P-wave Ω_b states: masses and pole residues[*]

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Abstract: In this study, we consider all *P*-wave Ω_b states represented by interpolating currents with a derivative and calculate the corresponding masses and pole residues using the QCD sum rule method. Because of the large uncertainties in our calculation compared with the small difference in the masses of the excited Ω_b states observed by the LHCb collaboration, it is necessary to study other properties of the *P*-wave Ω_b states represented by the interpolating currents investigated in the present work to gain a better understanding of the four excited Ω_b states observed by the LHCb collaboration.

I. INTRODUCTION

In 2017, the LHCb collaboration observed five narrow excited Ω_c states, i.e., $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3119)$, in the $\Xi_c^+ K^-$ mass spectrum [1]. Recently, they reported four excited Ω_b states in the $\Xi_b^0 K^-$ mass spectrum [2]:

$$
\begin{aligned}
\Omega_b(6316) &: m = 6315.64 \pm 0.31 \pm 0.07 \pm 0.50\,\text{MeV}, \quad \Gamma < 2.8\,\text{MeV}, \\
\Omega_b(6330) &: m = 6330.30 \pm 0.28 \pm 0.07 \pm 0.50\,\text{MeV}, \quad \Gamma < 3.1\,\text{MeV}, \\
\Omega_b(6340) &: m = 6339.71 \pm 0.26 \pm 0.05 \pm 0.50\,\text{MeV}, \quad \Gamma < 1.5\,\text{MeV}, \\
\Omega_b(6350) &: m = 6349.88 \pm 0.35 \pm 0.05 \pm 0.50\,\text{MeV}, \quad \Gamma = 1.4^{+1.0}_{-0.8} \pm 0.1\,\text{MeV}.
\end{aligned}
\tag{1}
$$

Following this experimental progresses, there have been many theoretical works concerning various properties of these excited Ω_Q ($Q = b, c$) states [3-26] and other excited heavy baryons [27-36].

Two kinds of excitations, the ρ-mode and λ-mode, exist in these excited Ω_Q states. The ρ-mode excitation is the excitation between two strange quarks, while the λ-mode one is the excitation between the strange diquark and bottom (charm) quark. In Ref. [37], the authors systematically considered all possible baryon currents with a derivative for internal ρ- and λ-mode excitations and studied the *P*-wave charmed baryons using the QCD sum rule method in the framework of heavy quark effective theory. In Refs. [12, 22], the authors studied these excited states using the QCD sum rule method in the framework of QCD.

In this paper, we construct the full QCD counterparts of the interpolating currents considered in Ref. [37] and study *P*-wave Ω_b excited states using the QCD sum rule method [38, 39]. The basic idea of the QCD sum rule method is that the correlation function of interpolating currents of hadrons can be represented in terms of hadronic parameters (the so-called hadronic side) and calculated at quark-gluon level by operator product expansion (OPE) (the so-called QCD side); then, by matching the two expressions, we can extract the physical quantities of the considered hadron.

The rest of the paper is organized as follows. In Sec. II, we construct the interpolating currents and derive the required sum rules. Sec. III is devoted to numerical analysis, and a short summary is given in Sec. IV. In Appendix B, OPE results are shown.

II. DERIVATION OF THE SUM RULES

A. Interpolating currents

Following Ref. [37], we introduce the symbols $[\Omega_b, j_l, s_l, \rho/\lambda]$ and $J^{\alpha_1\alpha_2\cdots\alpha_{j-\frac{1}{2}}}_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}$ to denote the P-wave Ω_b multiplets and the interpolating currents, respectively, where j is the total angular momentum; P is the parity; j_l and s_l are the total angular momentum and spin angular momentum of the light components, respectively; and $\rho(\lambda)$ denotes the $\rho(\lambda)$-mode excitations. The general interpolating currents of Ω_b baryons can be written as

$$J(x) \sim \epsilon^{abc}[s^{aT}(x)C\Gamma_1 s^b(x)]\Gamma_2 b^c(x), \qquad (2)$$

where a, b, and c are color indices, ϵ^{abc} is the totally antisymmetric tensor, C is the charge conjugation operator, T denotes the matrix transpose on the Dirac spinor indices, and $s(x)$ and $b(x)$ are the strange and bottom quark fields, respectively. The state function corresponding to the diquark $\epsilon^{abc}[s^{aT}(x)C\Gamma_1 s^b(x)]$ can be written as $|\text{color}\rangle\otimes|\text{flavor, spin, space}\rangle$ and should be antisymmetric under the interchange of the two strange quarks. Now, the color part and flavor part are antisymmetric and symmetric, respectively. The spin part is antisymmetric for the scalar diquark $\epsilon^{abc}[s^{aT}(x)C\gamma_5 s^b(x)]$ and symmetric for the axial-vector diquark $\epsilon^{abc}[s^{aT}(x)C\gamma_\mu s^b(x)]$. The spatial wave function is antisymmetric and symmetric corresponding to the the ρ-mode and λ-mode excitation, respectively. For example, if the spin angular momentum of the diquark is 0, the excitation in the Ω_b state should be the ρ-mode, and we have the baryon-multiplet $[\Omega_b, 1, 0, \rho]$. Consequently, the P-wave Ω_b states can be classified into four multiplets, i.e., $[\Omega_b, 1, 0, \rho]$, $[\Omega_b, 0, 1, \lambda]$, $[\Omega_b, 1, 1, \lambda]$, and $[\Omega_b, 2, 1, \lambda]$, and the corresponding interpolating currents are

- $[\Omega_b, 1, 0, \rho]$:

$$\begin{aligned}
J_{1/2,-,\Omega_b,1,0,\rho}(x) =&\, i\epsilon_{abc}\{[D_\mu s^T(x)]^a C\gamma_5 s^b(x) \\
&- s^T(x)C\gamma_5[D_\mu s(x)]^b\}\gamma^\mu\gamma_5 b^c(x),
\end{aligned}$$

$$\begin{aligned}
J^\alpha_{3/2,-,\Omega_b,1,0,\rho}(x) =&\, i\epsilon_{abc}\{[D_\mu s^T(x)]^a C\gamma_5 s^b(x) \\
&- s^T(x)C\gamma_5[D_\mu s(x)]^b\}\Gamma^{\alpha\mu} b^c(x),
\end{aligned} \qquad (3)$$

with $\Gamma^{\alpha\mu} = g^{\alpha\mu} - \frac{1}{4}\gamma^\alpha\gamma^\mu$,

- $[\Omega_b, 0, 1, \lambda]$:

$$\begin{aligned}
J_{1/2,-,\Omega_b,0,1,\lambda}(x) =&\, i\epsilon_{abc}\{[D_\mu s^T(x)]^a C\gamma^\mu s^b(x) \\
&+ s^T(x)C\gamma^\mu[D_\mu s(x)]^b\}b^c(x),
\end{aligned} \qquad (4)$$

- $[\Omega_b, 1, 1, \lambda]$:

$$\begin{aligned}
J_{1/2,-,\Omega_b,1,1,\lambda}(x) =&\, i\epsilon_{abc}\{[D_\mu s^T(x)]^a C\gamma_\nu s^b(x) \\
&+ s^T(x)C\gamma_\nu[D_\mu s(x)]^b\}\sigma^{\mu\nu} b^c(x),
\end{aligned}$$

$$\begin{aligned}
J^\alpha_{3/2,-,\Omega_b,1,1,\lambda}(x) =&\, i\epsilon_{abc}\{[D_\mu s^T(x)]^a C\gamma_\nu s^b(x) \\
&+ s^T(x)C\gamma_\nu[D_\mu s(x)]^b\}\Gamma_1^{\alpha\mu\nu} b^c(x),
\end{aligned} \qquad (5)$$

with $\Gamma_1^{\alpha\mu\nu} = \left(g^{\alpha\mu}\gamma^\nu - g^{\alpha\nu}\gamma^\mu - \frac{1}{4}\gamma^\alpha\gamma^\mu\gamma^\nu + \frac{1}{4}\gamma^\alpha\gamma^\nu\gamma^\mu\right)\gamma_5$,

