad (10)$$

with $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, δ being the Dirac phase, and the matrix Q containing the two Majorana phases, α_{21} and α_{31} . The best fit values of these parameters obtained from latest neutrino oscillation experiments are given in Table 1 [57].

Currently, the absolute value of neutrino masses have not been measured, and the strongest constraint is obtained from cosmological observation, $\Sigma m_i < 0.12$ eV [58]. To generate the observed neutrino masses successfully while also ensuring that the Yukawa couplings remain perturbative, it is required that $\nu_\Delta \gtrsim 0.05$ eV [28].

After EWSB, the massless gauge bosons acquire masses through the Higgs mechanism, and the 10 degrees of freedom in the scalar sector reduce to 7 physical scalar fields with definite mass: $H^{\pm\pm}, H^\pm, H^0, A^0, h^0$. $H^{\pm\pm}$ is simply $\Delta^{\pm\pm}$, $H^\pm$ is given by the mixing of $\Delta^\pm$ and $\phi^\pm$, and the mixing of Δ^0 and ϕ^0 gives rise to H^0, A^0 and the standard model Higgs h^0 [59–61].

B. Affleck-Dine leptogenesis through type II seesaw

As the minimal type II seesaw model cannot lead to successful thermal leptogenesis, the Affleck-Dine mechanism is taken into account. In the Affleck-Dine mechanism [62], a scalar field carrying nonzero baryon or lepton

number acquires a large vacuum expectation value along the flat direction of the potential in the early universe. With baryon or lepton number violating terms in the potential, the nontrivial angular motion in the phase of the scalar field during the evolution of the universe leads to the generation of baryon or lepton number asymmetry. Fortunately, the scalar field carrying lepton number required in the Affleck-Dine mechanism can be provided by the triplet Higgs Δ . Furthermore, the non-minimal coupling of the scalars H and Δ to gravity induces a flat potential required by the Affleck-Dine mechanism and a Starobinsky type inflation during the early universe [63–70]. This framework is called type II seesaw leptogenesis [27, 28, 71], and in this framework, the origins of neutrino mass, inflation, and baryon asymmetry may be explained simultaneously.

Considering the non-minimal couplings of Δ and H to gravity, the relevant Lagrangian can be written in the Jordan frame as

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset -\frac{1}{2}M_P^2 R - f(H, \Delta)R + g^{\mu\nu}(D_\mu H)^\dagger(D_\nu H) + g^{\mu\nu}\text{Tr}(D_\mu \Delta)^\dagger(D_\nu \Delta) - V(H, \Delta), \quad (11)$$

where R is the Ricci scalar. To simplify the analysis, we focus on the neutral components ϕ^0 and δ^0 and consider the non-minimal coupling to be

$$f(H, \Delta) = \xi_H |\phi^0|^2 + \xi_\Delta |\Delta^0|^2. \quad (12)$$

After a Weyl transformation, the Lagrangian can be written in the Einstein frame, where the gravitational portion is of Einstein-Hilbert form. The scalar potential in the Einstein frame is

$$V_E(H, \Delta) = \frac{M_P^4}{(M_P^2 + 2f(H, \Delta))^2} V(H, \Delta), \quad (13)$$

which exhibits a flat direction at the large field limit of ϕ^0 and Δ^0 . This flat direction can be recognized as a Starobinsky-like inflationary trajectory, and the inflaton is the mixing of Δ^0 and ϕ^0 .

During the inflationary evolution, the nontrivial motion of the angular direction of Δ^0 generates nonzero lepton number, which subsequently transfers to ordinary particles during reheating. After reheating and before electroweak phase transition, a part of net lepton number is converted to baryon number through the sphaleron process [72–75].

However, any lepton number violating processes in thermal equilibrium after reheating will washout the generated lepton asymmetry. Thus, it is required that the processes $LL \leftrightarrow HH$ and $HH \leftrightarrow \Delta$ are never in thermal equi-

Table 1. Best fit parameters (BFPs) and 1σ allowed ranges of the neutrino oscillation parameters for both the normal ordering (NO) and inverted ordering (IO) scenarios, derived from a global fit of the Super-Kamiokande atmospheric neutrino oscillation data by NuFit [57]. Note that $\Delta m_{3l}^2 \equiv \Delta m_{31}^2 > 0$ for NO, and $\Delta m_{3l}^2 \equiv \Delta m_{32}^2 < 0$ for IO.

BFP $\pm 1\sigma$	NO	IO
$\sin^2 \theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.304^{+0.013}_{-0.012}$
$\sin^2 \theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.575^{+0.016}_{-0.019}$
$\sin^2 \theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02238^{+0.00063}_{-0.00062}$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	$7.42^{+0.21}_{-0.20}$
$\frac{\Delta m_{3l}^2}{10^{-3} \text{ eV}^2}$	$+2.517^{+0.026}_{-0.028}$	$-2.498^{+0.028}_{-0.028}$

librium:

$$\Gamma|_{T=m_\Delta} = n\langle\sigma v\rangle \approx h^2\mu^2/m_\Delta < H|_{T=m_\Delta}, \quad (14)$$

$$\Gamma_{ID}(HH \leftrightarrow \Delta)|_{T=m_\Delta} \simeq \frac{\mu^2}{32\pi m_\Delta} < H|_{T=m_\Delta}, \quad (15)$$

where $H|_{T=m_\Delta} = \sqrt{\frac{\pi^2 g_*}{90}} \frac{m_\Delta^2}{M_p}$. Using $v_\Delta \simeq \frac{\mu v_{EW}^2}{2m_\Delta^2}$ and Eq. (15), the necessary condition to avoid washout effect is found to be

$$v_\Delta \lesssim 10^{-5} \text{ GeV} \left(\frac{m_\Delta}{1 \text{ TeV}} \right)^{-1/2}. \quad (16)$$

