Calculation and analysis of the magnetic field of a linearly tapered undulator *

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Abstract: There is an empirical formula describing the relationship between the peak magnetic field and the undulator structure parameters for a uniform-parameter hybrid undulator. In this paper, we investigate the relationship for a linearly tapered undulator through numerical calculation by using the code RADIA, and check it with the empirical formula. The results imply that this empirical formula is also effective for linearly tapered undulators at a big enough scope for the requirements of normal FEL experiments. Therefore, for a linearly tapered undulator, we can use the empirical formula to design the variation of the undulator gap. For the tapering rate demanded by normal FEL experiments, the gap of a linearly tapered undulator increases almost linearly, and the tapering rate will keep constant while adjusting the undulator gap with the same variation for each undulator period.

Key words: linearly tapered undulator, peak magnetic field, empirical formula

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1 Introduction

Free-electron lasers (FELs), such as oscillators, self-amplified spontaneous emission (SASE), and seeded harmonic generation (e.g. HGHG, EEHG) FELs, are capable of generating extremely high-brightness radiation from the THz to X-ray wavelengths [1–3]. However, for a uniform-parameter undulator, the FEL efficiency at saturation is roughly given by the FEL scaling parameter ρ [4], where ρ is typically of the order of 10^{-3}. Therefore, variable-parameter undulators were invented and in the beginning were focused on increasing the FEL radiation power.

With the rapid development of FEL theory and techniques in recent decades, many kinds of variable-parameter undulators have been proposed and used more and more in FEL facilities with many new purposes for both high gain and low gain FELs [5–8]. For example, tapered undulators have been utilized in SASE FELs to improve their poor temporal coherence. To achieve these purposes, people have developed many tapering strategies, such as step-tapering, linear tapering, several other complicated profiles presented in Refs. [9, 10], and so on. For a long undulator, tapering is usually implemented through multiple step-tapering, namely the undulator parameter is constant in each undulator module. However, in some cases there is only one or several undulator modules, for instance, in oscillator FELs or the insertion device in storage rings, thus we have to adjust the undulator gap from one period to another to obtain the tapering.

As we know, there is an empirical formula describing the relations between the peak magnetic field and the undulator structure parameters for a uniform-parameter hybrid undulator. In this paper, we take the linearly tapered undulator as an example, to investigate the relationship between the peak magnetic field and the varying undulator gap and check the results with the empirical formula.

2 The magnetic field of a uniform-parameter undulator

For a uniform-parameter hybrid undulator, the peak magnetic field B_0 on the axis can be given by the following empirical formula:

$$B_0=0.95a\mathrm{e}^{-b\frac{g}{\lambda_\mathrm{u}}+c(\frac{g}{\lambda_\mathrm{u}})^2},\ \ 0.07\leqslant\frac{g}{\lambda_\mathrm{u}}\leqslant0.7. \qquad (1)$$

Here, g and λ_u are the gap and period of the undulator respectively, and $a=0.55B_\mathrm{r}+2.835$, $b=-1.95B_\mathrm{r}+7.225$, $c=-1.3B_\mathrm{r}+2.97$, where B_r is the remanence of the permanent magnet. This formula was firstly given in Ref. [11], and was found to be a little larger ($<5\%$) than the result from practical measurement in our many years' experi-

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ence. Therefore, we slightly modified the formula by adding a coefficient 0.95 to ensure that we can achieve the object peak field when the undulator is constructed, such as in the undulator we have designed for the Shanghai DUV-FEL [12].

Another way to calculate the magnetic field of an undulator is simulation using the RADIA code [13]. We take a hybrid undulator composed of NdFeB magnet blocks and NdFeV magnet poles as an example, with the undulator structure parameters given in Table 1. In this undulator, the value of g/λ_{u} is about 0.41, which is in the applicable scope of the empirical formula. Using the parameters listed in Table 1, the magnetic field was simulated, giving the results shown in Fig. 1. From Fig. 1, the peak magnetic field on the axis is about $B_0 = 0.5884$ T, while from Eq. (1) it is 0.5698 T. The simulation result from RADIA is about 3% greater than that from Eq. (1).