- $[\Omega_b, 2, 1, \lambda]$:

$$\begin{aligned}
J^\alpha_{3/2,-,\Omega_b,2,1,\lambda}(x) =&\, i\epsilon_{abc}\{[D_\mu s^T(x)]^a C\gamma_\nu s^b(x) \\
&+ s^T(x)C\gamma_\nu[D_\mu s(x)]^b\}\Gamma_2^{\alpha\mu\nu} b^c(x),
\end{aligned}$$

$$\begin{aligned}
J^{\alpha_1\alpha_2}_{5/2,-,\Omega_b,2,1,\lambda}(x) =&\, i\epsilon_{abc}\{[D_\mu s^T(x)]^a C\gamma_\nu s^b(x) \\
&+ s^T(x)C\gamma_\nu[D_\mu s(x)]^b\}\Gamma^{\alpha_1\alpha_2\mu\nu} b^c(x),
\end{aligned} \qquad (6)$$

where

$$\Gamma_2^{\alpha\mu\nu} = \left(g^{\alpha\mu}\gamma^\nu + g^{\alpha\nu}\gamma^\mu - \frac{1}{2}g^{\mu\nu}\gamma^\alpha\right)\gamma_5, \qquad (7)$$

$$\begin{aligned}
\Gamma^{\alpha_1\alpha_2\mu\nu} =&\, g^{\alpha_1\mu}g^{\alpha_2\nu} + g^{\alpha_1\nu}g^{\alpha_2\mu} - \frac{1}{3}g^{\alpha_1\alpha_2}g^{\mu\nu} \\
&- \frac{1}{6}g^{\alpha_1\mu}\gamma^{\alpha_2}\gamma^\nu - \frac{1}{6}g^{\alpha_1\nu}\gamma^{\alpha_2}\gamma^\mu \\
&- \frac{1}{6}g^{\alpha_2\nu}\gamma^{\alpha_1}\gamma^\mu - \frac{1}{6}g^{\alpha_2\mu}\gamma^{\alpha_1}\gamma^\nu.
\end{aligned} \qquad (8)$$

In the above equations, $D_\mu(x) = \partial_\mu - ig_s A_\mu(x)$ is the gauge-covariant derivative; a, b, and c are color indices; C is the charge conjugation operator; T denotes the matrix transpose on the Dirac spinor indices; and $s(x)$ and $b(x)$ are the strange and bottom quark fields, respectively.

B. Sum rules

To obtain the mass sum rules for the P-wave excited Ω_b states, we begin with the following two-point correlation function of the interpolating currents constructed in the previous subsection,

$$\begin{aligned}
\Pi^{\alpha_1\alpha_2\cdots\alpha_{j-\frac{1}{2}}\beta_1\beta_2\cdots\beta_{j-\frac{1}{2}}}(p) =&\, i\int dx^4 e^{ipx}\langle 0 | T[J^{\alpha_1\alpha_2\cdots\alpha_{j-\frac{1}{2}}}_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}(x) \\
&\times \bar{J}^{\beta_1\beta_2\cdots\beta_{j-\frac{1}{2}}}_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}(0)] | 0\rangle.
\end{aligned} \qquad (9)$$

First, we need to phenomenologically represent the two-point correlation function (9) in terms of hadronic parameters. To this end, we insert a complete set of states with the same quantum numbers as the interpolating field, perform the integral over space-time coordinates, and fi-

nally obtain

$$
\begin{aligned}
\Pi^{(\text{Phy})\alpha_1\cdots\alpha_{j-\frac{1}{2}}\beta_1\cdots\beta_{j-\frac{1}{2}}}(p) =& \frac{1}{m_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}^2 - p^2} \\
& \times \langle 0|J_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}^{\alpha_1\cdots\alpha_{j-\frac{1}{2}}}|j,P,\Omega_b,j_l,s_l,\rho/\lambda,p\rangle \\
& \times \langle j,P,\Omega_b,j_l,s_l,\rho/\lambda,p|\bar{J}_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}^{\beta_1\cdots\beta_{j-\frac{1}{2}}}|0\rangle \\
& + \text{higher resonances.}
\end{aligned}
\tag{10}
$$

We parameterize the matrix element $\langle 0|J_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}^{\alpha_1\alpha_2\cdots\alpha_{j-\frac{1}{2}}}|j,P,\Omega_b,j_l,s_l,\rho/\lambda,p\rangle$ in terms of the current-hadron coupling constant (pole residue) $f_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}$ and spinor $u^{\alpha_1\alpha_2\cdots\alpha_{j-\frac{1}{2}}}(p)$,

$$
\begin{aligned}
& \langle 0|J_{j,P,\Omega_b,j_l,s_l,\rho/\lambda}^{\alpha_1\alpha_2\cdots\alpha_{j-\frac{1}{2}}}|j,P,\Omega_b,j_l,s_l,\rho/\lambda,p\rangle \\
& = f_{j,P,\Omega_b,j_l,s_l,\rho/\lambda} u^{\alpha_1\alpha_2\cdots\alpha_{j-\frac{1}{2}}}(p).
\end{aligned}
\tag{11}
$$

As a result, we have

- for spin-$\frac{1}{2}$ baryon:

$$
\Pi^{(\text{Phy})}(p) = \frac{f_{1/2}^2}{m_{1/2}^2 - p^2}(\not{p} + m_{1/2}) + \text{higher resonances}, \tag{12}
$$

- for spin-$\frac{3}{2}$ baryon:

$$
\begin{aligned}
\Pi^{(\text{Phy})\alpha_1\beta_1}(p) =& \frac{f_{3/2}^2}{m_{3/2}^2 - p^2}(\not{p} + m_{3/2})\left(-g^{\alpha_1\beta_1} + \frac{\gamma^{\alpha_1}\gamma^{\beta_1}}{3}\right. \\
& \left. + \frac{2p^{\alpha_1}p^{\beta_1}}{3m_{3/2}^2} - \frac{p^{\alpha_1}\gamma^{\beta_1} - p^{\beta_1}\gamma^{\alpha_1}}{3m_{3/2}}\right) \\
& + \text{higher resonances}, \tag{13}
\end{aligned}
$$

- for spin-$\frac{5}{2}$ baryon:

$$
\begin{aligned}
\Pi^{(\text{Phy})\alpha_1\alpha_2\beta_1\beta_2}(p) =& \frac{f_{5/2}^2}{m_{5/2}^2 - p^2}(\not{p} + m_{5/2})\left[\frac{\tilde{g}^{\alpha_1\beta_1}\tilde{g}^{\alpha_2\beta_2} + \tilde{g}^{\alpha_1\beta_2}\tilde{g}^{\alpha_2\beta_1}}{2} - \frac{\tilde{g}^{\alpha_1\alpha_2}\tilde{g}^{\beta_1\beta_2}}{5} - \frac{1}{10}\left(\gamma^{\alpha_1}\gamma^{\beta_1} + \frac{\gamma^{\alpha_1}p^{\beta_1} - \gamma^{\beta_1}p^{\alpha_1}}{m_{5/2}} - \frac{p^{\alpha_1}p^{\beta_1}}{m_{5/2}^2}\right)\tilde{g}^{\alpha_2\beta_2}\right. \\
& - \frac{1}{10}\left(\gamma^{\alpha_2}\gamma^{\beta_1} + \frac{\gamma^{\alpha_2}p^{\beta_1} - \gamma^{\beta_1}p^{\alpha_2}}{m_{5/2}} - \frac{p^{\alpha_2}p^{\beta_1}}{m_{5/2}^2}\right)\tilde{g}^{\alpha_1\beta_2} - \frac{1}{10}\left(\gamma^{\alpha_1}\gamma^{\beta_2} + \frac{\gamma^{\alpha_1}p^{\beta_2} - \gamma^{\beta_2}p^{\alpha_1}}{m_{5/2}} - \frac{p^{\alpha_1}p^{\beta_2}}{m_{5/2}^2}\right)\tilde{g}^{\alpha_2\beta_1} \\
& \left. - \frac{1}{10}\left(\gamma^{\alpha_2}\gamma^{\beta_2} + \frac{\gamma^{\alpha_2}p^{\beta_2} - \gamma^{\beta_2}p^{\alpha_2}}{m_{5/2}} - \frac{p^{\alpha_2}p^{\beta_2}}{m_{5/2}^2}\right)\tilde{g}^{\alpha_1\beta_1}\right] + \text{higher resonances}, \tag{14}
\end{aligned}
$$