For $m_\Delta \gtrsim 1 \text{ TeV}$ [56], we require $v_\Delta \lesssim 10 \text{ keV}$ to prevent the washout effect and achieve successful leptogenesis. In the limit of $v_\Delta \ll v_{EW}$ and $m_\Delta \gg v_{EW}$, the mass spectrum of the scalar sector except for the SM Higgs is approximately degenerate:

$$m_{H^{++}} \simeq m_{H^\pm} \simeq m_{H^0/A^0} \simeq m_\Delta. \quad (17)$$

III. LEPTON FLAVOR VIOLATION PROCESSES

In the type II seesaw model, the couplings of triplet Higgs scalar with the SM lepton sector lead to CLFV processes, and the most stringent constraints from experiments are set by the muon sector, including $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, and $\mu \rightarrow e$ conversion in nuclei [11, 12, 40]. The relevant parameters include the Yukawa couplings, mass scale of triplet scalar m_Δ , and lepton number violating coupling μ . Note that the Yukawa couplings can be parameterized by neutrino masses and the PMNS matrix, which are relevant in neutrino oscillation experiments. Meanwhile, the other two parameters are relevant to leptogenesis. Thus, in this section, we conduct an integral analysis of both parts of parameters and find that the constraints from CLFV experiments with the requirement of satisfying the current neutrino oscillation data lead to constraints on the parameters relevant to leptogenesis. Because the neutrino oscillation experiments have not determined all the parameters of neutrino masses and mixing, we adopt the Monte Carlo method to scan the parameter region to satisfy the neutrino data within the 3σ range. Finally, we obtain the most conservative constraints on the parameters of type II seesaw leptogenesis consistent with current neutrino data, as well as the future experimental reach of the parameter space.

In type II seesaw, the leading contribution to $\mu \rightarrow 3e$

amplitude is at the tree level, while the contributions to $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion arise at one-loop. The corresponding effective low energy LFV Lagrangian generated at one-loop contributing to the μ - e transition processes can be written as [40]

$$\begin{aligned} \mathcal{L}^{\text{eff}} = & -4 \frac{eG_F}{\sqrt{2}} (m_\mu A_R \bar{e} \sigma^{\alpha\beta} P_R \mu F_{\beta\alpha} + \text{h.c.}) \\ & - \frac{e^2 G_F}{\sqrt{2}} \left(A_L (-m_\mu^2) \bar{e} \gamma^\alpha P_L \mu \sum_{Q=u,d} q_Q \bar{Q} \gamma_\alpha Q + \text{h.c.} \right), \end{aligned} \quad (18)$$

where e is the proton charge, and q_Q is the electric charge of the up or down quark. The form factors $A_{R,L}$ are given by

$$A_R = -\frac{1}{\sqrt{2}G_F} \frac{(h^\dagger h)_{e\mu}}{48\pi^2} \left(\frac{1}{8m_{H^+}^2} + \frac{1}{m_{H^{++}}^2} \right), \quad (19)$$

$$\begin{aligned} A_L(q^2) = & -\frac{1}{\sqrt{2}G_F} \frac{h_{le}^* h_{l\mu}}{6\pi^2} \left(\frac{1}{12m_{H^+}^2} \right. \\ & \left. + \frac{1}{m_{H^{++}}^2} f\left(\frac{-q^2}{m_{H^{++}}^2}, \frac{m_l^2}{m_{H^{++}}^2}\right) \right), \end{aligned} \quad (20)$$

where m_l is the mass of the charged lepton $l, l = e, \mu, \tau$. The loop function $f(r, s_l)$ is well known:

$$\begin{aligned} f(r, s_l) = & \frac{4s_l}{r} + \log(s_l) \\ & + \left(1 - \frac{2s_l}{r} \right) \sqrt{1 + \frac{4s_l}{r}} \log \frac{\sqrt{r} + \sqrt{r+4s_l}}{\sqrt{r} - \sqrt{r+4s_l}}. \end{aligned} \quad (21)$$

In the limit in which the charged lepton masses $m_l \ll m_{H^{++}}$, one has $f(r, s_l) \simeq \log r = \log(m_\mu^2/m_{H^{++}}^2)$.

B. $\mu \rightarrow e\gamma$ decay

Using Eqs. (18) and (19), we get the branching ratio of $\mu \rightarrow e\gamma$ decay:

$$\begin{aligned} \text{BR}(\mu \rightarrow e\gamma) \simeq & 384\pi^2 (4\pi\alpha_{em}) |A_R|^2 \\ = & \frac{\alpha_{em}}{192\pi} \frac{|(h^\dagger h)_{e\mu}|^2}{G_F^2} \left(\frac{1}{m_{H^+}^2} + \frac{8}{m_{H^{++}}^2} \right)^2, \end{aligned} \quad (22)$$

where α_{em} is the fine structure constant, and G_F is the Fermi constant.

At present, the strongest constraint on this process is given by the MEG collaboration [47], with an upper limit on the branching ratio:

$$\text{BR}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}, \quad 90\% \text{ C.L.} \quad (23)$$

For $m_{H^+} \cong m_{H^{++}} \equiv m_\Delta$, using Eqs. (22) and (23), we can derive a bound on the Yukawa couplings:

$$\left| (h^\dagger h)_{e\mu} \right| < 1.6 \times 10^{-4} \left(\frac{m_\Delta}{800 \text{ GeV}} \right)^2, \quad (24)$$

from which we can derive the following m_Δ dependent lower bound on the triplet Higgs parameter μ :

$$\mu > 1.7 \times 10^{-6} \text{ GeV} \frac{\sqrt{\left| (m^\dagger m)_{e\mu} \right|}}{1 \text{ eV}} \frac{m_\Delta}{800 \text{ GeV}}, \quad (25)$$

where we have used Eq. (6). From Eq. (8), it is straightforward to obtain

$$\begin{aligned} \left| (m^\dagger m)_{e\mu} \right| &= \left| c_{13} (\Delta m_{21}^2 c_{12} c_{23} s_{12} \right. \\ &\quad \left. + e^{-i\delta} (\Delta m_{31}^2 - \Delta m_{21}^2 s_{12}^2) s_{13} s_{23} \right) \left| \right. \\ &= \left| U_{e2} U_{2\mu}^\dagger \Delta m_{21}^2 + U_{e3} U_{3\mu}^\dagger \Delta m_{31}^2 \right|, \end{aligned} \quad (26)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. From the above equation, we can see that $\left| (m^\dagger m)_{e\mu} \right|$ is independent of the Majorana neutrino phases and the unknown absolute neutrino masses, varying only with the δ phase once the mixing angles and mass differences are fixed (see Fig. 1).

