Chinese Physics C Vol. 42, No. 1 (2018) 013102

How can the neutrino interact with the electromagnetic field?*

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Abstract: Maxwell electrodynamics in the fixed Minkowski space-time background can be described in an equivalent way in a curved Riemannian geometry that depends on the electromagnetic field and that we call the electromagnetic metric (e-metric for short). After showing such geometric equivalence we investigate the possibility that new processes dependent on the e-metric are allowed. In particular, for very high values of the field, a direct coupling of uncharged particles to the electromagnetic field may appear.

Keywords: neutrino, electromagnetic field, effective metric

PACS: 11.10.Lm, 03.50.-z **DOI:** 10.1088/1674-1137/42/1/013102

1 Introductory comments

In Ref. [1] Shabad and Usov have considered the maximum possible value of a magnetic field without revising QED. They arrived at the value 10^{42} Gauss. This value is only a few orders of magnitude less than the more speculative value of 10^{47} Gauss that should be obtained by the hypothetical super conducting cosmic strings.

Whatever its origin, it is almost compelling the belief that nature has provided a mechanism by means of which the electromagnetic field cannot attain an unlimited value, according to the ideas in the pioneering work of Born and Infeld[1)] (see also an analogous argument concerning the existence of a maximum of the gravitational field in Refs. [3, 4]).

The existence of such an upper bound is necessary for the introduction of an electromagnetic metric (e-metric) as we will describe in this paper. We then show that the dynamics of Maxwell electrodynamics in Minkowski space-time can be equivalently described as a non-linear electrodynamics in a curved space-time endowed with another metric written in terms of the electromagnetic field itself. In this representation, the linear Maxwell theory becomes non-linear.

Why change the simplicity of Maxwell linear theory as described in Minkowski geometry for a non-linear description? This change could be justified only if some new insight is produced by means of the introduction of the e-metric. This is precisely what we will analyze in this paper, under the hypothesis that this e-metric is universal, that is, it is perceived by all charged or uncharged bodies. The characterization of such an e-metric opens a new window to couple (charged or uncharged) particles of any kind to the electromagnetic field. We will present a direct example of such coupling for the case of an uncharged fermion, a neutrino for instance.

Before entering into this subject let us give a short pedagogical review of other ideas concerning the uses of an effective electromagnetic metric as proposed by different authors to deal with non-linear electrodynamics.

2 Introduction

The first example of the use of an effective metric to describe electromagnetic waves appeared in early 1923 due to W. Gordon [5]. In modern language, it states that the propagation of an electromagnetic wave in a moving dielectric characterized by a refractive index n and 4-velocity u^{μ} obeys the Hamilton-Jacobi equation

$$g^{\mu\nu}k_{\mu}k_{\nu}=0$$

where

$$g^{\mu\nu}=\eta^{\mu\nu}+(n^2-1)u^{\mu}u^{\nu} \tag{1}$$

is the effective metric. Note that in such an optical approximation only photons move in geodesics of $g^{\mu\nu}$: all other particles of the dielectric live in the Minkowski background. After that, the method of effective metric was largely used to describe the propagation of electromagnetic waves in non-linear theories (see for instance Refs. [6, 7, 10]).

Received 19 September 2017, Published online 29 November 2017

* Supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), FAPERJ (Fundação do Amparo Pesquisa do Rio de Janeiro, FINEP (Financiadora de Estudos e Projetos) and Coordenação do Aperfeiçoamento do Pessoal do Ensino Superior (CAPES)

1) In this theory an upper value for the electromagnetic field appears. See Ref. [2]

2.1 Causal structure of non-linear electrodynamics controlled by the effective metric

Just for completeness and to fix notation let us review briefly the Hadamard analysis [11] of the propagation of the discontinuities for non-linear electrodynamics in theories such that the Lagrangian depends only on the invariant $F \equiv F_{\mu\nu}F^{\mu\nu}$ of the form $L(F)$. The corresponding equation of motion takes the form

$$(L_F F^{\mu\nu})_{,\nu}=0, \tag{2}$$

where a comma means partial derivative and L_F represents the derivative $L_F \equiv \partial L/\partial F$. In the Hadamard method we start by defining Σ as the surface of discontinuity of the field by the equation

$$\Sigma(x^\mu)=constant.$$

The discontinuity of any function f is given by the relation

$$[f(x)]_\Sigma=\lim_{\epsilon\to 0^+}\big(f(x+\epsilon)-f(x-\epsilon)\big). \tag{3}$$

According to Hadamard the discontinuity of the field and its first derivatives are

$$[F_{\mu\nu}]_\Sigma=0, \quad [F_{\mu\nu,\lambda}]_\Sigma=f_{\mu\nu}k_\lambda, \tag{4}$$

where the wave vector k_λ is orthogonal to Σ, that is, $k_\lambda=\Sigma_{,\lambda}$ and $f_{\mu\nu}$ represents the intensity of the discontinuity of the field.