where we have used the following formulas

$$
\sum_s u(p,s)\bar{u}(p,s) = \not{p} + m_{1/2}, \tag{15}
$$

$$
\sum_s u^{\alpha_1}(p,s)\bar{u}^{\beta_1}(p,s) = (\not{p} + m_{3/2})\left(-g^{\alpha_1\beta_1} + \frac{\gamma^{\alpha_1}\gamma^{\beta_1}}{3} + \frac{2p^{\alpha_1}p^{\beta_1}}{3m_{3/2}^2} - \frac{p^{\alpha_1}\gamma^{\beta_1} - p^{\beta_1}\gamma^{\alpha_1}}{3m_{3/2}}\right), \tag{16}
$$

$$
\begin{aligned}
\sum_s u^{\alpha_1\alpha_2}(p,s)\bar{u}^{\beta_1\beta_2}(p,s) =& (\not{p} + m_{5/2})\left[\frac{\tilde{g}^{\alpha_1\beta_1}\tilde{g}^{\alpha_2\beta_2} + \tilde{g}^{\alpha_1\beta_2}\tilde{g}^{\alpha_2\beta_1}}{2} - \frac{\tilde{g}^{\alpha_1\alpha_2}\tilde{g}^{\beta_1\beta_2}}{5} - \frac{1}{10}\left(\gamma^{\alpha_1}\gamma^{\beta_1} + \frac{\gamma^{\alpha_1}p^{\beta_1} - \gamma^{\beta_1}p^{\alpha_1}}{m_{5/2}} - \frac{p^{\alpha_1}p^{\beta_1}}{m_{5/2}^2}\right)\tilde{g}^{\alpha_2\beta_2}\right. \\
& - \frac{1}{10}\left(\gamma^{\alpha_2}\gamma^{\beta_1} + \frac{\gamma^{\alpha_2}p^{\beta_1} - \gamma^{\beta_1}p^{\alpha_2}}{m_{5/2}} - \frac{p^{\alpha_2}p^{\beta_1}}{m_{5/2}^2}\right)\tilde{g}^{\alpha_1\beta_2} - \frac{1}{10}\left(\gamma^{\alpha_1}\gamma^{\beta_2} + \frac{\gamma^{\alpha_1}p^{\beta_2} - \gamma^{\beta_2}p^{\alpha_1}}{m_{5/2}} - \frac{p^{\alpha_1}p^{\beta_2}}{m_{5/2}^2}\right)\tilde{g}^{\alpha_2\beta_1} \\
& \left. - \frac{1}{10}\left(\gamma^{\alpha_2}\gamma^{\beta_2} + \frac{\gamma^{\alpha_2}p^{\beta_2} - \gamma^{\beta_2}p^{\alpha_2}}{m_{5/2}} - \frac{p^{\alpha_2}p^{\beta_2}}{m_{5/2}^2}\right)\tilde{g}^{\alpha_1\beta_1}\right],
\end{aligned}
\tag{17}
$$

with $\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$.

Conversely, the correlation function (9) can be calculated theoretically via the OPE method at the quark-gluon level. We take the current $J_{1/2,-,\Omega_b,1,0,\rho}(x)$ as an example to illustrate the involved technologies. Inserting the interpolating current $J_{1/2,-,\Omega_b,1,0,\rho}(x)$ (3) into the correlation function (9) and contracting the relevant quark fields using Wick's theorem, we find

$$
\begin{aligned}
\Pi^{(\mathrm{OPE})}(p) = {}& -4i\epsilon_{abc}\epsilon_{a'b'c'}\int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}px}\gamma_\mu\gamma_5 S_{cc'}^{(b)}(x)\gamma_{\mu'}\gamma_5 \\
& \times \Big\{ \mathrm{Tr}\big[\gamma_5 S_{bb'}^{(s)}(x)\gamma_5 C\partial_x^\mu \partial_y^{\mu'} S_{aa'}^{(s)T}(x-y)C\big] - \mathrm{Tr}\big[\gamma_5\partial_x^\mu S_{bb'}^{(s)}(x)\gamma_5 C\partial_y^{\mu'} S_{aa'}^{(s)T}(x-y)C\big]\Big\}_{y=0} \\
& +4\epsilon_{abc}\epsilon_{a'b'c'}\int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}px} g_s A^{\mu ad}(x)\gamma_\mu\gamma_5 S_{cc'}^{(b)}(x)\gamma_{\mu'}\gamma_5 \\
& \times \Big\{ \mathrm{Tr}\big[\gamma_5\partial_y^{\mu'} S_{bb'}^{(s)}(x-y)\gamma_5 CS_{da'}^{(s)T}(x)C\big] - \mathrm{Tr}\big[\gamma_5 S_{bb'}^{(s)}(x)\gamma_5 C\partial_y^{\mu'} S_{da'}^{(s)T}(x-y)C\big]\Big\}_{y=0} \\
& + \frac{\langle 0|g_s \bar{s}\sigma\cdot Gs|0\rangle}{96}\epsilon_{abc}\epsilon_{a'b'c'}\int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}px} g_s\gamma_\mu\gamma_5 S_{cc'}^{(b)}(x)\gamma_{\mu'}\gamma_5 \\
& \times \Big(\frac{\lambda_n}{2}\Big)^{ad}\Big\{\Big(\frac{\lambda_n}{2}\Big)^{da'} x_\nu \mathrm{Tr}\big[\gamma_5\partial_y^{\mu'} S_{bb'}^{(s)}(x-y)\gamma_5\sigma^{\mu\nu}\big] - \Big(\frac{\lambda_n}{2}\Big)^{bb'} x_\nu \mathrm{Tr}\big[\gamma_5\partial_y^{\mu'} S_{da'}^{(s)}(x-y)\gamma_5\sigma^{\mu\nu}\big]\Big\}_{y=0},
\end{aligned}
\tag{18}
$$

where a, b, $\cdots$ are color indices, $\lambda^n, n = 1,2,\cdots,8$ are the Gell-Mann matrix, $A^{\mu ad}(x) = A^{n\mu}(x)\big(\frac{\lambda_n}{2}\big)^{ad}$ is the gluon field, g_s is the strong interaction constant, and $S^{(b)}(x)$ and $S^{(s)}(x)$ are the full bottom- and strange-quark propagators, respectively, whose expressions are given in Appendix A. Inserting the expressions for the full quark propagators into (18) and performing the involved integrals, we have

$$
\Pi^{(\mathrm{OPE})}(p) = \slashed{p}\left(\int_{(m_b+2m_s)^2}^\infty \mathrm{d}s \frac{\rho(s)}{s-p^2} + \frac{m_s^2\langle 0|\bar{s}s|0\rangle^2}{12(m_b^2-p^2)}\right) + \text{other Lorentz structures},
\tag{19}
$$

where $\rho(s)$ is the QCD spectral density

$$
\begin{aligned}
\rho(s) = {}& -\frac{3}{64\pi^4}\int_{a_{\min}}^1 \mathrm{d}a \frac{(1-a)^3}{a^2}(m_b^2-as)^3 + \frac{3m_s^2}{16\pi^4}\int_{a_{\min}}^1 \mathrm{d}a \frac{(1-a)^2}{a}(m_b^2-as)^2 \\
& -\frac{3m_s\langle 0|\bar{s}s|0\rangle}{4\pi^2}\int_{a_{\min}}^1 \mathrm{d}a(1-a)(m_b^2-as) - \frac{m_b^2\langle 0|g_s^2 GG|0\rangle}{256\pi^4}\int_{a_{\min}}^1 \mathrm{d}a\frac{(1-a)^3}{a^2} \\
& -\frac{5\langle 0|g_s^2 GG|0\rangle}{256\pi^4}\int_{a_{\min}}^1 \mathrm{d}a(1-a)(m_b^2-as) - \frac{m_s^2\langle 0|g_s^2 GG|0\rangle}{192\pi^4}(1-a_{\min})^2 - \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{96\pi^2 M_B^2}(1-a_{\min}),
\end{aligned}
\tag{20}
$$

with $a_{\min} = m_b^2/s$; here, m_s is the mass of the strange quark, m_b is the mass of the bottom quark. and M_B^2 is the Borel parameter, introduced to make the Borel transform in the next step.