As illustrated by the scattered points in Fig. 1, we scanned the whole neutrino mass and mixing parameters consistent with neutrino oscillation experiments using the Monte Carlo method. The scattered points are obtained

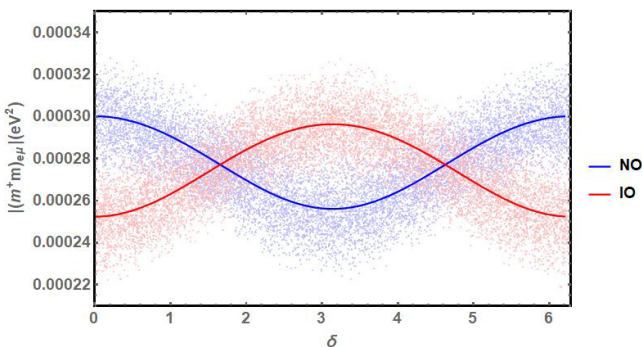


Fig. 1. (color online) $\left| (m^\dagger m)_{e\mu} \right|$ parameter, being independent of the lightest neutrino mass and Majorana phases, as a function of Dirac phase for the NO (Blue) and IO (Red) scenarios once the mixing angles and mass differences are fixed by the best fit values in Table 1. The scattered points are obtained by varying the neutrino oscillation parameters within their corresponding 3σ intervals and giving random values to the Dirac phase.

by varying all the neutrino oscillation parameters within the corresponding 3σ intervals and allowing for arbitrary values of the Dirac phase in $[0, 2\pi]$ with a uniform distribution of random sampling. To obtain the most conservative constraints, the minimum of $\left| (m^\dagger m)_{e\mu} \right|$ obtained in the Monte Carlo simulation is adopted:

$$\left| (m^\dagger m)_{e\mu} \right|_{\min} \sim 2.2 \times 10^{-4} \text{ eV}^2, \quad (27)$$

which is almost the same for both NO and IO. Substituting the above result into Eq. (25), we get

$$\mu > 2.5 \times 10^{-8} \text{ GeV} \frac{m_\Delta}{800 \text{ GeV}}, \quad (28)$$

as shown by the green lines in Figs. 6 and 7.

The upcoming experiment MEG II, which is an upgrade of the MEG experiment, promises to reach a sensitivity of [51]

$$\text{BR}(\mu \rightarrow e\gamma) < 6 \times 10^{-14}, \quad (29)$$

which is one order of magnitude better than the current limit. The expected sensitivity on the parameter space is

$$\mu > 4.9 \times 10^{-8} \text{ GeV} \frac{m_\Delta}{800 \text{ GeV}}, \quad (30)$$

as shown by the green dashed lines in the Figs. 6 and 7.

B. $\mu \rightarrow 3e$ decay

The leading order contribution to the $\mu \rightarrow 3e$ decay amplitude is at the tree level, generated by the diagram with exchange of a virtual doubly-charged Higgs boson Δ^{++} . The branching ratio can be easily calculated:

$$\text{BR}(\mu \rightarrow 3e) = \frac{1}{G_F^2} \frac{\left| (h^\dagger)_{ee} (h)_{\mu e} \right|^2}{m_{\Delta^{++}}^4} = \frac{1}{G_F^2 m_{\Delta^{++}}^4} \frac{\left| m_{ee}^* m_{\mu e} \right|^2}{16 v_\Delta^4}, \quad (31)$$

where we have used Eq. (7) in the last step and

$$\begin{aligned} \left| m_{ee}^* m_{\mu e} \right| &= c_{13} \left[m_1 c_{12}^2 c_{13}^2 + e^{i\alpha_{21}} m_2 c_{13}^2 s_{12}^2 + e^{i(\alpha_{31}-2\delta)} m_3 s_{13}^2 \right] \\ &\quad \times \left[e^{-i(\alpha_{31}-\delta)} m_3 s_{13} s_{23} - m_1 c_{12} (c_{23} s_{12} + e^{-i\delta} c_{12} s_{13} s_{23}) \right. \\ &\quad \left. + e^{-i\alpha_{21}} m_2 s_{12} (c_{12} c_{23} - e^{-i\delta} s_{12} s_{13} s_{23}) \right]. \end{aligned} \quad (32)$$

From the above equation, we can also see that $\left| m_{ee}^* m_{\mu e} \right|$ is dependent on the lightest neutrino mass, Dirac phase, and Majorana neutrino phases once the mixing angles and

mass differences are fixed (see Figs. 2 and 3).

First, we consider the dependence of $|m_{ee}|$ on the CP phases. To do this, we consider the CP phase parameter sets that maximize and minimize the $|m_{ee}^* m_{\mu e}|$ parameter. In the NO scenario, taking $m_1 \lesssim 10^{-4}$ eV, the $|m_{ee}|$ component is found to be minimized when the CP phases satisfy $\alpha_{21} - \alpha_{31} + 2\delta = \pi$ and maximized when this relation is equal to 0. However, for $m_1 \gtrsim 10^{-4}$ eV, one can have $|m_{ee}| = 0$ for specific values of m_1 if CP phases α_{21} and $\alpha_{31} - 2\delta$ satisfy that $\alpha_{21} = \pi$ and $(\alpha_{31} - 2\delta) = 0$ or π . As illustrated in Fig. 2, for $[\delta_{CP}, \alpha_{21}, \alpha_{31}] = [0, \pi, 0]$, we have $|m_{ee}| = 0$ at $m_1 \approx 2.25 \times 10^{-3}$ eV, while in the case of $[0, \pi, \pi]$, we have $|m_{ee}| = 0$ at $m_1 \approx 6.31 \times 10^{-3}$ eV.

