

Theoretical approaches to alpha decay half-lives of super-heavy nuclei

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Abstract: We consider the systematics of α -decay half-lives of super-heavy nuclei versus the decay energy and the total α -kinetic energy. We calculate the half-lives using the experimental Q_α values. The computed half-lives are compared with the experimental data and with existing empirical estimates and are found to be in good agreement. Also, we obtain α -preformation factors from the ratio between theoretical and experimental results for some super-heavy nuclei and evaluate the standard deviation. The results indicate the acceptability of the approach.

Keywords: alpha decay, super heavy nuclei (SHN), half-life, preformation factor

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1 Introduction

The first correlation of the empirical formula was predicted by Geiger and Nuttall [1] and shaped the experimental values of $\log_{10}(T_{1/2})$ vs $Q_\alpha^{1/2}$. Independently, Gamow [2] and Gurney and Condon [3] analyzed the one-body problem for α -decay (AD) and derived the known Geiger-Nuttall (GN) correlation from first principles of quantum mechanics that formulated a function of the halftime, the energy Q_α and the proton number of daughter nucleus Z_d . Viola and Seaborg (VS) [4] considered the intercept parameters' linear dependence on the charge number of the daughter nucleus by the work of Gallagher and Rasmussen [4, 5]. A linear relation between the Geiger-Nuttall law, Z_d and Q_α quantity was considered by Brown [6] to be the best representation for describing the AD properties of super-heavy nuclei (SHN). Royer (R) [7] suggested another empirical formula for AD half-life (HL) where $\log(T_\alpha)$ depends on the decay energy, the atomic mass number and the charge number of the parent nucleus. Dong et al [8] derived an expression of Q_α value based on the liquid drop model, which can be used as an input to quantitatively predict the half-lives of unknown nuclei. AD typically occurs in the heaviest nuclides. Alpha particles were described in the investigations of radioactivity by Ernest Rutherford in 1899 [9] and Gamow had interpreted the theory of alpha decay (quantum penetration of α particles) via tunneling in 1928. The alpha particle is trapped in a potential well by the nucleus. There are many theoretical and experimental approaches which have investigated AD, α cluster radioactivity models and SHN, such as those presented in Refs. [10–28]. The first system-

atics of α -decay properties of SHN was performed by studying the half-life versus kinetic energy (KE) correlations in terms of atomic number (Z) and mass number (A). The AD HL obtained from clustering and scattering amplitudes given by self-consistent nuclear models for the nuclear shell structure and reaction dynamics for SHN with $Z = 104 - 120$ were reported in Ref. [21]. Budaca and Silisteanu studied the AD of SHN within the framework of the shell-model rate theory, and also calculated the HLs and resonance scattering amplitudes with self-consistent models for nuclear structure and reaction dynamics [29]. Silisteanu et al. solved the radial Schrödinger equation for coupled channel problems with outgoing asymptotic and resonance conditions to estimate the alpha-emission rates of ground and excited states of the heaviest elements [30, 32]. Earlier, the nuclear shell model (NSM) predicted that the next magic proton number beyond $Z = 82$ would be $Z = 114$. Recent microscopic nuclear theories suggest a magic island around $Z = 120, 124, \text{ or } 126$ and $N = 184$. The heavy elements with $Z = 107 - 112$ have been successfully synthesized at GSI, Darmstadt and both theoretical and experimental facets of SHN have been extensively discussed [33–35]. The life-times of several isotopes of heavy elements with $Z = 102 - 120$ were used to calculate the quantum mechanical tunneling probability in a Wentzel-Kramers-Brillouin (WKB) framework and microscopic nucleus-nucleus potential with the DD (density dependent) M3Y effective nuclear interaction [36, 37]. This manuscript is organized as follows. Section 2 gives a brief description of the empirical approach to AD HL for isotopes of SHN. In Section 3 the penetration probability is summarized and the standard deviation evaluated.

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In Section 4 results and discussion are provided, and the conclusion of the work is given as Section 5.