Table 1. Main parameters of the undulator.

period length/mm	44
period number	30
gap/mm	18
remanence of magnet blocks/T	1.2
height of magnetic blocks/mm	45
width of magnetic blocks/mm	17
height of magnetic poles/mm	25
width of magnetic poles/mm	5

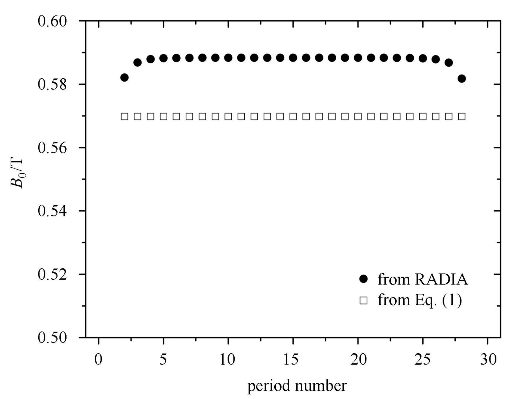

Fig. 1. The magnetic field B_y on the z axis of a uniform-parameter hybrid undulator.

3 The magnetic fields of tapered undulators

In this section we will numerically calculate the magnetic fields of tapered undulators and compare the results with those from Eq. (1). It is worth pointing out that we concentrate on the peak magnetic field on the axis rather than the fringe field or the field of integration or any other point. Firstly, we investigate the magnetic fields of undulators with linearly increasing gap. Then we solve the variations of gap with the undulator length $g(z)$ from Eq. (1) to obtain different tapering amplitude rates, and calculate the magnetic fields of undulators with gap function $g(z)$. Then we add the same variation Δg to each period of a tapered undulator and calculate the peak magnetic field to find out whether the tapering rate can stay constant or not.

3.1 Undulator with linearly increasing gap

One should notice that tapering normally means the decreasing of undulator strength K with the undulator length. Here we investigate the peak magnetic fields on the axis of undulators with a linearly increasing gap. The gap is fixed to be 18 mm in the first period and then it is increased linearly. We have considered many cases of different gap variations (Δgap), and two typical cases are shown in Fig. 2. When Δgap = 12 mm, the value of g/λ_{u} at the end of the undulator is about 0.7, which is the upper limit of the applicable scope of the empirical formula. Therefore, from Fig. 2 the results from the empirical formula also agree very well with those from simulations in the applicable scope of the empirical formula. When Δgap=3 mm, the peak magnetic field varies about 20% in the 1.32 m undulator length, which is very big for the FEL applications in practice, and it is obvious that the peak magnetic field also decreases linearly when the gap varies linearly.

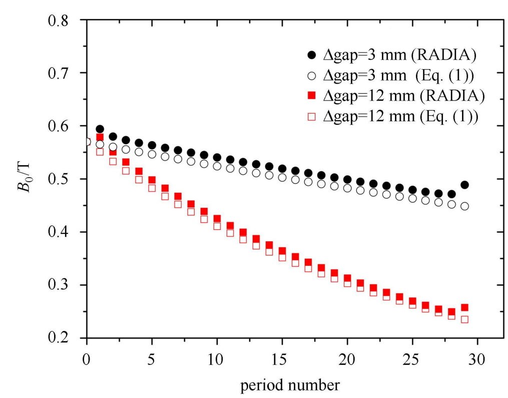

Fig. 2. (color online) The peak magnetic field B_0 on the axis from the RADIA code and Eq. (1) for undulators with linearly increasing gap.

3.2 Linear tapering

For a short linearly tapered undulator, how should we design the gap variation? In this subsection we solve the gap variation $g(z)$ from Eq. (1) to obtain the object tapering rate. As shown in Fig. 3, the variations of gap for two tapering rates ($\Delta B_0/B_0$=1% and 6%) are presented. One can easily see that the gap also increases linearly. Then using this gap variation, we simulate the magnetic field and show the results in Fig. 4 comparing the peak magnetic field with that from Eq. (1). Similar to that of the uniform-parameter undulator, the difference between the results from RADIA and Eq. (1) is about 3%, and the peak magnetic field has a very good linear relation with the undulator length.