If the light neutrino mass spectrum has inverted ordering ($m_3 \ll m_1 < m_2$), we have $|m_{ee}| \gtrsim \sqrt{|\Delta m_{31}^2| + m_3^2} \times \cos 2\theta_{12} \gtrsim 1.96 \times 10^{-2}$ eV with $m_3 = 0$ for the best fit parameters. In addition, the appearance of zeros is also caused by the variation of the $|m_{\mu e}|$ component with the CP phases. In the NO scenario, the minimal value of $|m_{\mu e}|$ is obtained for $\alpha_{21} = \pi$, $\delta = 0$ or π , and $\alpha_{31} = 0$ or π . As shown in Fig. 2, for $[0, \pi, 0]$, the zero occurs at $m_1 \approx 7.2 \times 10^{-3}$ eV, while for $[\pi, \pi, \pi]$, we have $|m_{\mu e}| = 0$ at $m_1 \approx 8.0 \times 10^{-3}$ eV. Meanwhile, the maximal value of $|m_{\mu e}|$ satisfies that $\alpha_{31} - \alpha_{21} = \delta$ and $\delta = \pi$, and we get $|m_{\mu e}| < 8.1 \times 10^{-3}$ eV.

In the IO scenario with negligible $m_3 \approx 0$, the maximal value of $|m_{\mu e}|$ corresponds to $\delta = 0$ and $\alpha_{21} = \pi$ and is given by $\max(|m_{\mu e}|) \approx \sqrt{|\Delta m_{31}^2|} c_{13} (c_{23} \sin 2\theta_{12} + s_{23} s_{13} \cos 2\theta_{12}) \approx 3.19 \times 10^{-2}$ eV for the best fit parameters. The component $|m_{\mu e}|$ is strongly suppressed (see Fig. 3), i.e., we have $|m_{\mu e}| \ll \max(|m_{\mu e}|)$ when $\delta \approx \pi/2$, and the Majorana phase α_{21} , which is determined by the equation

$$c_{23} c_{12} s_{12} \sin \alpha_{21} \approx (c_{12}^2 + s_{12}^2 \cos \alpha_{21}) s_{23} s_{13}, \quad (33)$$

is derived as $\alpha_{21} \approx 0.375$ for the best fit parameters in Table 1.

The properties of $|m_{ee}|$ and $|m_{\mu e}|$ described above allow us to understand most of the specific features of the dependence of $|m_{ee}^* m_{\mu e}|$ on the neutrino mass ordering and CP phases. In the NO scenario with negligible $m_1 \approx 0$, the maximum of $|m_{ee}^* m_{\mu e}|$ is obtained for $\alpha_{31} - \alpha_{21} = \delta = 0$, that is, $[\delta, \alpha_{21}, \alpha_{31}] = [0, 0, 0]$, and Eq. (32) becomes

$$\begin{aligned} \max(|m_{ee}^* m_{\mu e}|) &= |(m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2) \\ &\quad \times c_{13} (m_2 s_{12} (c_{12} c_{23} - s_{12} s_{23} s_{13}) + m_3 s_{23} s_{13})|, \end{aligned} \quad (34)$$

where the best fit parameters of the neutrino oscillation parameters are given in Table 1, from which we can get the maximum of $|m_{ee}^* m_{\mu e}|$ as approximately 2.81×10^{-5} eV².

Meanwhile, in the IO scenario with $m_3 \approx 0$, the maximum of $|m_{ee}^* m_{\mu e}|$ is achieved when $\delta = 0$ and $\alpha_{21} = \pi$:

$$\begin{aligned} \max(|m_{ee}^* m_{\mu e}|) &\approx |\Delta m_{31}^2| c_{13}^2 \left(\frac{1}{2} c_{23} \sin 4\theta_{12} + s_{23} s_{13} \cos^2 2\theta_{12} \right) \\ &\approx 6.1 \times 10^{-4} \text{ eV}^2, \end{aligned} \quad (35)$$

while the minimal case occurs for the parameter set satisfying Eq. (33), from which we can get the minimal value of $|m_{ee}^* m_{\mu e}| \approx 9.9 \times 10^{-6}$ eV².

Finally, the scattered points in Figs. 2 and 3 are obtained by varying all the neutrino oscillation parameters within the corresponding 3σ intervals and allowing for arbitrary values of the Dirac and Majorana phases in $[0, 2\pi]$. The most conservative constraints from the $\mu \rightarrow 3e$ process of our model can be derived from the minimal value of $|m_{ee}^* m_{\mu e}|$. In the IO scenario, we obtain

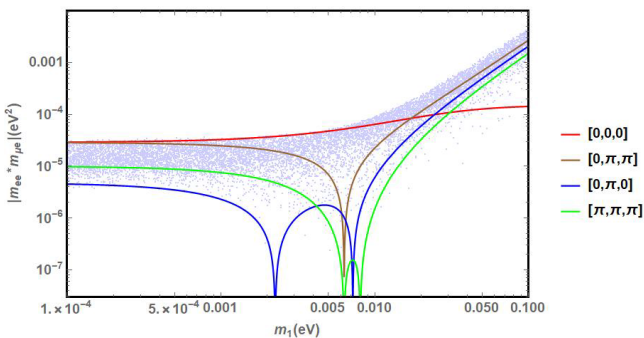


Fig. 2. (color online) Dependence of $|m_{ee}^* m_{\mu e}|$ on the lightest neutrino mass m_1 in the NO scenario, for four sets of values for the Dirac and two Majorana phases, $[\delta, \alpha_{21}, \alpha_{31}]$. All the best fit values of the neutrino oscillation parameters are given in Table 1. The scattered points are obtained by varying the neutrino oscillation parameters within their corresponding 3σ intervals and giving random values to the Dirac and Majorana phases.