Let us consider the discontinuity of the equations of motion (2). We have

$$L_F f^{\mu\nu}k_\nu+2\chi L_{FF}F^{\mu\nu}k_\nu=0, \tag{5}$$

where χ is defined by

$$F^{\alpha\beta}f_{\alpha\beta}\equiv\chi.$$

Taking the discontinuity of the identity

$$F_{\mu\nu,\lambda}+F_{\nu\lambda,\mu}+F_{\lambda\mu,\nu}=0$$

and multiplying by $F^{\mu\nu}k^\lambda$, it follows that

$$\chi k^2+F^{\mu\nu}f_{\nu\lambda}k^\lambda k_\mu+F^{\mu\nu}f_{\lambda\mu}k^\lambda k_\nu=0. \tag{6}$$

Then, from Eq. (5), for $\chi\neq 0$,

$$L_F k^2-4L_{FF}F^{\mu\alpha}F_\alpha{}^\lambda k_\mu k_\lambda=0, \tag{7}$$

which can be written as

$$g^{\mu\nu}k_\mu k_\nu=0,$$

where the effective metric is

$$g^{\mu\nu}=L_F\eta^{\mu\nu}-4L_{FF}F^{\mu\alpha}F_\alpha{}^\nu. \tag{8}$$

We note that once k_ν is a gradient this equation implies that energetic photons in a non-linear electrodynamics controlled by the Lagrangian $L=L(F)$ do not propagate along the null cones of the geometry of the background, but instead follow null geodesics in another metric given by Eq. (8).

It is not difficult to generalize the above procedure for the case where the Lagrangian depends not only on F but also on the other invariant G given by

$$G=F^*_{\mu\nu}F^{\mu\nu}=\frac{1}{2}\eta^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}$$

where $\eta^{\alpha\beta\mu\nu}=-\varepsilon^{\alpha\beta\mu\nu}/\sqrt{-\eta}$ and η is the determinant of $\eta_{\mu\nu}$.

For later reference we present the case of Born-Infeld theory [2] (see also Einstein concerning an analogous maximum value for the gravitational field in Ref. [3]). The Lagrangian is given by

$$L=\beta^2\left(1-\sqrt{U}\right) \tag{9}$$

where

$$U\equiv 1+\frac{F}{2\beta^2}-\frac{G^2}{16\beta^4}$$

and β is a constant.

The discontinuities (waves) of this theory are geodesics in the effective metric given by (up to a non-relevant conformal term)

$$g_{\mu\nu}=\eta_{\mu\nu}-\frac{1}{\beta^2}\Phi_{\mu\nu} \tag{10}$$

where

$$\Phi_{\mu\nu}=F_\mu{}^\lambda F_{\lambda\nu}.$$

Born and Infeld showed that the Lagrangian (9) can be written in terms of a non-symmetric quantity

$$C_{\mu\nu}\equiv\eta_{\mu\nu}+F_{\mu\nu}$$

and it takes the form

$$L\sim det C_{\mu\nu}.$$

However, there is another way to re-write the Born-Infeld Lagrangian in terms of a symmetric tensor that is precisely the effective metric that controls the propagation of its discontinuities.

Indeed, it is not difficult [12] to show that the Born-Infeld Lagrangian may be written as

$$L_{BI}=\beta^2\left[1-\frac{1}{2}(\det g_{\mu\nu})^{\frac{1}{8}}\right]. \tag{11}$$

This expression shows that Born-Infeld dynamics may be given in terms of a functional of the determinant of the effective metric. We can then state the following self-consistent result: the Born-Infeld dynamics for the electromagnetic field is given by the extremum of the determinant of the corresponding effective metric that controls the causal structure of the nonlinear photons generated by this theory.

This effective metric procedure was then used as a tool to mimic certain configurations of gravity in the realm of general relativity (see for instance Ref. [10]). However, it is possible to go beyond such limited uses and analyze not only the properties of waves but properties of the dynamics of the electromagnetic field itself. In the next section we show how it is possible to describe all properties (not only the propagation of the waves) of the electrodynamics in the realm of the effective metric. In a subsequent section we will go one step further and investigate some consequences of accepting a deeper role for the effective metric, making the hypothesis that it has a true universal character.

3 Uses of the effective metric: the bridge formulation

In the present section we focus on the statement [13] according to which the standard Maxwell electrodynamics in a Minkowski background can alternatively be described as a non-linear theory described in a curved space endowed with a metric constructed with the electromagnetic field itself (the e-geometry for short). These two dynamics are one and the same. We present a rather simple and direct proof of this.

Let S_M represent the free part of Maxwell's action in Minkowski space-time:

$$S_M=-\int\sqrt{-\eta}F\,\mathrm{d}^4x$$

where η is the determinant of the Minkowski metric,

$$F=F_{\mu\nu}F^{\mu\nu}=F_{\alpha\mu}F_{\beta\nu}\eta^{\alpha\beta}\eta^{\mu\nu}.$$