2 The empirical approach

The Geiger-Nuttall (GN) law is given by $\log_{10} T_{1/2(\alpha)}^{GN} = aQ_{\alpha}^{-1/2} + b$, where a and b are the coefficients which are determined by fitting experimental data for each isotopic chain and Q_{α} (MeV) is the total energy of the α -decay process (α -decay Q value). The decay Q_{α} values for measured super-heavy α emitters can be obtained from the measured α -particle kinetic energy (KE) E_{α} using the following expression:

$$Q_{\alpha} = \frac{A_p}{A_p - 4} E_{\alpha}^{\text{exp}} + [6.53(Z_d - 2)^{7/5} - 8.0(Z_d - 2)^{2/5}] \times 10^{-5} (\text{MeV}), \quad (1)$$

where the first and second terms are the standard recoil and an electron shielding correction in a systematic manner, respectively, as suggested by Perlman and Rasmussen [38], A and Z are the mass and atomic numbers of the parent nucleus [39, 51], E_{α}^{exp} is the measured kinetic energy of α -particles, and the last term in Eq. (1) is the screening energy.

Dong et al proposed a formula for the α -decay Q values of SHN based on a liquid drop model (LDM) [8, 40–42]. We have calculated the $Q_{\alpha}^{\text{theor}}$ value using the equation,

$$Q_{\alpha}^{\text{theor}} = \alpha Z A^{-4/3} (3A - Z) + \beta \left(\frac{N - Z}{A} \right)^2 + \gamma \left[\frac{|N - 152|}{N} - \frac{|N - 154|}{N - 2} \right] + \delta \left[\frac{|Z - 110|}{Z} - \frac{|N - 112|}{Z - 2} \right] + \varepsilon. \quad (2)$$

Here Z , N and A are the proton, neutron and mass numbers of the parent nuclei, respectively. The first two terms are the contributions from the LDM Coulomb energy and symmetry energy, respectively, while the next two account for the neutron and proton shell effects of $N = 152$ and $Z = 110$ respectively. The parameters involved were determined in [43–46] by fitting $N = 154$ experimental Q_{α} data points and setting the values $\alpha = 0.9373$ MeV, $\beta = -99.3027$ MeV, $\gamma = 16.0363$ MeV, $\delta = -21.5983$ MeV and $\varepsilon = -27.4530$ MeV [8]. We now analyze three phenomenological formulas of the empirical formulas for half-life systematics of SHN.

The first is the Royer (R) formula [7] which can be written as

$$\log_{10} T_{1/2(\alpha)}^R = aZ_p Q_{\alpha}^{-1/2} + bA_p^{1.6} Z^{1/2} + c, \quad (3)$$

where a , b and c are adjustable parameters that refer to each (Z_p, N_p) parity of the parent nucleus combination, which we denote as even-even (e-e), odd-even (o-e), even-odd (e-o) and odd-odd (o-o). These parameters were given in Ref. [7] and listed here in Table 1.

The second formula is the well-known Viola-Seaborg (VS) formula [4], which is written

$$\log_{10} T_{1/2(\alpha)}^{\text{VS}} = (aZ_p + b)Q_{\alpha}^{-1/2} + (cZ_p + d) + h_{Z-N}^{\text{VS}}, \quad (4)$$

where Z_p is the charge number of the parent nucleus, h_{Z-N}^{VS} is an even-odd hindrance term, and a , b , c and d are fitting parameters. The parameters used are taken from Ref. [47], see also Table 1. The hindrance term values were obtained from the original paper of Viola and Seaborg. Other sets of parameters are constantly provided by fits on updated and new experimental data [48] or different sets of highly precise data [49].