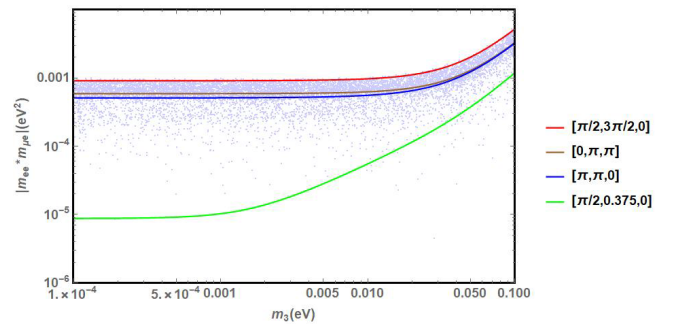


Fig. 3. (color online) Same as Fig. 2 but for the IO scenario.

$$|m_{ee}^* m_{\mu e}|_{\min}(\text{IO}) \simeq 1.1 \times 10^{-5} \text{ eV}^2. \quad (36)$$

In the NO scenario, however, in some parameter region, $|m_{ee}^* m_{\mu e}|$ may vanish; thus, this part of parameter region is not constrained by $\mu \rightarrow 3e$ experiments. Therefore, in the NO scenario, we focus on the parameter region with $m_1 \lesssim 10^{-3} \text{ eV}$ and obtain

$$|m_{ee}^* m_{\mu e}|_{\min}(\text{NO}) \simeq 2.4 \times 10^{-6} \text{ eV}^2, \quad m_1 \lesssim 10^{-3} \text{ eV}. \quad (37)$$

The current best upper bound on the $\mu \rightarrow 3e$ branching ratio is derived from the results of the SINDRUM experiment [48]:

$$\text{BR}(\mu \rightarrow 3e) < 10^{-12}, \quad 90\% \text{C.L.} \quad (38)$$

From the present limit and Eq. (31), one can obtain the following constraint on $|(h^\dagger)_{ee}(h)_{\mu e}|$:

$$|(h^\dagger)_{ee}(h)_{\mu e}| < 7.5 \times 10^{-6} \left(\frac{m_\Delta}{800 \text{ GeV}} \right)^2, \quad (39)$$

from which, using Eq. (6), we also get the lower bound on the cubic coupling μ :

$$\mu > 7.7 \times 10^{-6} \text{ GeV} \frac{\sqrt{|m_{ee}^* m_{\mu e}|}}{1 \text{ eV}} \frac{m_\Delta}{800 \text{ GeV}}. \quad (40)$$

Using Eq. (40) and the Monte Carlo results, we determine the lower bounds on coupling term μ from the current SINDRUM experiment:

$$\mu > 1.2(2.6) \times 10^{-8} \text{ GeV} \frac{m_\Delta}{800 \text{ GeV}}, \quad (41)$$

where the parentheses represent the case of IO. Note that, from the above equation, we can get the parameter space between the cubic coupling μ term and mass of triplet Higgs from the $\mu \rightarrow 3e$ decay process in the case of the normal ordering and inverted ordering for the neutrino mass, as shown by the red lines in Figs. 6 and 7.

The upcoming experiment known as Mu3e promises to deliver significantly improved sensitivity to the $\mu \rightarrow 3e$ process, potentially probing the branching ratio by an additional four orders of magnitude, that is [52],

$$\text{BR}_{\text{Mu3e}}(\mu \rightarrow 3e) < 10^{-16}, \quad (42)$$

which will provide the corresponding limit on the Yukawa couplings,

$$|(h^\dagger)_{ee}(h)_{\mu e}| < 7.5 \times 10^{-8} \left(\frac{m_\Delta}{800 \text{ GeV}} \right)^2, \quad (43)$$

and the cubic coupling μ ,

$$\mu > 7.7 \times 10^{-5} \text{ GeV} \frac{\sqrt{|m_{ee}^* m_{\mu e}|}}{1 \text{ eV}} \frac{m_\Delta}{800 \text{ GeV}}, \quad (44)$$

providing an order of magnitude improvement to the sensitivity to μ .

Similarly, using Eq. (44) and the Monte Carlo results, the future expected lower bounds on the cubic coupling μ from the Mu3e experiment can be determined:

$$\mu > 1.2(2.6) \times 10^{-7} \text{ GeV} \frac{m_\Delta}{800 \text{ GeV}}, \quad (45)$$

where the parentheses represent the case of IO. From Eq. (45) as well, we can get the parameter space between the cubic coupling μ term and mass of triplet Higgs from the $\mu \rightarrow 3e$ decay process in the case of the normal ordering and inverted ordering for the neutrino mass, as shown by the red dashed lines in Figs. 6 and 7.

C. $\mu - e$ conversion in nuclei

Finally, we consider the $\mu - e$ conversion in a generic nucleus $\mathcal{N}$. From the interaction Lagrangian in Eq. (18), we get the conversion rate of the $\mu \rightarrow e$ conversion in the nuclei process in the type-II seesaw scenario:

$$\text{CR}(\mu \mathcal{N} \rightarrow e \mathcal{N}) \cong (4\pi\alpha_{em})^2 \frac{2G_F^2}{\Gamma_{\text{capt}}} \left| A_R \frac{D}{\sqrt{4\pi\alpha_{em}}} + (2q_u + q_d) A_L V^{(p)} \right|^2, \quad (46)$$

where A_R and A_L are given by Eqs. (19) and (20), respectively. In Eq. (46), the parameters D and $V^{(p)}$ present overlap integrals of the muon and electron wave functions, and Γ_{capt} is the experimentally known total muon capture rate [76].