From an analogy to the description of the propagation of waves in non-linear electrodynamics, we are led to consider the electromagnetic metric as[1)]

$$\hat{e}^{\mu\nu}=\eta^{\mu\nu}-\frac{1}{\beta^2}\Phi^{\mu\nu} \tag{12}$$

in which the constant β^2 has the dimension of $[F]$ and $\Phi^{\mu\nu}$ is the same as that in the preceding equations, that is $\Phi_{\mu\nu}=F_{\mu}{}^{\lambda}F_{\lambda\nu}$. The determinant of the e-metric is given by

$$\sqrt{-\hat{e}}=\frac{\sqrt{-\eta}}{U}$$

where

$$U=1+\frac{F}{2\beta^2}-\frac{G^2}{16\beta^4}.$$

We note that this expression is present in Born-Infeld theory. Thus we can apply a similar analysis to that made by these authors concerning the constant β. Indeed, in the case of a pure electrostatic field, $U=1-E^2/\beta^2$. From the form of the determinant and to keep the geometry (Eq. 12) well-behaved, the quantity U should not change sign. This yields a limitation for the field, that is

$$E^2<\beta^2$$

similar to the maximum value of the field that was hypothesized by Born and Infeld (see Ref. [2]). From now on we will attribute to the constant β the meaning of the maximum possible value for the field.

For later application, we make explicit the identity

$$\Phi_{\mu}{}^{\alpha}\Phi_{\alpha\nu}=\frac{1}{16}G^2\eta_{\mu\nu}-\frac{1}{2}F\Phi_{\mu\nu}.$$

The next step is to assume that the e-metric is Riemannian, that is, there is a covariant derivative constructed with this metric in such way that it obeys the metricity condition

$$\hat{e}^{\mu\nu}{}_{;\alpha}=0.$$

This allows us to define a connection

$$\hat{\Gamma}^{\alpha}_{\mu\nu}=\frac{1}{2}\hat{e}^{\alpha\lambda}\left(\hat{e}_{\lambda\mu,\nu}+\hat{e}_{\lambda\nu,\mu}-\hat{e}_{\mu\nu,\lambda}\right)$$

where a comma means the ordinary derivative. We use a hat to characterize objects that are represented in the e-metric. The inverse metric defined as

$$\hat{e}^{\alpha\nu}\hat{e}_{\nu\beta}=\delta^{\alpha}_{\beta}$$

is given by

$$\hat{e}_{\mu\nu}=A\eta_{\mu\nu}+B\Phi_{\mu\nu}$$

where

$$A=\frac{2+F/\beta^2}{2U}, \tag{13}$$

$$B=\frac{1}{\beta^2U}. \tag{14}$$

Remember that for any non-hat tensorial quantity all properties concerning the relationship between covariant and contra-variant indices are performed by the Minkowski metric $\eta_{\mu\nu}$. The effect of the map from Minkowski geometry into the e-metric does not change the covariant form of the electromagnetic tensor, that is

$$\hat{F}_{\mu\nu}=F_{\mu\nu}.$$

It then follows that for the contra-variant form

$$\hat{F}^{\mu\nu}=\hat{e}^{\mu\alpha}\hat{e}^{\nu\beta}F_{\alpha\beta}=(\Sigma F^{\mu\nu}+\Pi F^{*\mu\nu}) \tag{15}$$

Consequently the associated invariants $\hat{F}$ and $\hat{G}$ are related to their counterparts in Minkowski space by

$$\hat{F}=(\Sigma F+\Pi G), \tag{16}$$

$$\hat{G}=UG, \tag{17}$$

where

$$\Sigma=1+\frac{F}{\beta^2}+\frac{1}{4\beta^4}\left(F^2+\frac{G^2}{4}\right), \tag{18}$$

and

$$\Pi=\frac{1}{2\beta^2}G\left(\frac{1}{4\beta^2}F+1\right). \tag{19}$$

1) It has been shown [7] that waves in non-linear electrodynamics are described as geodesics in a generic e-metric of the form $a\eta^{\mu\nu}+b\Phi^{\mu\nu}$ where a and b were taken as functions of the invariants F and G.

The inverse expressions of formulas (15), (16) and (17) that relate F and G to their correspondents in the e-metric are very involved. However, we can obtain valuable information if we restrict our calculations up to order $O(1/\beta^2)$. We find

$$F^{\mu\nu}\approx\hat{F}^{\mu\nu}-\frac{1}{\beta^2}\left[\hat{F}\hat{F}^{\mu\nu}+\frac{\hat{G}}{2}{}^*\hat{F}^{\mu\nu}\right] \tag{20}$$

Thus

$$F\approx\hat{F}-\frac{1}{\beta^2}(\hat{F}^2+\frac{1}{2}\hat{G}^2)$$

$$G\approx\hat{G}-\frac{1}{2\beta^2}(\hat{F}\hat{G}).$$

With these preliminaries we are prepared to prove the equivalence of dynamics contained in the following proposition: the dynamics of Maxwell electrodynamics in Minkowski space-time can be equivalently described as a non-linear electrodynamics in a curved space-time endowed with another metric written in terms of the electromagnetic field itself.