Finally, we match the phenomenological side (12) and the QCD representation (19) for the Lorentz structure $\slashed{p}$,

$$
\frac{f_{1/2,-,\Omega_b,1,0,\rho}^2}{m_{1/2,-,\Omega_b,1,0,\rho}^2 - p^2} + \text{higher resonances} = \int_{(m_b+2m_s)^2}^\infty \mathrm{d}s \frac{\rho(s)}{s-p^2} + \frac{m_s^2\langle 0|\bar{s}s|0\rangle^2}{12(m_b^2-p^2)},
\tag{21}
$$

According to the quark-hadron duality, the higher resonances can be approximated by the QCD spectral density above some effective threshold $s_0^{1/2,-,\Omega_b,1,0,\rho}$,

$$\frac{f^2_{1/2,-,\Omega_b,1,0,\rho}}{m^2_{1/2,-,\Omega_b,1,0,\rho} - p^2} + \int_{s_0^{1/2,-,\Omega_b,1,0,\rho}}^{\infty} \mathrm{d}s \frac{\rho(s)}{s-p^2} = \int_{(m_b+2m_s)^2}^{\infty} \mathrm{d}s \frac{\rho(s)}{s-p^2} + \frac{m_s^2 \langle 0|\bar{s}s|0\rangle^2}{12(m_b^2 - p^2)}. \tag{22}$$

Subtracting the contributions of the excited and continuum states, we obtain

$$\frac{f^2_{1/2,-,\Omega_b,1,0,\rho}}{m^2_{1/2,-,\Omega_b,1,0,\rho} - p^2} = \int_{(m_b+2m_s)^2}^{s_0^{1/2,-,\Omega_b,1,0,\rho}} \mathrm{d}s \frac{\rho(s)}{s-p^2} + \frac{m_s^2 \langle 0|\bar{s}s|0\rangle^2}{12(m_b^2 - p^2)}, \tag{23}$$

To improve the convergence of the OPE series and suppress the contributions from the excited and continuum states, it is necessary to make a Borel transform. As a result, we have

$$f^2_{1/2,-,\Omega_b,1,0,\rho}\mathrm{e}^{-m^2_{1/2,-,\Omega_b,1,0,\rho}/M_B^2} = \int_{(m_b+2m_s)^2}^{s_0^{1/2,-,\Omega_b,1,0,\rho}} \mathrm{d}s\rho(s)\mathrm{e}^{-s/M_B^2} + \frac{m_s^2 \langle 0|\bar{s}s|0\rangle^2}{12}\mathrm{e}^{-m_b^2/M_B^2}, \tag{24}$$

where M_B^2 is the Borel parameter. Applying the operator $-\frac{\mathrm{d}}{\mathrm{d}(1/M_B^2)}$ to (24) and dividing the resulting equation with (24), we obtain the mass sum rule

$$m^2_{1/2,-,\Omega_b,1,0,\rho} = \frac{-\dfrac{\mathrm{d}}{\mathrm{d}(1/M_B^2)}\left(\displaystyle\int_{(m_b+2m_s)^2}^{s_0^{1/2,-,\Omega_b,1,0,\rho}} \mathrm{d}s\rho(s)\mathrm{e}^{-s/M_B^2} + \frac{m_s^2 \langle 0|\bar{s}s|0\rangle^2}{12}\mathrm{e}^{-m_b^2/M_B^2}\right)}{\displaystyle\int_{(m_b+2m_s)^2}^{s_0^{1/2,-,\Omega_b,1,0,\rho}} \mathrm{d}s\rho(s)\mathrm{e}^{-s/M_B^2} + \frac{m_s^2 \langle 0|\bar{s}s|0\rangle^2}{12}\mathrm{e}^{-m_b^2/M_B^2}}. \tag{25}$$

In Sec. III, we will numerically analyze (25) and (24) and estimate the values of the mass $m_{1/2,-,\Omega_b,1,0,\rho}$ and the pole residue $f_{1/2,-,\Omega_b,1,0,\rho}$.

For other interpolating currents, we do the same analysis, and the corresponding OPE results are given in Appendix B.

III. NUMERICAL ANALYSIS

The sum rule (25) contains some parameters, various condensates, and quark masses, whose values are presented in Table 1. The values of m_b and m_s are the $\overline{MS}$ values. In addition to these parameters, we need to determine the working intervals of the threshold parameter $s_0^{j,P,\Omega_b,j_l,s_l,\rho/\lambda}$ and the Borel mass M_B^2 in which the masses and pole residues are stable. We take the continuum threshold to be approximately $m_{j,P,\Omega_b,j_l,s_l,\rho/\lambda} + (0.7 \pm 0.1)$ GeV, while the Borel parameter is determined by demanding that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from higher dimensional operators are small.

We define two quantities: the ratio of the pole contribution to the total contribution (Pole Contribution, abbreviated PC) and the ratio of the highest dimensional term in the OPE series to the total OPE series (Convergence, abbreviated CVG), as follows,

$$\mathrm{PC} \equiv \frac{\displaystyle\int_{(m_b+2m_s)^2}^{s_0^{j,P,\Omega_b,j_l,s_l,\rho/\lambda}} \mathrm{d}s\rho(s)\mathrm{e}^{-\frac{s}{M_B^2}}}{\displaystyle\int_{(m_b+2m_s)^2}^{\infty} \mathrm{d}s\rho(s)\mathrm{e}^{-\frac{s}{M_B^2}}},$$

$$\mathrm{CVG} \equiv \frac{\displaystyle\int_{(m_b+2m_s)^2}^{s_0^{j,P,\Omega_b,j_l,s_l,\rho/\lambda}} \mathrm{d}s\rho^{(d=7)}(s)\mathrm{e}^{-\frac{s}{M_B^2}}}{\displaystyle\int_{(m_b+2m_s)^2}^{s_0^{j,P,\Omega_b,j_l,s_l,\rho/\lambda}} \mathrm{d}s\rho(s)\mathrm{e}^{-\frac{s}{M_B^2}}}, \tag{26}$$

where $\rho^{(d=7)}(s)$ are the terms proportional to $\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle$ in the spectral density.

For the current $J_{1/2,-,\Omega_b,1,0,\rho}(x)$, the numerical results

Table 1. Input parameters required for calculations.

