

# Effect of nuclear deformation on direct capture reactions

FAN Guang-Wei(樊广伟)<sup>1;1)</sup> CAI Xiao-Lu(蔡晓鹭)<sup>2</sup> M. Fukuda<sup>3</sup> HAN Ti-Fei(韩体飞)<sup>1</sup>  
 LI Xue-Chao(李学超)<sup>1</sup> REN Zhong-Zhou(任中洲)<sup>4</sup> XU Wang(徐望)<sup>2</sup>

<sup>1</sup> School of Chemical Engineering, Anhui University of Science and Technology, Huainan 232001, China

<sup>2</sup> Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China

<sup>3</sup> Department of Physics, Osaka University, Osaka 560-0043, Japan

<sup>4</sup> Department of Physics, Nanjing University, Nanjing 210008, China

**Abstract:** Direct radiative capture processes are well described by a spherical potential model. Since most nuclei are not spherical, and in order for the model to explain direct radiative captures more accurately, the effect of nuclear deformation has been analyzed with q-deformed Woods-Saxon potential in this work. The results imply that nuclear deformation largely affects the direct radiative capture, and it should be taken into account when discussing direct capture reactions.

**Key words:** potential model, direct radiative capture reactions, cross sections, deformation effect

**PACS:** 25.40.Lw, 26.35.+c **DOI:** 10.1088/1674-1137/38/4/044101

## 1 Introduction

Low energy nuclear direct radiative capture reactions play a crucial role in big bang nucleosynthesis, main path stellar evolution, element synthesis at supernova sites, X-ray bursts etc., since the cross sections of the reaction are often necessary for investigating the astrophysical entities [1–3]. However, some reactions occur at energies that are almost not directly accessible in terrestrial laboratories. Furthermore, some reactions are practically impossible to be directly measured, such as the  ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ , because no  ${}^8\text{Li}$  or neutron target exists [4]. These necessitate theoretical extrapolations from higher to lower energies, or pure theoretical calculations with no firm experimental basis.

The direct radiative capture process represents a transition for the projectile from an initial continuum state to a final bound state via interaction with the electromagnetic field. The reaction selects those projectiles from the appropriate partial waves with orbital angular momenta that can jump into final orbits by the emission of  $\gamma$  ray of multipolarity  $L$ . In order to calculate the direct radiative capture cross sections, one needs to solve many body problems for bound and continuum states of relevance to the capture process. There are several levels of difficulty in attacking this problem. The simplest solution, among theories which have been developed to overcome the difficulties, such as microscopic cluster model [5],  $R$ -matrix [6], etc., is based on a potential model to solve the initial and final state. The model treats the

direct radiative capture reaction as a core plus a valence part interacting via a potential (for example, Woods-Saxon (WS)). Thus, the cross sections are sensitive to the potential [7–9].

Because nuclear deformation largely affects its corresponding potential, and since most nuclei are not spherical (for instance, the famous nuclei  ${}^8\text{Li}$  and  ${}^8\text{B}$  ( ${}^7\text{Li}(n, \gamma){}^8\text{Li}$  and  ${}^7\text{Be}(p, \gamma){}^8\text{B}$ ) [10]) in this article nuclear deformation will be added to the potential model and its effect will be analyzed by the calculation of the cross sections of  ${}^7\text{Li}(n, \gamma){}^8\text{Li}$ . In the following section, a brief framework of the potential model will be presented. Deformation effect analyzed by the calculation of cross sections of  ${}^7\text{Li}(n, \gamma){}^8\text{Li}$  is shown in Section 3, which is followed by a summary in Section 4.