Let us consider the action defined in the e-space:

$$\hat{S}=\int\sqrt{-\hat{e}}\hat{L}\mathrm{d}^4x \tag{21}$$

To write this action in terms of the Minkowski metric we must insert in it the functions of the determinant of the e-metric and the Lagrangian $\hat{L}$ in terms of the quantities F and G. Take the expression of the action of Maxwell dynamics in Minkowski geometry:

$$I_M=-\frac{1}{4}\int\sqrt{-\eta}F\mathrm{d}^4x$$

Using the bridge formulas, we obtain up to order $O(1/\beta^2)$

$$I_M=-\int\sqrt{-\hat{e}}\left(\frac{1}{4}\hat{F}-\frac{4}{32\beta^2}[\hat{F}^2+\hat{G}^2]\right)\mathrm{d}^4x$$

which is the form of Maxwell electrodynamics as it appears when represented in the e-metric space. It is natural that in this space, endowed with a metric that depends on the electromagnetic field, Maxwell linear theory appears in a non-linear form. This equivalence is not restricted to Maxwell theory but it can be displayed for any non-linear theory. Let us take the case of Born-Infeld dynamics. In the standard formulation in the rigid space-time described by Minkowski geometry, its action is given by

$$I_{BI}=-\int\sqrt{-\eta}\beta^2(1-\sqrt{U})\mathrm{d}^4x$$

and using the bridge formulas we obtain up to order $O(1/\beta^2)$

$$I_{BI}=-\int\sqrt{-\hat{e}}\left(\frac{1}{4}\hat{F}-\frac{5}{32\beta^2}[\hat{F}^2+\hat{G}^2]\right)\mathrm{d}^4x$$

We note that the non-linearities of Maxwell and Born-Infeld dynamics in the e-representation are quite similar. Indeed, the first non-linear terms of $O(1/\beta^2)$ have the same structure but are multiplied by factors 4/32 and 5/32 respectively.

Summarizing, we can state that from the above expressions it follows that Maxwell's dynamics can be described in two equivalent ways:

1) Standard formulation in the Minkowski background:

$$\left(\sqrt{-\eta}F^{\mu\nu}\right)_{,\nu}=0.$$

2) Alternative formulation in the e-metric:

$$\left(\sqrt{-\hat{e}}\left[\frac{U}{\Sigma}\hat{F}^{\mu\nu}-\frac{\Pi}{\Sigma}{}^*\hat{F}^{\mu\nu}\right]\right)_{,\nu}=0$$

where in the expressions of Σ and Π given by Eqs. (18, 19), F and G must be given in terms of $\hat{F}$,$\hat{G}$.

Using the approximation up to order $1/\beta^2$ we obtain

$$\left(\sqrt{-\hat{e}}\left[\hat{F}^{\mu\nu}-\frac{1}{2\beta^2}(\hat{F}\hat{F}^{\mu\nu}+\hat{G}{}^*\hat{F}^{\mu\nu})\right]\right)_{,\nu}\approx0$$

We note that in the e-metric formulation one must use the covariant derivative. Up to order $O(1/\beta^2)$ we have the relation

$$\hat{F}^{\mu\nu}{}_{;\nu}\approx\hat{F}^{\mu\nu}{}_{,\nu}-\frac{1}{2\beta^2}\hat{F}^{\mu\nu}\hat{F}_{,\nu}$$

where a comma (,) represents a simple derivative and (;) represents a covariant derivative.

3.1 The source

Let us now consider the case in which a current is introduced, changing the dynamics into the form

$$\frac{1}{\sqrt{-\eta}}\left(\sqrt{-\eta}F^{\mu\nu}\right)_{,\nu}=J^\mu \tag{22}$$

This can be written in terms of the e-metric in the approximation we are considering as

$$\frac{\left(1-\dfrac{\hat{F}}{2\beta^2}\right)}{\sqrt{-\hat{e}}}\left[\sqrt{-\hat{e}}\left(\hat{F}^{\mu\nu}-\frac{1}{2\beta^2}(\hat{F}\hat{F}^{\mu\nu}+\hat{G}{}^*\hat{F}^{\mu\nu})\right)\right]_{,\nu}\approx J^\mu. \tag{23}$$

or, using the covariant derivative (;) it becomes

$$\left(\hat{F}^{\mu\nu}-\frac{1}{2\beta^2}(\hat{F}\hat{F}^{\mu\nu}+\hat{G}{}^*\hat{F}^{\mu\nu})\right)_{;\nu}\approx\hat{J}^\mu, \tag{24}$$

where

$$\hat{J}^\mu=(1+\frac{\hat{F}}{2\beta^2})J^\mu.$$

The conservation of the current can thus be expressed either in the Minkowski background

$$J^\mu_{,\mu}=0$$

or equivalently in the e-metric as

$$\hat{J}^{\mu}_{;\mu}=0$$

using the covariant derivative of the e-metric.

Let us pause and consider what we have achieved. We have shown that there are two equivalent formulations of Maxwell electrodynamics, to wit:

1) Maxwell linear action in a given frozen metric, say, Minkowski geometry $\eta_{\mu\nu}$;

2) A non-linear action written in terms of an electromagnetic metric described in terms of the electromagnetic tensor.

The description of electrodynamics in either one of these approaches is just a matter of taste. This equivalence is valid for any non-linear electrodynamics that preserves gauge invariance. Besides, the background does not need to be restricted to Minkowski space-time but may be any metric $g_{\mu\nu}$. Thus, not only can electromagnetic waves be described in terms of the effective e-metric, but the dynamics of the electromagnetic field itself can be described in the e-metric framework.