Table 1. Parameters taken from the original references for the VS [4], R [7] and mB1 and mB2 [51] formulas.

h		a				b				c				d	
VS	mB1	VS	mB1	R	mB2	VS	mB1	R	mB2	VS	mB1	R	mB2	VS	
e-e	-	-	-	1.6672	10.8238	-1.2216	0.5966	-	-	-26.3843	-56.9785	-	-	-	
e-o	0.1.066	0.4666	1.5744	13.0705	1.4763	14.7747	-23.392	0.5182	-1.3523	0.5021	-0.2746	-47.8867	-15.8306	-49.7080	-33.9069
o-e	0.772	0.6001	-	-	1.1499	11.1462	-	-	-1.0402	0.5110	-	-	-12.6186	-39.0096	-
o-o	1.114	0.820	-	-	1.2451	14.7405	-	-	-1.2134	0.4666	-	-	-11.1310	-41.7227	-

The third formula is the Brown formula, obtained from the semi-classical (WKB) approximation and fit to the experimental data [50, 51], and given by

$$\log T_{\alpha}^{\text{B}} = 9.54Z_d^{0.6} Q_{\alpha}^{-1/2} - 51.37. \quad (5)$$

The parameters are determined by fitting to the available experimental data from [29]. Budaca et al. expressed the modified Brown formula with comparison and fitting of the VS and R formulas. The first modified Brown

fit (mB1) will have the parameters a , b and c parity-independent with an additional hindrance term differentiated by parity [51]

$$\log T_{\alpha}^{\text{mB1}} = a(Z_p - 2)^b Q_{\alpha}^{-1/2} + c + h_{Z-N}^{\text{mB1}}. \quad (6)$$

The parameters a , b and c , parity and hindrance terms are for the mB1 formula. The modified Brown formula (mB2) in Ref. [51] is chosen as:

$$\log T_{\alpha}^{\text{mB2}} = a_{Z-N} (Z_p - 2)^{b_{Z-N}} Q_{\alpha}^{-1/2} + c_{Z-N}. \quad (7)$$

The a , b and c parameters are shown in Table 1. The parameters of the empirical formulae are usually determined by fitting to a large amount of data, which may yield significant errors. This means that the relation between half-life, reaction energy and number of constituent nucleons is in fact quite complicated [51].

3 α -preformation factor

Lovas et al discussed the microscopic theory of alpha cluster radioactivity decay in 1998. The preformation probability is defined in quantum mechanics for a two-cluster component in the bound initial state of the parent nucleus [52]. It describes the influences of the different nuclear structure of properties of the parent, for instance the isospin asymmetry of the even-even nuclei [53], shell closure from ground and isomeric states and pairing effects [54, 55], and a double folding procedure using M3Y plus Coulomb two-body forces of the quadrupole deformations [56, 57]. Furthermore, several theoretical and experimental efforts have been made to calculate the preformation factor S_α [58–61]. The preformation factor S_α is obtained from the ratio of the calculated and the experimental half-lives. Also, S_α can be used for the prediction of half-lives of unknown super-heavy nuclei in a consistent way. The preformation factor may be also obtained from the work of Mohr

[62] reporting $S_\alpha = T_{1/2(\alpha)}^{\text{cal}}(s)/T_{1/2(\alpha)}^{\text{exp}}(s)$. Below, we plot the ratio $T_{1/2(\alpha)}^{\text{cal}}/T_{1/2(\alpha)}^{\text{exp}}$ versus the neutron number of the daughter nucleus (N_d). $\log_{10}(T_{1/2(\alpha)}^{\text{exp}}(s))$ values are reported in Table (2).

4 Results and discussion

We used two fitting schemes with the well-known empirical correlations Viola-Seaborg (VS) and Royer (R) formulas, and compared the results with the two modified versions of the Brown (B) formula, mB1 and mB2 [51]. In Table 2 we have calculated the half-lives for some super-heavy nuclei. The first, second and third columns represent the mass, proton and neutron numbers of the parent. The fourth, fifth and sixth column is the decay energy (Q_α) in MeV from Eq. (1) taken from [24], the theoretical decay energy from Eq. (2) [8, 40-42] and the alpha kinetic energy (E_α) in Oganessian et al. [24]. The seventh to eleventh columns are the experimental and the calculated half-lives with VS [4], R [7], mB1 and mB2 [51], respectively. We calculated the α -decay half-lives by comparing with the empirical formula for the SHN. For example, the half-life ^{267}Rf of VS value is 3.02180, which is better than R = 2.3343, mB1 = 2.3154 and mB2 = 2.4781. For ^{285}Cn , however, the mB2 value is 1.5072, which is better than VS = 2.5463, R = 1.8603 and mB1 = 1.4495.