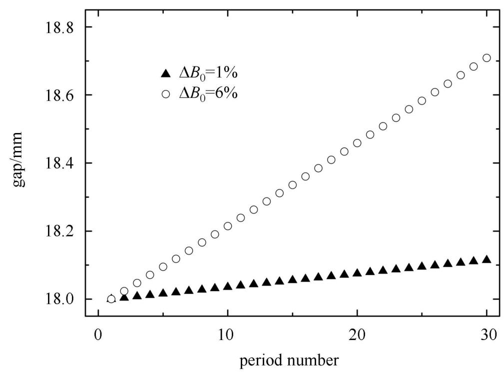

Fig. 3. The variation of gap solved from Eq. (1) for obtaining the object tapering rate.

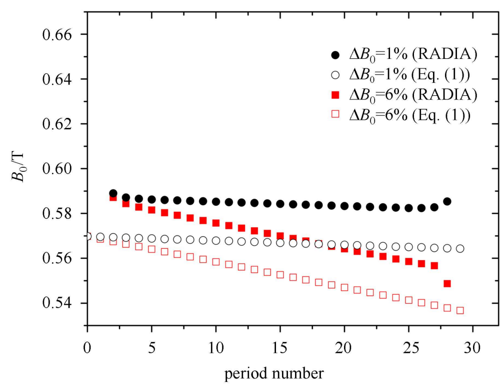

Fig. 4. (color online) The peak magnetic field B_0 from the RADIA code and Eq. (1) for undulators with the gap shown in Fig. 3.

3.3 Linear tapering with gap variation

In FEL applications, we usually need to adjust the undulator gap to change the radiation wavelength. Normally, the magnet blocks and poles are installed on the mechanical bracket and there is usually only one drive motor or two motors located at both ends. The undulator blocks and poles at one side can be considered as a whole, and in FEL experiments, the gap can only be adjusted by the same variation simultaneously for each period. Under this condition, it is worth investigating the variation of the tapering rate when the gap varies.

We take the case of $\Delta B_0/B_0=6\%$ in the last subsection as an example. We add the same variation Δg to each period and calculate the magnetic field. Fig. 5 shows the results of $\Delta g=0$, ±2, ±4 mm. The five curves in Fig. 5 are almost parallel, which means that the tapering amplitude rate is almost constant. In this example, the maximal adjustment of the peak magnetic field is about 42%, which is big enough for normal FEL experiments.

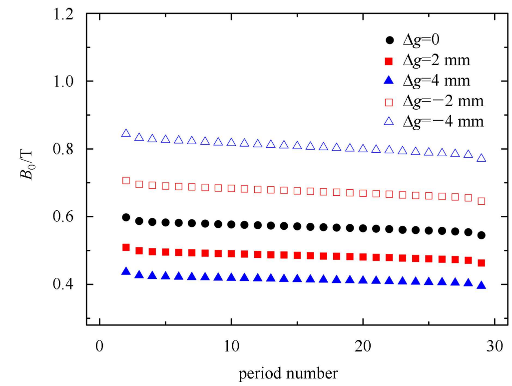

Fig. 5. (color online) The peak magnetic field B_0 from the RADIA code for the case of $\Delta B_0/B_0=6\%$ with Δg=0, ±2, ±4 mm.

4 Conclusions

In this paper we have numerically simulated the magnetic field of a linearly tapered undulator and compared the results with those from the empirical formula that has been proved to be correct for uniform-parameter undulators. The comparison results imply that the empirical formula is also effective for linearly tapered undulators with a big enough scope ($\Delta B_0/B_0<20\%$ in 1.32m undulator length) for the requirements of normal FEL experiments. For a linearly tapered undulator, we can use the empirical formula to design the variation of the undulator gap. Based on the results above, for the tapering rate demanded by normal FEL experiments, the gap of a linearly tapered undulator increases almost linearly, and the tapering rate will keep constant when the undulator gap is adjusted with the same variation for each undulator period.

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