All this is straightforward. As mentioned earlier, changing from the simplicity of Maxwell linear theory as described in Minkowski geometry to a non-linear description can only be justified if some new insight is produced by means of the introduction of the e-metric. This is precisely what we will analyze in the rest of this paper.

We start such a proposal by suggesting that the e-metric be treated as universal and that it not only describes the dynamics of the electromagnetic field but influences all kind of bodies, all kind of fields and particles. In this case, what about uncharged bodies? Should they interact with the e-metric? Should this mean that uncharged bodies can interact with the electromagnetic field? This unexpected result becomes a true possibility due to the equivalence between the description of the electromagnetic field in a fixed Minkowski geometry and in the space-time controlled by the e-metric. It is in the realm of the analysis of test particles that we will explore such an idea here.

The world of elementary particles shows that there are particles that do not interact directly with the electromagnetic field in the Standard Model. Another way to state this fact is to say that these particles do not have electric charge. This is an obvious statement in the standard gauge framework. However, in the e-metric formulation such an identification is no longer true. We can consider, as a working hypothesis, that it may be possible to generate a direct interaction of a neutral particle, say a neutrino, and the electromagnetic field (see however for a different approach the interesting works Refs. [8] [9]). The theory of gravity suggests a way to realize this. Indeed, we have learned from general relativity that the existence of a non-flat metric changes the properties of measuring distances and times in an universal way. Otherwise it is just a convenient tool to describe certain particular properties, as is the case of Gordon's analysis of propagation of light in moving dielectrics or in Unruh's sound propagation [10].

Thus it seems worth investigating the consequences of the hypothesis of universality of the e-metric, representing the modification it produces on all kind of matter embedded in an electromagnetic field.

In this vein, charged particles acquire two channels of interaction: the standard one (that needs a charge to establish the contact of the body with the electromagnetic field) and another one mediated by a modification of the geometry, a process that is made theoretically possible only after the introduction of the e-metric associated with the electromagnetic field. Assuming universality of the geometric structure implies that uncharged bodies interact with the electromagnetic field only through the e-metric channel.

Then an immediate question arises: how can this proposal be reconciled with the observation that there exist particles which are inert in an electromagnetic field? The answer should be found by analysis of the intensity of the field. Indeed, the effects of such a geometric formulation are important only for very high values of the field, on the verge of its maximum possible value, which we call β.

All the new results in the present work rely upon the existence of such an upper limit. If this value is taken to be infinite, then the e-metric becomes identical to the Minkowski background and we recover that uncharged bodies do not interact with the electromagnetic field. In this vein, the new properties concerning the possibility of a new channel of interaction with the electromagnetic field do not conflict with standard observations in ordinary values for the electromagnetic field. Only for very high fields do some new effects appear, depending upon the smallness of the quantity $1/\beta$ that controls the intensity of the e-metric. In other words, we have shown that the presence of an equivalent metric associated with the space-time can be used alternatively to describe the electromagnetic field. Having defined this electromagnetic metric, two ways to couple fields to the electromagnetic field appear:

1) The standard minimal coupling principle, which changes the derivative ∂_μ into the form $\partial_\mu - \mathrm{i}eA_\mu$;

2) Minimal coupling with the e-metric, which changes ∂_μ into a covariant derivative ∇_μ defined in terms of the connection of the metric.

The possibility of describing electrodynamics in terms of a modification of the metric of space-time opens a new scenario to investigate the interaction of particles of any kind with the electromagnetic field. Let us explore this possibility for a fermion.

4 Influence of the electromagnetic field on charged and uncharged fermions

In Minkowski space-time a free fermion is described by Dirac's equation:

$$\mathrm{i}\gamma^{\mu}\partial_{\mu}\Psi - m\Psi = 0,$$

where we take $\hbar = c = 1$. In the presence of an electromagnetic field, the gauge principle states that the interaction of a fermion endowed with charge e is provided by the substitution

$$\partial_{\mu} \to \partial_{\mu} - \mathrm{i}eA_{\mu}.$$

This form of coupling requires the existence of a charge e. Equivalently, only charged particles interact with the electromagnetic field. However, the possibility of treating the electromagnetic interaction in terms of a modification of the metric opens another theoretical treatment that is worth examining.

First of all we recognize that the existence of a dimensionless metric allows the possibility to couple any particle to the electromagnetic field through the minimal coupling principle in a similar way as is done in the gravitational interaction in the framework of general relativity. The starting point concerns the definition of an internal connection $\hat{\Gamma}_{\mu}$, according to Fock and Ivanenko. Thus the minimal coupling of an uncharged fermion to an electromagnetic field takes the form

$$\mathrm{i}\hat{\gamma}^{\mu}\hat{\nabla}_{\mu}\Psi - m\Psi = 0, \tag{25}$$

where the internal covariant derivative is given by

$$\hat{\nabla}_{\mu}\Psi = \partial_{\mu}\Psi - \hat{\Gamma}_{\mu}\Psi$$

through the Fock-Ivanenko coefficient defined by [14]

$$\hat{\Gamma}_{\mu} = -\frac{1}{8}\left(\hat{\gamma}^{\lambda}\hat{\gamma}_{\lambda,\mu} - \hat{\gamma}_{\lambda,\mu}\hat{\gamma}^{\lambda} - \hat{\Gamma}^{\varrho}{}_{\mu\alpha}(\hat{\gamma}^{\alpha}\hat{\gamma}_{\varrho} - \hat{\gamma}_{\varrho}\hat{\gamma}^{\alpha}).\right) \tag{26}$$