Parameter	Value
$\langle \bar{s}s \rangle$	$(0.8 \pm 0.1)\langle \bar{q}q \rangle$
$\langle \bar{q}q \rangle$	$-(0.24 \pm 0.01)^3 \mathrm{GeV}^3$
$\langle g_s\bar{s}\sigma Gs \rangle$	$(0.8 \pm 0.1)\langle \bar{s}s \rangle\mathrm{GeV}^2$
$\langle g_s^2GG \rangle$	$0.88 \pm 0.25 \mathrm{~GeV}^4$
m_b	$(4.18 \pm 0.03)\mathrm{GeV}$[40]
m_s	$(0.095 \pm 0.005)\mathrm{GeV}$[40]

are shown in Fig. 1. In Fig. 1(a), we compare the various condensate contributions as functions of M_B^2 with $s_0^{1/2,-,\Omega_b,1,0,\rho} = 6.95^2 \text{ GeV}^2$. From the figure, it is clear that the OPE has good convergence. Fig. 1(b) shows PC and CVG varying with M_B^2 at $s_0^{1/2,-,\Omega_b,1,0,\rho} = 6.95^2 \text{ GeV}^2$. The figure shows that the requirement PC $\geqslant 50\%$ gives $M_B^2 \leqslant 5.5 \text{ GeV}^2$. The dependences of the mass $m_{1/2,-,\Omega_b,1,0,\rho}$ and the pole residue $f_{1/2,-,\Omega_b,1,0,\rho}$ on the Borel

parameter M_B^2 are depicted in Fig. 1(c) and (d) at three different values of $s_0^{1/2,-,\Omega_b,1,0,\rho}$, respectively. It is obvious that the mass and the pole residue are stable in the interval $4.5 \text{ GeV}^2 \leqslant M_B^2 \leqslant 5.5 \text{ GeV}^2$. The mass and the pole residue are estimated to be $m_{1/2,-,\Omega_b,1,0,\rho} = (6.28^{+0.11}_{-0.10})\text{GeV}$ and $f_{1/2,-,\Omega_b,1,0,\rho} = (0.35 \pm 0.06)\text{GeV}^4$, respectively.

For other interpolating currents, the same analysis can be performed. We summarize our results in Table 2 and
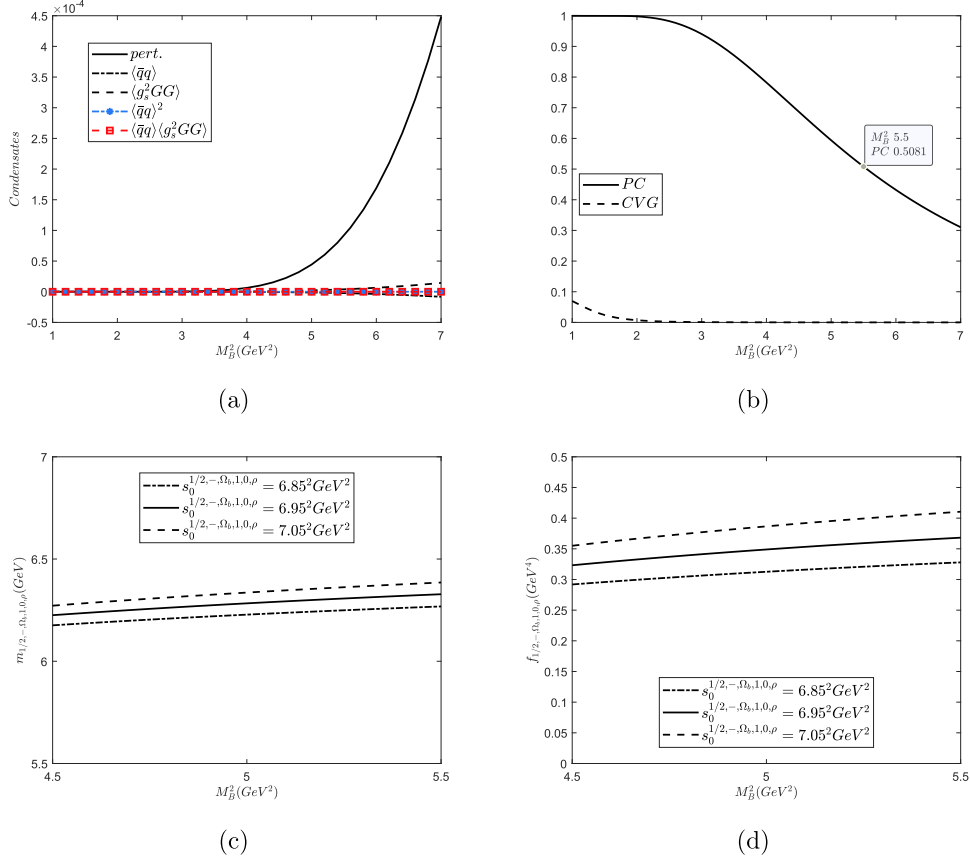
Fig. 1. (color online) For the interpolating current $J_{1/2,-,\Omega_b,1,0,\rho}(x)$: (a) denotes the various condensate contributions as functions of M_B^2 with $s_0^{1/2,-,\Omega_b,1,0,\rho} = 6.95^2 \text{ GeV}^2$; (b) represents PC and CVG varying with M_B^2 at $s_0^{1/2,-,\Omega_b,1,0,\rho} = 6.95^2 \text{ GeV}^2$; (c) and (d) depict the dependence of the mass and the pole residue on M_B^2 with three different values of $s_0^{1/2,-,\Omega_b,1,0,\rho}$, respectively.

Table 2. Masses and pole residues of the P-wave excited Ω_b states.

Multiples	Baryons (j^P)	Masses/GeV		Pole residues/GeV^4
		This work	Ref. [20]	
$[\Omega_b, 1, 0, \rho]$	$\Omega_b(\frac{1}{2}^-)$	$6.28^{+0.10}_{-0.10}$	$6.32^{+0.12}_{-0.10}$	0.35 ± 0.06
	$\Omega_b(\frac{3}{2}^-)$	$6.31^{+0.10}_{-0.11}$	$6.32^{+0.12}_{-0.10}$	0.19 ± 0.03
$[\Omega_b, 0, 1, \lambda]$	$\Omega_b(\frac{1}{2}^-)$	$5.75^{+0.05}_{-0.02}$	6.34 ± 0.11	$0.0183^{+0.0013}_{-0.0007}$
$[\Omega_b, 1, 1, \lambda]$	$\Omega_b(\frac{1}{2}^-)$	$6.33^{+0.10}_{-0.11}$	$6.34^{+0.09}_{-0.08}$	0.62 ± 0.10
	$\Omega_b(\frac{3}{2}^-)$	$6.37^{+0.10}_{-0.11}$	$6.34^{+0.09}_{-0.08}$	$0.36^{+0.06}_{-0.05}$
$[\Omega_b, 2, 1, \lambda]$	$\Omega_b(\frac{3}{2}^-)$	$6.34^{+0.09}_{-0.10}$	$6.35^{+0.13}_{-0.11}$	0.71 ± 0.11
	$\Omega_b(\frac{5}{2}^-)$	$6.54^{+0.07}_{-0.08}$	$6.36^{+0.13}_{-0.11}$	0.15 ± 0.02
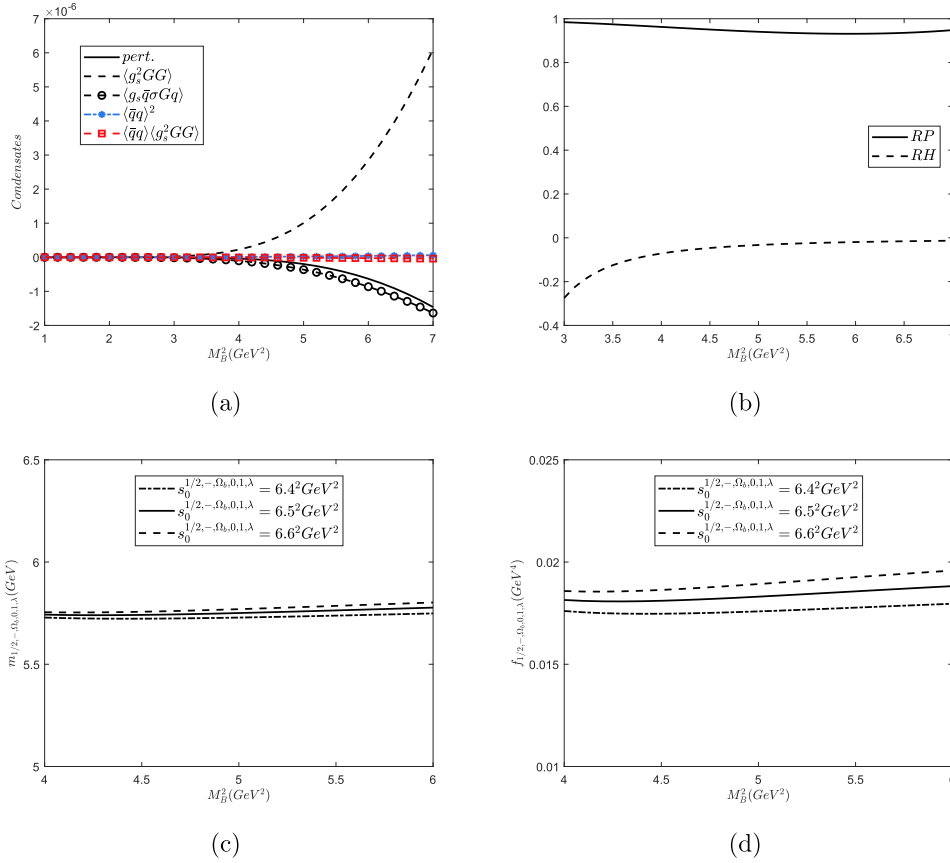
Fig. 2. (color online) For the interpolating current $J_{1/2,-,\Omega_b,0,1,\lambda}(x)$: (a) denotes the various condensate contributions as functions of M_B^2 with $s_0^{1/2,-,\Omega_b,0,1,\lambda} = 6.5^2$ GeV^2; (b) represents *RP* and *RH* varying with M_B^2 at $s_0^{1/2,-,\Omega_b,0,1,\lambda} = 6.5^2$ GeV^2; (c) and (d) depict the dependence of the mass and the pole residue on M_B^2 with three different values of $s_0^{1/2,-,\Omega_b,0,1,\lambda}$, respectively.