Table 2. Logarithm α decay half-lives for SHN with various theoretical estimations and comparison with the results obtained by VS, R, the two versions of mB empirical formulas and experimental data.

A_p	Z_p	N_p	$Q_\alpha(\text{MeV})[24]$	$Q^{\text{theor}}(\text{MeV})[7]$	$E_\alpha(\text{MeV})[24]$	$\log(T_{1/2\alpha}(s))$				
						exp	VS	R	mB1	mB2
^{267}Rf	104	163	≤ 8.22	7.85	—	3.9180	3.0295	2.3343	2.3154	2.4781
^{271}Sg	106	165	8.65 ± 0.08	8.50	8.53 ± 0.08	2.1583	2.5463	1.8603	1.7543	1.8731
^{275}Hs	108	167	9.44 ± 0.07	9.15	9.30 ± 0.07	0.8239	0.8313	0.1576	0.1537	0.1789
^{279}Ds	110	169	9.84 ± 0.06	9.80	9.70 ± 0.06	-0.7447	0.5061	-0.1485	-0.2319	-0.2403
^{281}Ds	110	170	≤ 9.05	9.65	9.00527	0.9822	1.3765	0.0869	0.9449	1.3941
^{282}Cn	112	170	≤ 10.82	9.798	10.7741	-3.3010	-2.9445	-4.1132	-2.8041	-3.0098
^{283}Cn	112	171	9.67 ± 0.06	9.65	9.54 ± 0.06	0.6020	1.4590	0.8263	0.4687	0.4793
^{284}Cn	112	172	≤ 9.85	9.50	9.804097	-0.9956	-0.3740	-1.6237	-0.6427	-0.4225
^{285}Cn	112	173	9.29 ± 0.06	9.36	9.16 ± 0.06	1.5314	2.6255	1.9576	1.4495	1.5072
^{286}Fl	114	172	10.35 ± 0.06	10.45	10.20 ± 0.06	-0.7958	-0.8446	-2.0527	-1.1306	-0.9273
^{287}Fl	114	173	10.16 ± 0.06	10.30	10.02 ± 0.06	-0.2924	-0.2847	-0.2466	0.1155	0.7222
^{288}Fl	114	174	10.09 ± 0.07	10.16	9.95 ± 0.07	-0.0969	-0.1444	-1.4020	-0.5472	-0.2280
^{289}Fl	114	175	9.96 ± 0.06	10.02	9.82 ± 0.06	0.4313	1.2960	0.6539	0.2313	0.2160
^{290}Lv	116	174	11.00 ± 0.08	11.08	10.85 ± 0.08	-1.8239	-1.9466	-3.1346	-2.1267	-2.0454
^{291}Lv	116	175	10.89 ± 0.07	10.95	10.74 ± 0.07	-2.2006	-0.5974	-1.1729	-1.4263	-1.5344
^{292}Lv	116	176	10.80 ± 0.07	10.80	10.66 ± 0.07	-1.7447	-1.4544	-2.6886	-1.7204	-1.5575
^{293}Lv	116	177	10.67 ± 0.06	10.67	10.53 ± 0.06	-1.2757	-0.0445	-0.6556	-0.9699	-1.0563
^{294}Og	118	176	11.81 ± 0.06	11.71	11.65 ± 0.06	-2.744	-3.3125	-4.4707	-3.3173	-3.4016