From the form of the metric,

$$\hat{e}^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{\beta^2}\Phi^{\mu\nu}$$

where

$$\Phi^{\mu\nu} = F^{\mu\alpha}F_{\alpha}{}^{\nu}.$$

Note that the gamma matrices satisfy the relation

$$\hat{\gamma}^{\mu}\hat{\gamma}^{\nu} + \hat{\gamma}^{\nu}\hat{\gamma}^{\mu} = 2\hat{e}^{\mu\nu}\mathbb{I}$$

where $\mathbb{I}$ is the identity of the Clifford algebra. We can then write $\hat{\gamma}^{\mu}$ in terms of the constant matrices of the Minkowski background γ^{μ}:

$$\hat{\gamma}^{\mu} = P^{\mu}{}_{\alpha}\gamma^{\alpha} \tag{27}$$

where

$$P^{\mu}{}_{\alpha} = \delta^{\mu}{}_{\alpha} - \frac{1}{\beta}F^{\mu}{}_{\alpha}$$

and

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}\mathbb{I}$$

The inverse covariant expression $\hat{\gamma}_{\mu} = \hat{e}_{\mu\nu}\hat{\gamma}^{\nu}$ is given by

$$\hat{\gamma}_{\mu} = \mathbb{M}_{\mu\alpha}\gamma^{\alpha}$$

where

$$\mathbb{M}_{\mu\alpha} = A\eta_{\mu\alpha} - \frac{1}{\beta}\left(A - \frac{F}{2\beta^2 U}\right)F_{\mu\alpha} + \frac{G}{4\beta^3 U}F^{*}_{\mu\alpha} + \frac{1}{\beta^2 U}\Phi_{\mu\alpha},$$

and the quantity A was defined in the inverse expression of the metric Eq. (13). Note that $\mathbb{M}_{\mu\alpha}$ has no definite symmetry. Using this in the expression for the spinor connection, we obtain

$$-8\hat{\Gamma}_{\mu} = P^{\lambda}{}_{\alpha}\left(\mathbb{M}_{\lambda\beta,\mu} - \hat{\Gamma}^{\varrho}_{\mu\lambda}\mathbb{M}_{\varrho\beta}\right)\Sigma^{\alpha\beta}, \tag{28}$$

where $\Sigma^{\alpha\beta} = \gamma^{\alpha}\gamma^{\beta} - \gamma^{\beta}\gamma^{\alpha}$. Now, we have

$$\hat{\gamma}^{\mu}\hat{\Gamma}_{\mu} = -\frac{1}{8}\left(\gamma^{\mu} - \frac{1}{\beta}F^{\mu}{}_{\varepsilon}\gamma^{\varepsilon}\right)Y_{\alpha\beta\mu}\Sigma^{\alpha\beta}$$

where

$$Y_{\alpha\beta\mu} = P^{\lambda}{}_{\alpha}\left(\mathbb{M}_{\lambda\beta,\mu} - \hat{\Gamma}^{\varrho}_{\mu\lambda}\mathbb{M}_{\varrho\beta}\right).$$

Using the identity

$$\gamma_{\mu}\Sigma_{\varrho\nu} = 2\eta_{\mu\varrho}\gamma_{\nu} - 2\eta_{\mu\nu}\gamma_{\varrho} + 2\mathrm{i}\varepsilon_{\mu\varrho\nu\sigma}\gamma^{5}\gamma^{\sigma}, \tag{29}$$

we set the equation for the fermion in a more suitable form.

We are interested in describing such a new form of interaction in the standard Minkowski background. To simplify our task we limit ourselves to $O(1/\beta^2)$. Let us list the approximations of the relations that we need to obtain the form of the covariant derivative and the determinant of the e-metric. We have:

$$\sqrt{-\hat{e}} \approx \sqrt{-\eta}(1 - \frac{F}{2\beta^2}) \approx \sqrt{-\eta}(1 - \frac{\hat{F}}{2\beta^2})$$

$$G \approx \hat{G} - \frac{\hat{F}\hat{G}}{2\beta^2}$$

$$A = \frac{2 + F/\beta^2}{2U} \approx 1$$

$$\mathbb{M}_{\mu\nu} \approx \eta_{\mu\nu} - \frac{1}{\beta}F_{\mu\nu} + \frac{1}{\beta^2}\Phi_{\mu\nu}$$

$$\hat{e}_{\mu\nu} \approx \eta_{\mu\nu} + \frac{1}{\beta^2}\Phi_{\mu\nu}$$

$$\hat{\gamma}_{\mu} \approx \gamma_{\mu} - \frac{1}{\beta}F_{\mu}{}^{\alpha}\gamma_{\alpha} + \frac{1}{\beta^2}\Phi_{\mu}{}^{\alpha}\gamma_{\alpha}$$

$$8\hat{\Gamma}_{\mu} \approx \left(\frac{1}{\beta}F_{\alpha\beta,\mu} - \frac{1}{\beta^2}(F_{\lambda\alpha}F^{\lambda}{}_{\beta,\mu} + \Phi_{\alpha\mu,\beta})\right)\Sigma^{\alpha\beta}.$$