Considering a light nucleus, one has $D \simeq 8 \sqrt{4\pi\alpha_{em}} V^{(p)}$ with a good approximation, with the vector type overlap integral of the proton, $V^{(p)}$, given by

$$V^{(p)} \simeq \frac{1}{4\pi} m_\mu^{5/2} \alpha_{em}^{3/2} Z_{\text{eff}}^2 Z^{1/2} F(-m_\mu^2), \quad (47)$$

where $F(q^2)$ is the nuclear form factor at momentum transfer squared $q^2 = -m_\mu^2$, m_μ is the muon mass, and Z_{eff} is an effective atomic charge. Using the result for D quoted above and Eqs. (19), (20), and (47), the conversion rate can be written as

$$\begin{aligned} \text{CR}(\mu N \rightarrow e N) \cong & \frac{\alpha_{em}^5}{36\pi^4} \frac{m_\mu^5}{\Gamma_{\text{capt}}} Z_{eff}^4 Z F^2(-m_\mu^2) \\ & \times \left| (h^\dagger h)_{e\mu} \left[\frac{5}{24m_{H^+}^2} + \frac{1}{m_{H^{++}}^2} \right] \right. \\ & \left. + \frac{1}{m_{H^{++}}^2} \sum_{l=e,\mu,\tau} h_{el}^\dagger f(r, s_l) h_{l\mu} \right|^2. \quad (48) \end{aligned}$$

The two nuclei that were used in the past and are of interest in the future in $\mu-e$ conversion experiments are $^{48}_{22}\text{Ti}$ and $^{27}_{13}\text{Al}$. The relevant parameters involved in the calculation of $\mu-e$ conversion are given in Table 2.

Thus, assuming $m_{H^+} \simeq m_{H^{++}} = m_\Delta$, we have $\text{CR}(\mu N \rightarrow e N) \propto |C_{\mu e}^{(II)}|^2$, where

$$C_{\mu e}^{(II)} \equiv \frac{1}{4v_\Delta^2} \left[\frac{29}{24} (m^\dagger m)_{e\mu} + \sum_{l=e,\mu,\tau} m_{el}^\dagger f(r, s_l) m_{l\mu} \right], \quad (49)$$

and Eq. (7) is employed. In the above equation, the first term is given by Eq. (26), and the form of $f(r, s_l)$ is given by Eq. (21). Using Eq. (49), we can get the dependence of $4v_\Delta^2 |C_{\mu e}^{(II)}|$ on the lightest neutrino mass, Dirac phase, and Majorana phases in the PMNS matrix once the mixing angles and mass differences are fixed, as depicted in Figs. 4 and 5.

For $m_\Delta = 800 (2 \times 10^6)$ GeV and the NO scenario with $m_1 \ll 10^{-3}$, the maximal value of $4v_\Delta^2 |C_{\mu e}^{(II)}|$ occurs in $[\delta, \alpha_{21}, \alpha_{31}] = [0, 0, 0]$, and at the maximum, we have $4v_\Delta^2 |C_{\mu e}^{(II)}| \simeq 3.78(8.04) \times 10^{-3}$ eV (see Fig. 4) for the best fit parameters given in Table 1.

As analyzed in Section II, the fact that the loop function is negative when m_Δ varies from 800 GeV to 2×10^6 GeV allows us to get the suppression of the conversion rate. For values of the CP phases $[\delta, \alpha_{21}, \alpha_{31}] = [0, \pi, 0]$ and $m_\Delta = 800 (2 \times 10^6)$ GeV, $4v_\Delta^2 |C_{\mu e}^{(II)}|$ goes through zero at $m_1 \simeq 0.0252(0.0439)$ eV (see Fig. 4).

Meanwhile, in the IO case with negligible m_3 ($m_3 \ll 10^{-3}$), the maximal value of $4v_\Delta^2 |C_{\mu e}^{(II)}|$ occurs for $[\delta, \alpha_{21}, \alpha_{31}] = [\pi/2, 3\pi/2, 0]$, and at the maximum in this case, we have $4v_\Delta^2 |C_{\mu e}^{(II)}| \simeq 7.9(12.2) \times 10^{-3}$ for $m_\Delta = 800 (2 \times 10^6)$ GeV. The minimum of $4v_\Delta^2 |C_{\mu e}^{(II)}|$ for small m_3 can be found when the CP phases take $[\pi, \pi, 0]$. At the minimum for $m_\Delta = 800 (2 \times 10^6)$ GeV, we get $4v_\Delta^2 |C_{\mu e}^{(II)}| \simeq 1.3(6.04) \times 10^{-3}$ eV. Taking the CP phase

Table 2. Nuclear parameters relevant to $\mu-e$ conversion in $^{48}_{22}\text{Ti}$ and $^{27}_{13}\text{Al}$ [40, 76].

N	Z_{eff}	$Dm_\mu^{-5/2}$	$V^{(p)} m_\mu^{-5/2}$	$\Gamma_{\text{capt}} (10^6 \text{ s}^{-1})$	$F(-m_\mu^2)$
$^{48}_{22}\text{Ti}$	17.6	0.0864	0.0396	2.590	0.54
$^{27}_{13}\text{Al}$	11.62	0.0362	0.0161	0.7054	0.64

parameter set $[\pi, \pi, 0]$, one can get a strong suppression of the conversion ratio for $m_3 \simeq 0.012(0.038)$ eV and $m_\Delta = 800 (2 \times 10^6)$ GeV (see Fig. 5).

Finally, the scattered points in Figs. 4 and 5 are obtained by varying all the neutrino oscillation parameters within the corresponding 3σ intervals and allowing for arbitrary values of the Dirac and Majorana phases in $[0, 2\pi]$. Because the value of $4v_\Delta^2 |C_{\mu e}^{(II)}|$ vanishes at some part of the parameter region, we consider only the region with $m_1(m_3) \lesssim 10^{-3}$ eV and obtain $4v_\Delta^2 |C_{\mu e}^{(II)}|_{\text{min}} \simeq 2.5 \times 10^{-3} (5.5 \times 10^{-5})$ eV² for $m_\Delta = 800$ GeV and $4v_\Delta^2 |C_{\mu e}^{(II)}|_{\text{min}} \simeq 6(3.9) \times 10^{-3}$ eV² for $m_\Delta = 2 \times 10^6$ GeV in the NO(IO) scenario.