A straight line gives a good fit to the preformation factor versus the neutron number of the nucleus. For example, we obtained the S_{α}^{mB2} value 1.00 for ^{293}Cn . The values for S_{α}^{VS} , S_{α}^R and S_{α}^{mB1} are obtained as 0.1, 0.58 and 0.83, respectively. To judge the agreement between the experimental and calculated values, we have evaluated the standard deviation, σ , for the α -decay half-lives. The standard deviation is given by [63],

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N [\log_{10} T_{(1/2\alpha)_i}^{\text{theor}} - \log_{10} T_{(1/2\alpha)_i}^{\text{exp}}]^2} \quad (8)$$

The results are summarized in Table 3.

Table 3. Standard deviation obtained with the Royer formula with the values of parameters taken from Ref. [10].

σ (Eq. (8))			
VS	R	mB1	mB2
0.7808	0.9655	0.5675	0.5709

In Fig. 1 we plot the preformation factor (S_{α}) for VS, V, mB1 and mB2 vs. versus neutron number of the daughter.

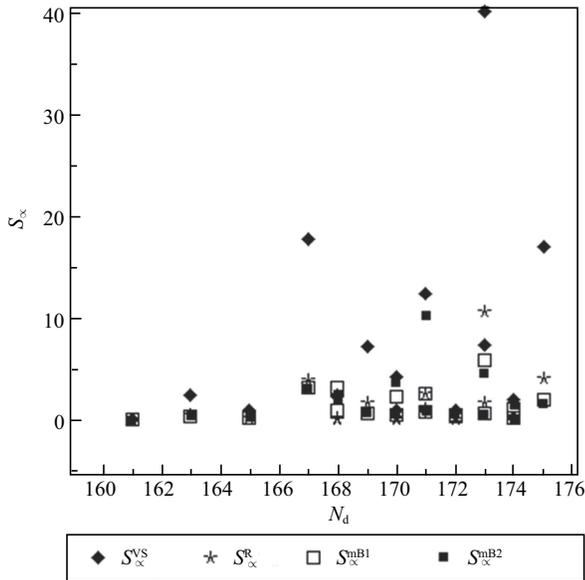


Fig. 1. $T_{1/2(\alpha)}^x/T_{1/2(\alpha)}^{\text{exp}}$ (preformation factors S_{α}^x for several super heavy α -emitters) versus the neutron number of the daughter nucleus (N_d).

In Fig. 2 we show how $\log T_{1/2}(\text{s})$ increases when decay energy increases. The behavior is in complete agreement with the Geiger-Nuttall rule.

In Fig. 3 we show separately for the S_{α} versus N_d that the VS values are better than R, mB1 and mB2 for some SHN.