compare the obtained masses with the results in Ref. [20] estimated using the QCD sum rule method in the framework of heavy quark effective theory. It is clear that they are in agreement with each other within the inherent uncertainties of the QCD sum rule method, except for the multiplet $[\Omega_b, 0, 1, \lambda]$. We should provide some arguments regarding the result of the interpolating current $J_{1/2,-,\Omega_b,0,1,\lambda}(x)$ shown in Fig. 2. From Eqs. (B3) and (B4), we can see that all terms of the OPE series are proportional to the strange quark mass m_s or m_s^2, except for the second term in (B4). As a result, the gluon-condensate term is much larger than the other terms, and OPE is invalid in this case. Moreover, the corresponding mass and pole residue are much lower than the others. All in all, our model can not give reasonable results in this case.

IV. CONCLUSION

In this paper, we consider all *P*-wave Ω_b states represented by interpolating currents with a derivative and calculate the corresponding masses and pole residues us-

ing the QCD sum rule method. The results are summarized in Table 2. Because of the large uncertainties in our calculation compared with the small difference in the masses of the excited Ω_b states observed by the LHCb collaboration, it is necessary to study other properties of the *P*-wave Ω_b states represented by the interpolating currents investigated in the present work to gain a better understanding of the four excited Ω_b states observed by the LHCb collaboration. For example, we could study their decay widths. Our results in this paper are necessary input parameters when studying their decay widths using the QCD sum rule method or light-cone sum rule method.

APPENDIX A: QUARK PROPAGATORS

The full quark propagators are

$$S_{ij}^q(x) = \frac{i\,\slashed{k}}{2\pi^2 x^4}\delta_{ij} - \frac{m_q}{4\pi^2 x^2}\delta_{ij} - \frac{\langle\bar{q}q\rangle}{12}\delta_{ij} + i\frac{\langle\bar{q}q\rangle}{48}m_q\,\slashed{k}\delta_{ij} - \frac{x^2}{192}\langle g_s\bar{q}\sigma Gq\rangle\delta_{ij}$$

$$+ i\frac{x^2\,\slashed{k}}{1152}m_q\langle g_s\bar{q}\sigma Gq\rangle\delta_{ij} - i\frac{g_s t_{ij}^a G_{\mu\nu}^a}{32\pi^2 x^2}(\slashed{k}\sigma^{\mu\nu} + \sigma^{\mu\nu}\,\slashed{k}) + \cdots \tag{A1}$$

for light quarks, and

$$S_{ij}^Q(x) = i\int\frac{\mathrm{d}^4k}{(2\pi)^4}e^{-ikx}\left[\frac{\slashed{k}+m_Q}{k^2-m_Q^2}\delta_{ij} - \frac{g_s t_{ij}^a G_{\mu\nu}^a}{4}\frac{\sigma^{\mu\nu}(\slashed{k}+m_Q)+(\slashed{k}+m_Q)\sigma^{\mu\nu}}{(k^2-m_Q^2)^2} + \frac{\langle g_s^2 GG\rangle}{12}\delta_{ij}m_Q\frac{k^2+m_Q\,\slashed{k}}{(k^2-m_Q^2)^4} + \cdots\right] \tag{A2}$$

for heavy quarks. In these expressions, $t^a = \dfrac{\lambda^a}{2}$ and λ^a are the Gell-Mann matrices, g_s is the strong interaction coupling constant, and i, j are color indices.

APPENDIX B: SPECTRAL DENSITIES

We choose the Lorentz structures $\slashed{p}$, $\slashed{p}g^{\alpha\beta}$, and $\slashed{p}g^{\alpha_1\alpha_2}g^{\beta_1\beta_2}$ to obtain the sum rules for spin-1/2, spin-3/2, and spin-5/2 baryons, respectively. In this appendix, we will give the corresponding OPE results.

For the interpolating current $J_{3/2,-,\Omega_b,1,0,\rho}^\alpha(x)$,

$$\Pi^{(\mathrm{OPE})\alpha\beta}(p) = \slashed{p}g^{\alpha\beta}\left(\int_{(m_b+2m_s)^2}^\infty \mathrm{d}s\frac{\rho(s)}{s-p^2} - \frac{m_s^2\langle 0|\bar{s}s|0\rangle^2}{24(m_b^2-p^2)}\right) + \text{other Lorentz structures}, \tag{B1}$$

where $\rho(s)$ is the QCD spectral density,

$$\rho(s) = \frac{1}{384\pi^4}\int_{a_{\min}}^1 \mathrm{d}a\frac{(1-a)^3(4+a)}{a^2}(m_b^2-as)^3 - \frac{m_s^2}{64\pi^4}\int_{a_{\min}}^1 \mathrm{d}a\frac{(1-a)^2(2+a)}{a}(m_b^2-as)^2$$

$$+ \frac{m_s\langle 0|\bar{s}s|0\rangle}{8\pi^2}\int_{a_{\min}}^1 \mathrm{d}a\,a(1-a)(m_b^2-as) + \frac{m_b^2\langle 0|g_s^2 GG|0\rangle}{4608\pi^4}\int_{a_{\min}}^1 \mathrm{d}a\frac{(1-a)^3(4+a)}{a^2}$$

$$+ \frac{\langle 0|g_s^2 GG|0\rangle}{512\pi^4}\int_{a_{\min}}^1 \mathrm{d}a(1-a)(2+a)(m_b^2-as) + \frac{m_s^2\langle 0|g_s^2 GG|0\rangle}{2304\pi^4}(1-a_{\min})^2(2+a_{\min})$$

$$+ \frac{m_s\langle 0|g_s\bar{s}\sigma\cdot Gs|0\rangle}{24\pi^2}\int_{a_{\min}}^1 \mathrm{d}a\,a(2-a) + \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{576\pi^2 M_B^2}a_{\min}(1-a_{\min}). \tag{B2}$$