From these expressions we obtain

$$\hat{\Gamma}^{\varrho}{}_{\mu\alpha}(\hat{\gamma}^{\alpha}\hat{\gamma}_{\varrho} - \hat{\gamma}_{\varrho}\hat{\gamma}^{\alpha}) \approx \frac{1}{\beta^2}\Phi_{\alpha\mu,\beta}\Sigma^{\alpha\beta}.$$

Then the equation of the uncharged fermion up to $O(1/\beta)$ written in the Minkowski representation takes the form

$$\mathrm{i}\gamma^{\mu}\partial_{\mu}\Psi+\frac{\mathrm{i}}{\beta}F^{\mu\nu}\gamma_{\mu}\partial_{\nu}\Psi+\frac{\mathrm{i}}{2\beta}\partial_{\alpha}F_{\mu}{}^{\alpha}\gamma^{\mu}\Psi-m\Psi=0. \quad (30)$$

Without any limit for the electromagnetic field, that is, in the present framework, to take the limit $\beta\to\infty$, the extra coupling terms disappear. This is the case of standard Maxwell theory: no limit for the energy of the field and consequently no electromagnetic interaction for the uncharged particle. At this point one should consider the possible limits on the correction that the term in β induces. A good reference for this is give in Refs. [8] and [9].

5 Back-reaction: the influence of uncharged fermions on the dynamics of the electromagnetic field

In the previous section we described the influence of the electromagnetic field on uncharged fermions. Let us now consider the back-reaction that affects the dynamics of the electromagnetic field. To simplify our presentation (and due to the fact that the value of β is very high) we will limit our analysis to first order $O(1/\beta.)$

We set the total action $S=S_{EM}+S_{\Psi}$. Assuming the minimal coupling of the fermion with the e-metric provides the following action for the fermion:

$$S_{\Psi}=\int\sqrt{-\hat{e}}\left(\frac{\mathrm{i}}{2}\bar{\Psi}\hat{\gamma}^{\mu}\hat{\nabla}_{\mu}\Psi-\frac{\mathrm{i}}{2}\hat{\nabla}_{\mu}\bar{\Psi}\hat{\gamma}^{\mu}\Psi-m\bar{\Psi}\Psi\right). \quad (31)$$

The hypothesis that matter of any kind (charged or not) interacts with the electromagnetic metric changes the dynamics of the field. Let us re-write this dynamics as it appears in the Minkowski representation. We obtain in $O(1/\beta)$

$$S_{\Psi}\approx S_D+S_{int} \quad (32)$$

where

$$S_D=\int\sqrt{-\eta}\left(\frac{\mathrm{i}}{2}\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi-\frac{\mathrm{i}}{2}\partial_{\mu}\bar{\Psi}\gamma^{\mu}\Psi-m\bar{\Psi}\Psi\right)$$

$$S_{int}\approx-\frac{\mathrm{i}}{2\beta}\int\sqrt{-\eta}F^{\mu\nu}Z_{\mu\nu}$$

and we have defined

$$Z_{\mu\nu}=\bar{\Psi}\gamma_{\nu}\partial_{\mu}\Psi-\partial_{\mu}\bar{\Psi}\gamma_{\nu}\Psi.$$

We note that $\mathrm{i}Z_{\mu\nu}$ is hermitian. The net effect of order $O(1/\beta)$ of the uncharged fermion is to generate an interaction provided by[1)]

$$L_{em}\approx\frac{\mathrm{i}}{2\beta}F^{\mu\nu}Z_{\mu\nu}.$$

Thus the corresponding dynamics of the electromagnetic field acquires the form

$$F^{\mu\nu}{}_{,\nu}=J^{\mu}+\frac{\mathrm{i}}{\beta}Z^{[\mu\nu]}{}_{,\nu}.$$

The term dependent on the current J^{μ} exists only for charged bodies, although the very small extra term is universal.

6 Final comments

The description of Maxwell electrodynamics in terms of a special modification of the metric of space-time opens a new possibility to describe the interaction of particles with the electromagnetic field besides the standard gauge form $\partial_{\mu}-\mathrm{i}eA_{\mu}$, a procedure that can be used only for charged particles (see also Ref. [16]).