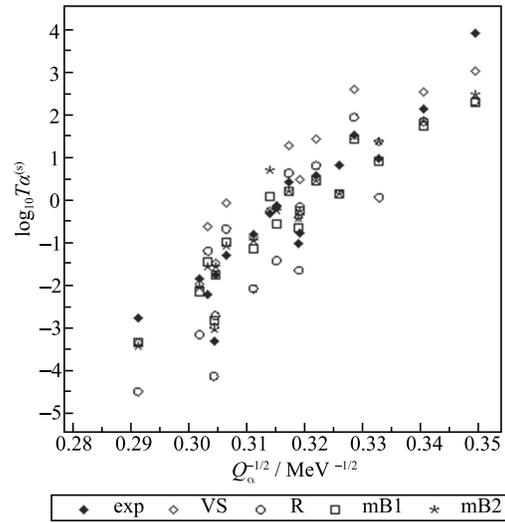
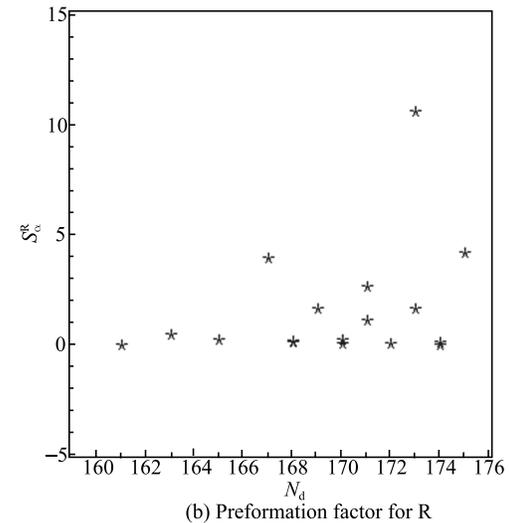
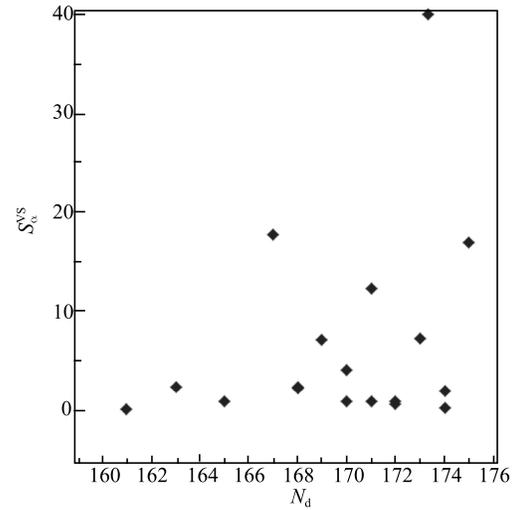
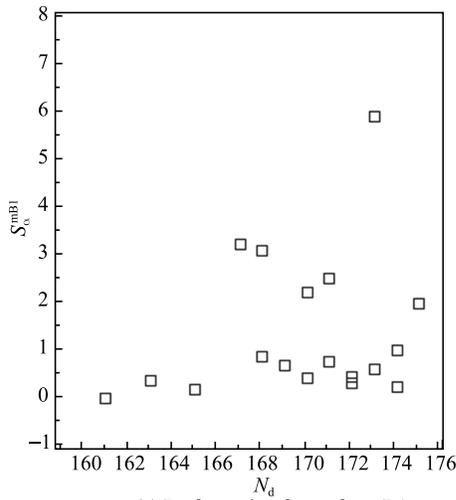
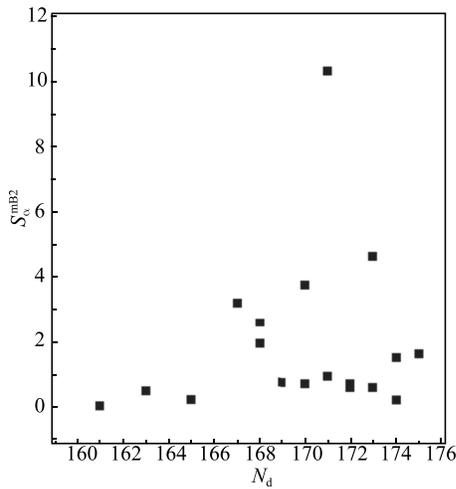


Fig. 2. Calculated $\log_{10} T_{1/2(\alpha)}$ versus the effective decay energy $Q_{(\alpha)}^{-1/2}/\text{MeV}^{-1/2}$.