1. $\mu-e$ conversion in Ti nuclei

Currently, the best constraints on the conversion rate are provided by the SINDRUM II experiment, which utilizes Ti nuclei [50]:

$$\text{CR}(\mu N \rightarrow e N) < 4.3 \times 10^{-12}, \quad 90\% \text{C.L.} \quad (50)$$

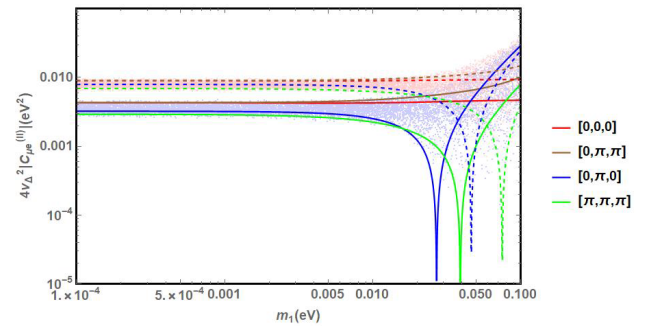


Fig. 4. (color online) Dependence of $4v_\Delta^2 |C_{\mu e}^{(II)}|$ on the lightest neutrino mass m_1 in the case of NO, for four sets of values of the Dirac phase and two Majorana phases, $[\delta_{CP}, \alpha_{21}, \alpha_{31}]$ and $m_\Delta = 800(2 \times 10^6)$ GeV, indicated by plain (dashed) curves. The figure is presented for the best fit values of the neutrino oscillation parameters given in Table 1. The scattered points are obtained by varying the neutrino oscillation parameters within their corresponding 3σ intervals and giving random values to the Dirac and Majorana phases.

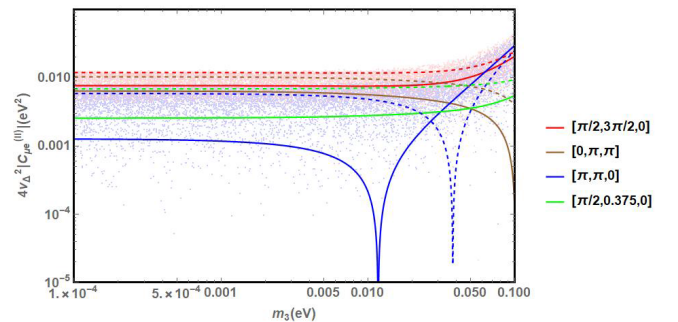


Fig. 5. (color online) Same as Fig. 4 but for the IO scenario.

The constraint translates to the following limit:

$$|C_{\mu e}^{(II)}| < 7.94 \times 10^{-3} \left(\frac{m_\Delta}{800 \text{ GeV}} \right)^2, \quad (51)$$

with a corresponding limit on the cubic coupling μ ,

$$\mu > 2.4 \times 10^{-7} \text{ GeV} \frac{\sqrt{4v_\Delta^2 |C_{\mu e}^{(II)}|}}{1 \text{ eV}} \frac{m_\Delta}{800 \text{ GeV}} \quad (52)$$

Using Eq. (52) and the Monte Carlo method, the current most conservative limits of $\mu \rightarrow e$ conversion in Ti nuclei can be obtained, as indicated by the blue lines in Figs. 6 and 7.

2. $\mu - e$ conversion in Al nuclei

The upcoming COMET and Mu2e experiments,

which search for $\mu - e$ conversion in the field of Al nuclei, is aiming to improve the current limit by several orders of magnitude [53, 54]:

$$\begin{aligned} \text{CR}_{\text{COMET}}(\mu N \rightarrow e N) &< 7 \times 10^{-15}, \\ \text{CR}_{\text{Mu2e}}(\mu N \rightarrow e N) &< 6.2 \times 10^{-16}. \end{aligned} \quad (53)$$

We consider the better sensitivity provided by Mu2e, which can be translated into the following sensitivity:

$$|C_{\mu e}^{(II)}| < 1.2 \times 10^{-4} \left(\frac{m_\Delta}{800 \text{ GeV}} \right)^2, \quad (54)$$

with a corresponding limit on the cubic coupling μ ,

$$\mu > 1.9 \times 10^{-6} \text{ GeV} \frac{\sqrt{4v_\Delta^2 |C_{\mu e}^{(II)}|}}{1 \text{ eV}} \frac{m_\Delta}{800 \text{ GeV}}. \quad (55)$$