## 2 Framework of the potential model

Since the potential model is a standard model, we will briefly outline its theoretical formalism in this paper. The details are referred to Refs. [11, 12]. Within the model, the capture cross section of the capture reaction  $a(x, \gamma)b$  is given by

$$\sigma_{a \rightarrow b}^{(\text{EL})} = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{k_\gamma^{2L+1}}{\hbar v} \frac{1}{2s+1} \frac{1}{2I+1} e_{\text{EL}}^2 \sum |Q_{a \rightarrow b}^{(\text{EL})}|^2, \quad (1)$$

where  $k_\gamma = \frac{\varepsilon_\gamma}{\eta c}$  is the wave number corresponding to a  $\gamma$  ray energy,  $\varepsilon_\gamma$ , the  $e = -Z/A$  is the effective charge for

Received 24 June 2013

1) E-mail: gwfan@aust.edu.cn

©2014 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

neutrons. The term denotes the EL transition matrix element with electric multiplicity  $L$  given by

$$Q_{a \rightarrow b}^{(EL)} = \langle \Psi_a || T^{EL} || \Psi_b \rangle, \quad (2)$$

where  $T^{EL} = r^L Y_{LM}$  stands for the electric dipole operator. The matrix element can be explained as a product of three factors

$$Q_{a \rightarrow b}^{(EL)} = \tau_{a,b}^{EL} B_b A_{a,b}, \quad (3)$$

$$\tau_{a,b}^{EL} = \int \psi_{\text{scat}} r^L \psi_{\text{bound}} dr, \quad (4)$$

where the Eq. (4) denotes the overlap integral between radial components of the continuum or incoming particle wave function (WF)  $\psi_{\text{scat}}$  scattered by the a-x potential and bound state WF  $\psi_{\text{bound}}$  of b. The quantity  $B_b$  represents the fractional parentage coefficient for a single-particle configuration of the bound state (spectroscopic factor), the  $A_{a,b}$  denotes an angular momentum coupling coefficient.

For charged particles, the astrophysical  $S$ -factor for the direct capture from a continuum state to the bound state is defined as

$$S^{(c)}(E) = E \sigma_{a \rightarrow b}^{(EL)}(E) \exp[2\pi\eta(E)], \quad (5)$$

with  $\eta(E) = Z_a Z_b e^2 / \hbar v$ ,

where  $v$  is the relative velocity between a and b.

The WFs of  $\psi_{\text{scat}}$  and  $\psi_{\text{bound}}$  in the potential model are obtained by solving the scattering and bound state systems, respectively. Thus, the essential ingredients of the model are potentials used to generate WFs. The shape of potentials for the model is spherical, such as WS potential. This may cause potential uncertainties to describe cross sections because most nuclei are not spherical. Therefore, we modify the model with  $q$ -deformed WS potential instead of spherical one. The spherical WS potential is expressed as

$$V = \left( -V_0 + V_1(l.s) \frac{r_{l.s}^2}{r} \frac{d}{dr} \right) f(r), \quad (6)$$

where  $f(r) = [1 + \exp((r - R_c)/a_0)]$ ,  $a_0$  is the diffuseness parameter,  $R_c$  is the width of the potential, and  $r_{l.s}$ , the radius of spin-orbit potential [9, 12]. The depth  $V_0$  is adjusted to reproduce the experimental binding energy of the valence neutron or proton. The  $q$ -deformed WS potential is obtained by introducing a deformed coefficient  $q$  to  $f(r)$

$$f(r) = 1/[1 + q \exp((r - R_c)/a_0)], \quad (7)$$

where  $q (=0.67)$  is the real parameter that determines the shape (deformation) of the potential [13].

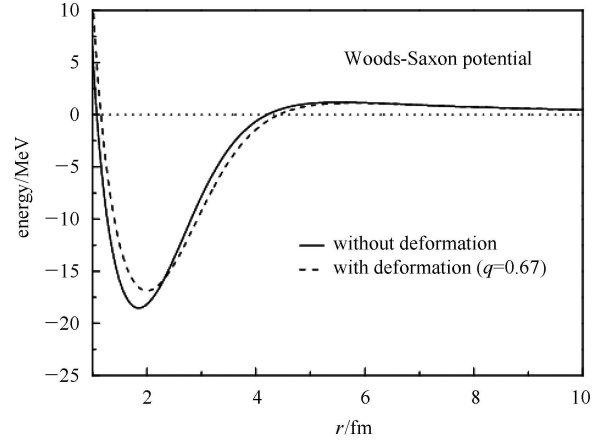


Fig. 1. The difference of potentials given by spherical and deformed WS.