For the interpolating current $J_{1/2,-,\Omega_b,0,1,\lambda}(x)$,

$$\Pi^{(\mathrm{OPE})}(p) = \slashed{p}\left(\int_{(m_b+2m_s)^2}^\infty \mathrm{d}s\frac{\rho(s)}{s-p^2} + \frac{2m_s^2\langle 0|\bar{s}s|0\rangle^2}{3(m_b^2-p^2)} + \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{192\pi^2(m_b^2-p^2)}\right) + \text{other Lorentz structures}, \tag{B3}$$

where $\rho(s)$ is the QCD spectral density,

$$\rho(s) = -\frac{3m_s^2}{32\pi^4}\int_{a_{\min}}^1 \mathrm{d}a\frac{(1-a)^2}{a}(m_b^2-as)^2 - \frac{\langle 0|g_s^2 GG|0\rangle}{128\pi^4}\int_{a_{\min}}^1 \mathrm{d}a(1-a)(m_b^2-as)$$

$$+ \frac{m_s^2\langle 0|g_s^2 GG|0\rangle}{384\pi^4}(1-a_{\min})^2 + \frac{3m_s\langle 0|g_s\bar{s}\sigma\cdot Gs|0\rangle}{16\pi^2}\int_{a_{\min}}^1 \mathrm{d}a\,a. \tag{B4}$$

For the interpolating current $J_{1/2,-,\Omega_b,1,1,\lambda}(x)$,

$$\Pi^{(\text{OPE})}(p) = \not{p}\left(\int_{(m_b+2m_s)^2}^{\infty} ds \frac{\rho(s)}{s-p^2} + \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{192\pi^2(m_b^2-p^2)}\right) + \text{other Lorentz structures}, \tag{B5}$$

where $\rho(s)$ is the QCD spectral density,

$$\begin{aligned}
\rho(s) = &-\frac{1}{8\pi^4}\int_{a_{\min}}^1 da \frac{(1-a)^3}{a^2}(m_b^2-as)^3 + \frac{27m_s^2}{32\pi^4}\int_{a_{\min}}^1 da \frac{(1-a)^2}{a}(m_b^2-as)^2 + \frac{3m_s\langle 0|\bar{s}s|0\rangle}{\pi^2}\int_{a_{\min}}^1 da(1-a)(m_b^2-as) \\
&- \frac{m_b^2\langle 0|g_s^2 GG|0\rangle}{96\pi^4}\int_{a_{\min}}^1 da\frac{(1-a)^3}{a^2} - \frac{3\langle 0|g_s^2 GG|0\rangle}{128\pi^4}\int_{a_{\min}}^1 da\frac{(1-a)^2}{a}(m_b^2-as) + \frac{\langle 0|g_s^2 GG|0\rangle}{128\pi^4}\int_{a_{\min}}^1 da(1-a)(m_b^2-as) \\
&- \frac{3m_s^2\langle 0|g_s^2 GG|0\rangle}{128\pi^4}(1-a_{\min})^2 - \frac{m_s\langle 0|g_s\bar{s}\sigma\cdot Gs|0\rangle}{16\pi^2}\int_{a_{\min}}^1 da(4-7a) + \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{24\pi^2 M_B^2}(1-a_{\min}) \\
&- \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{96\pi^2 s}a_{\min}.
\end{aligned} \tag{B6}$$

For the interpolating current $J^{\alpha}_{3/2,-,\Omega_b,1,1,\lambda}(x)$,

$$\Pi^{(\text{OPE})\alpha\beta}(p) = \not{p}g^{\alpha\beta}\left(\int_{(m_b+2m_s)^2}^{\infty} ds \frac{\rho(s)}{s-p^2} - \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{576\pi^2(m_b^2-p^2)}\right) + \text{other Lorentz structures}, \tag{B7}$$

where $\rho(s)$ is the QCD spectral density,

$$\begin{aligned}
\rho(s) = &\frac{1}{96\pi^4}\int_{a_{\min}}^1 da \frac{(1-a)^3(3+a)}{a^2}(m_b^2-as)^3 - \frac{3m_s^2}{32\pi^4}\int_{a_{\min}}^1 da\frac{(1-a)^2(2+a)}{a}(m_b^2-as)^2 \\
&- \frac{m_s\langle 0|\bar{s}s|0\rangle}{2\pi^2}\int_{a_{\min}}^1 da(1-a)(1+a)(m_b^2-as) + \frac{m_b^2\langle 0|g_s^2 GG|0\rangle}{1152\pi^4}\int_{a_{\min}}^1 da\frac{(1-a)^3(3+a)}{a^2} \\
&- \frac{\langle 0|g_s^2 GG|0\rangle}{768\pi^4}\int_{a_{\min}}^1 da\frac{(1-a)^2(4-a)}{a}(m_b^2-as) - \frac{\langle 0|g_s^2 GG|0\rangle}{768\pi^4}\int_{a_{\min}}^1 da(1-a)(1+a)(m_b^2-as) \\
&+ \frac{m_s^2\langle 0|g_s^2 GG|0\rangle}{384\pi^4}(1-a_{\min})^2(2+a_{\min}) - \frac{m_s\langle 0|g_s\bar{s}\sigma\cdot Gs|0\rangle}{48\pi^2}\int_{a_{\min}}^1 da(3-4a+4a^2) \\
&- \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{144\pi^2 M_B^2}(1-a_{\min})(1+a_{\min}) - \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{288\pi^2 s}a_{\min}(1-a_{\min}).
\end{aligned} \tag{B8}$$

For the interpolating current $J^{\alpha}_{3/2,-,\Omega_b,2,1,\lambda}(x)$,

$$\Pi^{(\text{OPE})\alpha\beta}(p) = \not{p}g^{\alpha\beta}\left(\int_{(m_b+2m_s)^2}^{\infty} ds \frac{\rho(s)}{s-p^2} + \frac{5m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2 GG|0\rangle}{576\pi^2(m_b^2-p^2)}\right) + \text{other Lorentz structures}, \tag{B9}$$

where $\rho(s)$ is the QCD spectral density,

$$\rho(s) = \frac{1}{96\pi^4} \int_{a_{\min}}^1 \mathrm{d}a \frac{(1-a)^3(7+13a)}{a^2}(m_b^2 - as)^3 - \frac{3m_s^2}{32\pi^4} \int_{a_{\min}}^1 \mathrm{d}a \frac{(1-a)^2(6+a)}{a}(m_b^2 - as)^2$$

$$- \frac{m_s\langle 0|\bar{s}s|0\rangle}{2\pi^2} \int_{a_{\min}}^1 \mathrm{d}a(1-a)(5-7a)(m_b^2 - as) + \frac{m_b^2\langle 0|g_s^2GG|0\rangle}{1152\pi^4} \int_{a_{\min}}^1 \mathrm{d}a \frac{(1-a)^3(7+13a)}{a^2}$$

$$- \frac{\langle 0|g_s^2GG|0\rangle}{384\pi^4} \int_{a_{\min}}^1 \mathrm{d}a(1-a)(2+a)(m_b^2 - as) + \frac{m_s^2\langle 0|g_s^2GG|0\rangle}{384\pi^4}(1-a_{\min})^2(6+a_{\min})$$

$$+ \frac{m_s\langle 0|g_s\bar{s}\sigma \cdot Gs|0\rangle}{48\pi^2} \int_{a_{\min}}^1 \mathrm{d}a(1-4a+18a^2) - \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle}{144\pi^2 M_B^2}(1-a_{\min})(5-7a_{\min})$$

$$- \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle}{288\pi^2 s}a_{\min}(1+3a_{\min}). \tag{B10}$$