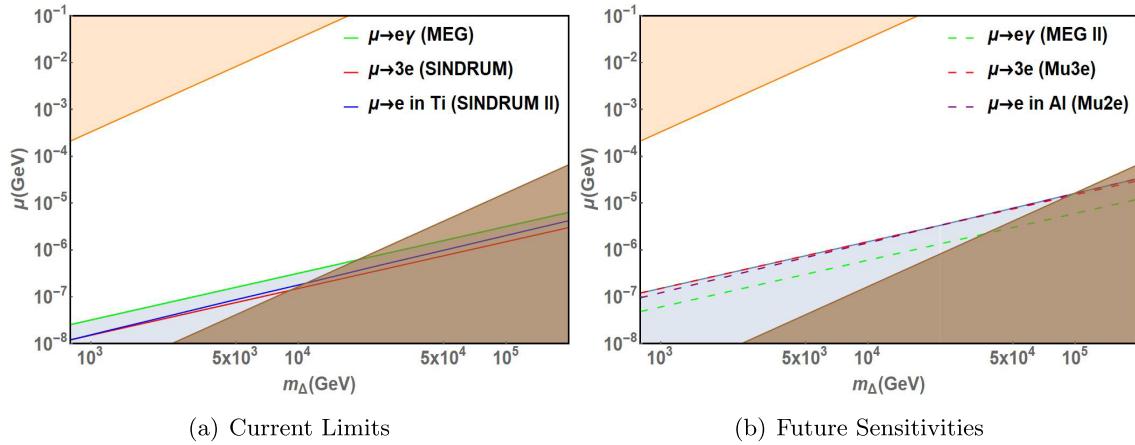


Fig. 6. (color online) Current limits (Left) and future experimental sensitivities (Right) are depicted for the most conservative values of $|(m^\dagger m)_{e\mu}|$, $|m_{ee}^\dagger m_{\mu e}|$, and $4v_\Delta^2 |C_{\mu e}^{(II)}|$ in the NO scenario. The neutrino parameters are given by the best fit parameters in Table 1 within their 3σ intervals. The constraints from requiring perturbative Yukawa couplings and avoiding the washout effect are depicted as the brown and orange regions, respectively.

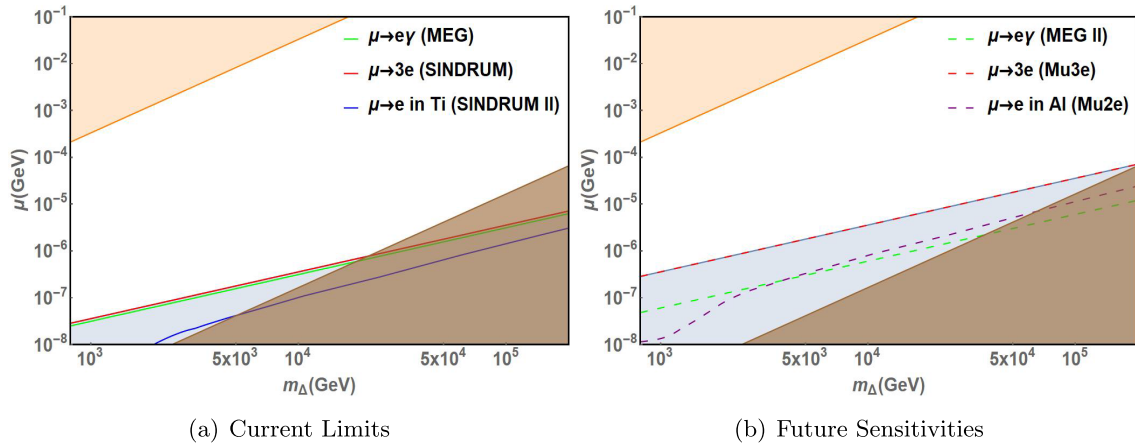


Fig. 7. (color online) Same as Fig. 6 but for the IO scenario.

Therefore, as previously calculated, using the Monte Carlo method, the future sensitivities of $\mu \rightarrow e$ conversion in Al nuclei are indicated by purple dashed lines in Figs. 6 and 7.

D. Current constraints and future sensitivities

In this subsection, we summarize our previous analyses and present the final results. The current constraints on the parameter space of m_Δ and μ from different CLFV processes are shown in Figs. 6 (a) and 7(a) for the NO and IO scenarios, respectively. As discussed in previous subsections, the constraints are derived by scanning the parameter region, consistent with the neutrino oscillation data from the Monte Carlo method, and adopting the points with minimal predicted decay rates, which are translated to the most conservative constraints on the model. Note that, for some value of the lightest neutrino mass, the constraints may vanish; therefore, here, we only impose the limit assuming $m_1 \lesssim 10^{-3}\text{eV}$. It is shown that, in the NO scenario, the most stringent constraint comes from $\mu \rightarrow e\gamma$ decay at MEG, while in the IO scenario, the most stringent constraint is derived from $\mu \rightarrow e\gamma$ decay at MEG and $\mu \rightarrow 3e$ decay at SINDRUM, which are close to each other. The constraints from requiring perturbative Yukawa couplings are depicted as the brown regions. The constraints derived from condition Eq. (16) to avoid the washout effect are also depicted as orange regions.

We also illustrate the sensitivities of future CLFV experiments in Figs. 6 (b) and 7 (b). In the NO scenario, $\mu \rightarrow 3e$ decay at Mu3e and $\mu \rightarrow e$ conversion in Al at Mu2e have comparable sensitivities, which are more

sensitive than $\mu \rightarrow e\gamma$ at MEG II. In the IO scenario, the most sensitive experiment will be Mu3e.

Compared to the previous study [43] in which the constraints were obtained at several benchmark points, we scanned the allowed neutrino parameter regions to obtain the most conservative constraints. In summary, although the parameter space is constrained by the washout effect, the perturbative condition, and various CLFV experiments, a vast region still survives even under the constraints from the upcoming CLFV experiments.

IV. DISCUSSION AND CONCLUSION

In this study, we focused on CLFV tests within the framework of type II seesaw leptogenesis. The key processes considered include $\mu^+ \rightarrow e^+\gamma$, $\mu^+ \rightarrow e^+e^-e^+$, and $\mu^-N \rightarrow e^-N$. We performed a comprehensive parameter space scan using the Monte Carlo method and derived conservative constraints on the parameters of type II seesaw leptogenesis that are consistent with current neutrino oscillation data. Our results indicate that, in the NO scenario, the most stringent constraint arises from $\mu \rightarrow e\gamma$, as measured by the MEG experiment. In the IO scenario, the strongest constraints are derived from $\mu \rightarrow e\gamma$ decay at MEG and $\mu \rightarrow 3e$ decay at SINDRUM, with both providing similar sensitivities. We also analyzed the potential sensitivities of upcoming CLFV experiments. For the NO scenario, $\mu \rightarrow 3e$ decay at Mu3e and $\mu \rightarrow e$ conversion in Al at Mu2e are predicted to have comparable sensitivities, both surpassing the sensitivity of $\mu \rightarrow e\gamma$ at MEG II. In the IO scenario, the most sensitive experiment is expected to be Mu3e.

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