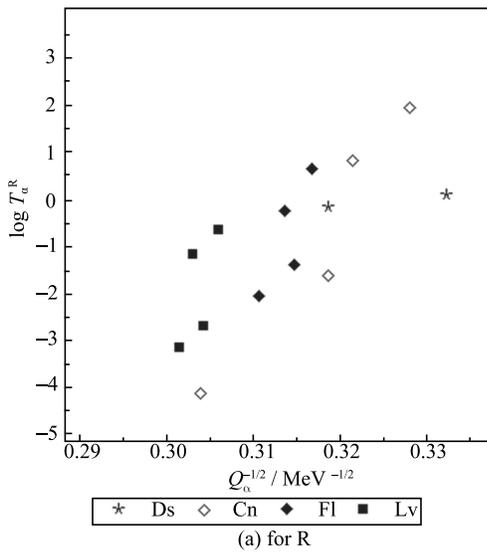


(c) Preformation factor for mB1

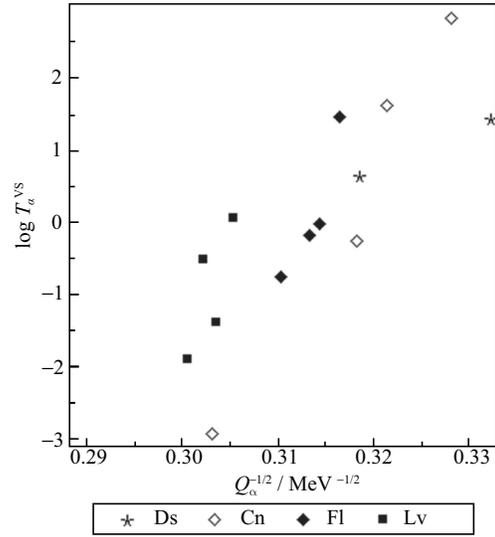


(d) Preformation factor for mB2

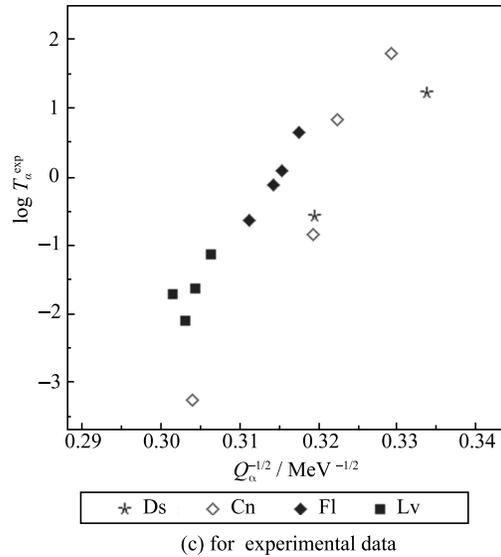
Fig. 3. Preformation factors S_{α}^x for SHN α -emitters versus the neutron number of the daughter nucleus (N_d).



(a) for R



(b) for VS



(c) for experimental data

Fig. 4. Calculated $\log_{10} T_{1/2(\alpha)}$ versus effective decay energy $Q_{(\alpha)}^{-1/2}$ for $Z_p = 110, 112, 114, 116$.

In Fig. 4, The calculated $\log_{10} T_{1/2(\alpha)}$ values are plotted versus the effective decay energy $Q_{\alpha}^{-1/2}(\text{MeV}^{-1/2})$ for Ds, Cn, Fl and Lv, showing the increasing behavior of $\log T_{1/2}$ for increasing effective decay energy. We have also compared the experimental and calculated data in Fig. 4. The results show an acceptable agreement with the experimental data. Indeed, the trend depicted in Fig. 4 for Ds, Cn, Fl and Lv does indicate a suitable correlation between the half-life and the α -energy available for decay, resembling a Geiger–Nuttall-like law.

5 Conclusion

In this manuscript, we considered the alpha decay half-lives for some super-heavy nuclei, such as Rf, Sg,

Hs, Ds, Cn, Fl, Lv and Og, and analyzed these using the Viola-Seaborg and Royer formulae and a new analysis in the Brown formula [51]. The computed half-life values were compared with the experimental data and indicate acceptable agreement with some of the systematic empirical correlations. From the ratio of the calculated and the experimental half-life, plotted versus N_d , a preformation factor for alpha decay is deduced. We depicted some

empirical and theoretical results compared with experimental data for SHNs. Finally, we calculated the standard deviation of the logarithm of the half-life, and the comparison models depicted in Table 3 were thus found to be 0.7808, 0.9655, 0.5675 and 0.5709 for VS, R, mB1 and mB2, respectively.

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