For the interpolating current $J_{5/2,-,\Omega_b,2,1,\lambda}^{\alpha_1\alpha_2}(x)$,

$$\Pi^{(\mathrm{OPE})\alpha_1\beta_1\beta_2}(p) = \not{p}g^{\alpha_1\alpha_2}g^{\beta_1\beta_2}\left(\int_{(m_b+2m_s)^2}^\infty \mathrm{d}s \frac{\rho(s)}{s-p^2} + \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle}{1728\pi^2(m_b^2 - p^2)} \right) + \text{other Lorentz structures}, \tag{B11}$$

where $\rho(s)$ is the QCD spectral density,

$$\rho(s) = \frac{1}{288\pi^4} \int_{a_{\min}}^1 \mathrm{d}a \frac{(1-a)^3(1+a)}{a}(m_b^2 - as)^3 - \frac{1}{288\pi^4} \int_{a_{\min}}^1 \mathrm{d}a \frac{(1-a)^4(1+2a)}{a}s(m_b^2 - as)^2$$

$$- \frac{1}{144\pi^4} \int_{a_{\min}}^1 \mathrm{d}a(1-a)^5 s^2(m_b^2 - as) - \frac{m_s^2}{32\pi^4} \int_{a_{\min}}^1 \mathrm{d}a(1-a)^2(m_b^2 - as)^2$$

$$+ \frac{m_s^2}{48\pi^4} \int_{a_{\min}}^1 \mathrm{d}a(1-a)^3 s(m_b^2 - as) - \frac{m_s\langle 0|\bar{s}s|0\rangle}{18\pi^2} \int_{a_{\min}}^1 \mathrm{d}a\,a(1-a)^2(3m_b^2 - (a+1)s)$$

$$+ \frac{m_b^2 m_s\langle 0|\bar{s}s|0\rangle}{18\pi^2}a_{\min}(1-a_{\min})^3 + \frac{m_b^2\langle 0|g_s^2GG|0\rangle}{3456\pi^4} \int_{a_{\min}}^1 \mathrm{d}a \frac{(1-a)^3(1+a)}{a}$$

$$- \frac{\langle 0|g_s^2GG|0\rangle}{1152\pi^4} \int_{a_{\min}}^1 \mathrm{d}a(1-a)(2-a^2)(m_b^2 - as) + \frac{\langle 0|g_s^2GG|0\rangle}{6912\pi^4} \int_{a_{\min}}^1 \mathrm{d}a(1-a)^2(3-4a^2)s$$

$$+ \frac{\langle 0|g_s^2GG|0\rangle s}{3456\pi^4}(1-a_{\min})^4 + \frac{m_b^2\langle 0|g_s^2GG|0\rangle}{3456\pi^4}a_{\min}(1-a_{\min})^3$$

$$- \frac{\langle 0|g_s^2GG|0\rangle s^2}{10368\pi^4 M_B^2}(1-a_{\min})^5 - \frac{m_s^2\langle 0|g_s^2GG|0\rangle}{3456\pi^4}(1-a_{\min})^2(1-4a_{\min})$$

$$+ \frac{m_s^2\langle 0|g_s^2GG|0\rangle s}{3456\pi^4 M_B^2}(1-a_{\min})^3 - \frac{m_s\langle 0|g_s\bar{s}\sigma \cdot Gs|0\rangle}{72\pi^2} \int_{a_{\min}}^1 \mathrm{d}a\,a^2(1-2a)$$

$$- \frac{m_s\langle 0|g_s\bar{s}\sigma \cdot Gs|0\rangle}{72\pi^2} \int_{a_{\min}}^1 \mathrm{d}a\,a(1-a) - \frac{m_s\langle 0|g_s\bar{s}\sigma \cdot Gs|0\rangle}{432\pi^2}a_{\min}(1-a_{\min})(1-8a_{\min})$$

$$- \frac{m_b^2 m_s\langle 0|g_s\bar{s}\sigma \cdot Gs|0\rangle}{108\pi^2 M_B^2}a_{\min}(1-a_{\min})^2 - \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle}{1296\pi^2 M_B^2}(1-a_{\min})^2(4+a_{\min})$$

$$+ \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle s}{1296\pi^2 M_B^4}(1-a_{\min})^2(5-4a_{\min}) - \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle s^2}{1296\pi^2 M_B^6}(1-a_{\min})^3$$

$$- \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle}{5184\pi^2 s}a_{\min} + \frac{m_s\langle 0|\bar{s}s|0\rangle\langle 0|g_s^2GG|0\rangle}{5184\pi^2 M_B^2}a_{\min}. \tag{B12}$$

In the above equations, $a_{\min} = m_b^2/s$, and M_B^2 is the Borel parameter.

References

[1]   R.Aaij *et al*. (LHCb Collaboration), Phys. Rev. Lett. **118**, 182001 (2017)

[2]   R. Aaij *et al*. (LHCb Collaboration), Phys. Rev. Lett. **124**, 082002 (2020)

[3]   S. S. Agaev, K. Azizi, and H. Sundu, Eur. Phys. Lett. **118**, 61001 (2017)

[4]   M. Karliner and J. L. Rosner, Phys. Rev. D **95**, 114012 (2017)

[5]   H. X. Chen, Q. Mao, W. Chen *et al*., Phys. Rev. D **95**, 094008 (2017)

[6]   G. Yang and J. L. Ping, Phys. Rev. D **97**, 034023 (2018)

[7]   K. L. Wang, L. Y. Xiao, X. H. Zhong *et al*., Phys. Rev. D **95**, 116010 (2017)

[8]   W. Wang and R. L. Zhu, Phys. Rev. D **96**, 014024 (2017)

[9]   H. Y. Cheng and C. W. Chiang, Phys. Rev. D **95**, 094018 (2017)

[10] M. Padmanath and N. Mathur, Phys. Rev. Lett. **119**, 042001 (2017)

[11] H. X. Huang, J. L. Ping, and F. Wang, Phys. Rev. D **97**, 034027 (2018)

[12] Z. G. Wang, Eur. Phys. J. C **77**, 325 (2017)

[13] Z. Zhao, D. D. Ye, and A. L. Zhang, Phys. Rev. D **95**, 114024 (2017)

[14] B. Chen and X. Liu, Phys. Rev. D **96**, 094015 (2017)

[15] S. S. Agaev, K. Azizi, and H. Sundu, Eur. Phys. J. C **77**, 395 (2017)

[16] C. S. An and H. Chen, Phys. Rev. D **96**, 034012 (2017)

[17] Z. G. Wang, X. N. Wei, and Z. H. Yan, Eur. Phys. J. C **77**, 832 (2017)

[18] Q. Mao, H. X. Chen, A. Hosaka *et al*., Phys. Rev. D **96**, 074021 (2017)

[19] W. Liang and Q. F. Lü, Eur. Phys. J. C **80**, 198 (2020)

[20] H. X. Chen, E. L. Cui, A. Hosaka *et al*., Eur. Phys. J. C **80**, 256 (2020)

[21] W. H. Liang and E. Oset, Phys. Rev. D **101**, 054033 (2020)

[22] Z. G. Wang, Int. J. Mod. Phys. A **35**, 2050043 (2020)

[23] L. Y. Xiao, K. L. Wang, M. S. Liu *et al*., Eur. Phys. J. C **80**, 279 (2020)

[24] H. Mutuk, Eur. Phys. J. A **56**, 146 (2020)

[25] H. M. Yang and H. X. Chen, Phys. Rev. D **101**, 114013 (2020)

[26] M. Karliner and J. L. Rosner, Phys. Rev. D **102**, 014027 (2020)

[27] K. Azizi, Y. Sarac, and H. Sundu, Phys. Rev. D **102**, 034007 (2020)

[28] L. Y. Xiao and X. H. Zhong, Phys. Rev. D **102**, 014009 (2020)

[29] K. L. Wang, L. Y. Xiao, and X. H. Zhong, Phys. Rev. D **102**, 034029 (2020)

[30] H. M. Yang, H. X. Chen, and Q. Mao, Phys. Rev. D **102**, 114009 (2020)

[31] H. M. Yang and H. X. Chen, Phys. Rev. D **104**, 034037 (2021)

[32] S. S. Agaev, K. Azizi, and H. Sundu, Eur. Phys. J. A **57**, 201 (2021)

[33] H. Q. Zhu, N. N. Ma, and Y. Huang, Eur. Phys. J. C **80**, 1184 (2020)

[34] H. J. Wang, Z. Y. Di, and Z. G. Wang, Commun. Theor. Phys. **73**, 035201 (2021)

[35] H. Q. Zhu and Y. Huang, Chin. Phys. C **44**, 083101 (2020)

[36] Z. G. Wang and H. J. Wang, Chin. Phys. C **45**, 013109 (2021)

[37] H. X. Chen, W. Chen, Q. Mao *et al*., Phys. Rev. D **91**, 054034 (2015)

[38] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979)

[39] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147**, 448 (1979)

[40] M. Tanabashi *et al*. (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)