In this paper we suggest that the electromagnetic metric could be used as a new channel of interaction for all particles with the electromagnetic field. This universality, which seems weird at first glance, could be reconciled with observations once the new effects appear only for extremely high values of the electromagnetic field. The formal existence of a regular electromagnetic metric requires the existence of an upper bound of the field,

1) A final remark concerning the quantity

$$Z_{\mu\nu}\equiv\bar{\Psi}\gamma_{\nu}\partial_{\mu}\Psi-\partial_{\mu}\bar{\Psi}\gamma_{\nu}\Psi.$$

The interaction of the spinor field with the two classical fields, gravity and electromagnetism, can be described using this second order tensor $Z_{\mu\nu}$. Gravity interacts with the symmetric part of $Z_{\mu\nu}$ and the electromagnetic field interacts with its antisymmetric part. Indeed, for gravity one sets the coupling of the gravitational field given by

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$

with the symmetric energy-momentum tensor $T_{\mu\nu}=Z_{\mu\nu}+Z_{\nu\mu}+$ hermitian conjugate (hc) under the form

$$L_{int}\approx\mathrm{i}h^{\mu\nu}Z_{\mu\nu}.$$

For the case of the electromagnetic interaction in the realm of the e-metric representation a similar structure occurs once the interaction of the fermion with the electromagnetic field takes the form

$$L_{em}\approx\mathrm{i}F^{\mu\nu}Z_{\mu\nu}.$$

which we named β. The present model is not able to provide a theoretical value for β[1]. However, the analysis described in Ref. [9] concerning electromagnetic effects on neutrinos can help in searching for this limit.

Finally, if there is no upper limit for the electromagnetic field, setting β to ∞, this corresponds to the standard formulation of electrodynamics. As there are abundant neutrinos in the universe, it should be interesting to analyze our present geometrical proposal in some astrophysical and cosmological situations, which could help fix limits for β, see for instance Ref. [1]. This is under investigation.

We would like to thank Ugo Moschella for a critical reading of a previous version of this paper.

Appendix A

Charged bodies in an electromagnetic field

Let us consider now the modifications of the electromagnetic force on a test particle endowed with velocity v_μ due to the presence of the associated e-metric.

We set

$$\hat{v}_\mu = v_\mu.$$

The contra-variant form is given by

$$\hat{v}^\mu = \hat{e}^{\mu\nu}\,\hat{v}_\nu = (1-\frac{E^2}{\beta^2})v^\mu - \frac{2}{\beta^2}q^\mu$$

where $q^\mu = \frac{1}{2}\eta^{\mu\nu\alpha\beta}E_\nu v_\alpha H_\beta$ is the heat flux (Poynting vector). Remember that for any non-hat tensorial quantity all properties concerning the relationship between covariant and contra-variant indices are performed by the Minkowski metric $\eta_{\mu\nu}$.

In the standard formulation of Maxwell's theory in Minkowski space-time, an uncharged particle does not interact with the electromagnetic field. The gauge principle that guides such an interaction needs a dimensional quantity, the charge, to implement this interaction. Thus, there is no room for coupling an uncharged particle directly to the electromagnetic field. However, from the knowledge of the e-metric a new possibility appears. In order to analyze the motion of the particle in the e-metric, we proceed from first principles and use a formal expression directly to analyze the acceleration suffered by a charged or an uncharged particle. We appeal to the intimate relationship between the field and the metric. The acceleration is given by

$$\hat{a}_\mu = \hat{v}_{\mu;\nu}\,\hat{v}^\nu = \left(\hat{v}_{\mu,\nu} - \hat{\Gamma}^\alpha_{\mu\nu}\,\hat{v}_\alpha\right)\hat{v}^\nu. \tag{A1}$$

We can then proceed and develop the connection to obtain

$$\hat{\Gamma}^\alpha_{\mu\nu}\,\hat{v}_\alpha\,\hat{v}^\nu = \frac{1}{2}\,\hat{e}_{\lambda\nu,\mu}\,\hat{v}^\lambda\,\hat{v}^\nu$$

Let us deal with the simplest case where we can set $q_\mu = 0$ and

$$v_{\mu,\nu} = a_\mu v_\nu.$$

Using the inverse metric $\hat{e}_{\mu\nu}$ we have

$$\hat{e}_{\lambda\nu,\mu}\,\hat{v}^\lambda\,\hat{v}^\nu = (1-\frac{E^2}{\beta^2})^2\left(X_{,\mu} - 2\gamma_{\lambda\nu}a^\lambda v^\nu v_\mu\right) \tag{A2}$$

where

$$X \equiv \frac{1}{U}\left(1+\frac{H^2}{\beta^2}\right).$$

Then the acceleration in the e-metric is given by:

$$\hat{a}_\mu = (1-\frac{E^2}{\beta^2})a_\mu - \frac{1}{2}(1-\frac{E^2}{\beta^2})^2 X_{,\mu}. \tag{A3}$$

where $a_\mu = qF_{\mu\nu}v^\nu$ is the acceleration in the limit $\beta\to\infty$ and q is the charge.

Then we can re-write the acceleration up to order $O(1/\beta^2)$ as

$$\hat{a}_\mu = \frac{q}{m}\,\hat{F}_{\mu\nu}\,\hat{v}^\nu - \frac{1}{2\beta^2}(E^2)_{,\mu} \tag{A4}$$

where we are using $c = 1$. This is the form of action of the electromagnetic field on a test particle. Note that correction of the Lorentz force appears only if the field is high enough, that is, if we cannot neglect terms of order E^2/β^2. For uncharged bodies only the second term of Eq. (A4) survives. If there is no limit for the values of the field and we can take $\beta\to\infty$, this formula reduces to the Lorentz force.

1) Besides the values described in Ref. [1], there are other forms of analysis as for instance in Ref. [15], which analyzes the critical field $E_{cr} = 2m^2c^3/e\hbar$.

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