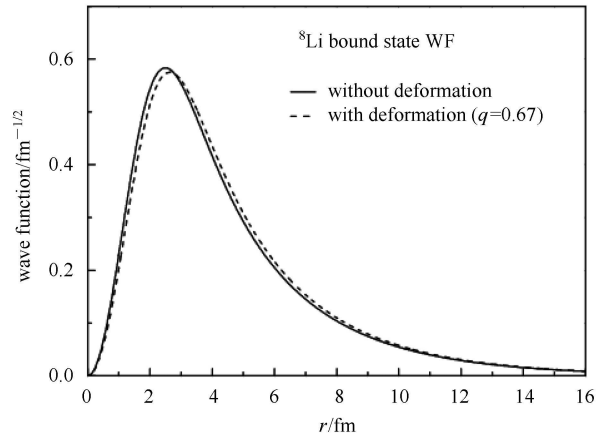


Fig. 2. The difference of the valence bound-state WF of  ${}^8\text{Li}$  between with and without deformation correction.

### 3 Calculations of ${}^7\text{Li}(n, \gamma){}^8\text{Li}$ reaction

${}^7\text{Li}(n, \gamma){}^8\text{Li}$  reaction is a key reaction in inhomogeneous big bang nucleosynthesis to jump  $A=8$  gap. It is also a representative calculation for theories. For  ${}^7\text{Li}(n, \gamma){}^8\text{Li}$ , the direct radiative capture of an  $s$ - or  $d$ - wave neutron by  ${}^7\text{Li}$ , leaving the  ${}^8\text{Li}$  compound nucleus in either the g.s. ( $J^\pi=2^+$ ) or the first excited state ( $J^\pi=1^+$ ) proceeds by an E1 transition. To discuss the reaction, we adopt the spherical and  $q$ -deformed WS potential with  $r_0=1.25$  fm,  $a_0=0.65$  fm. The depth of the potentials ( $V_0(\text{g.s.})$  and  $V_0(1\text{st})$ ) are adjusted to reproduce the corresponding binding energies,  $E_{\text{gs}}=2.033$  MeV and  $E_{1\text{st}}=1.052$  MeV. Potentials used to describe the scattering of the neutron by  ${}^7\text{Li}$  also have geometric parameters  $r_0=1.25$  fm and  $a_0=0.65$  fm. The well depths have been adjusted in order to reproduce experimental scattering length  $a_+ = -(3.63 \pm 0.05)$  fm and  $a_- = (0.87 \pm 0.05)$  fm for the two components of the channel spin  $s$  at thermal

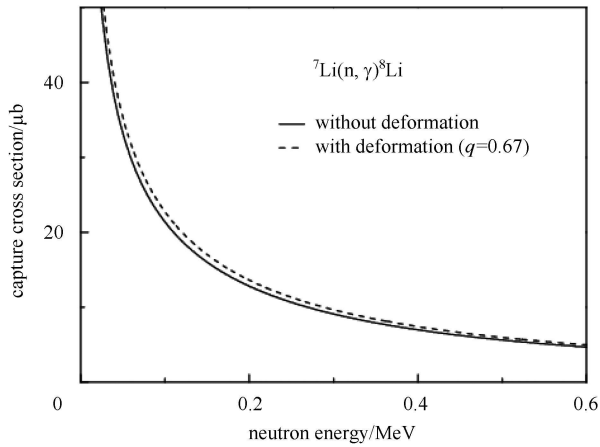


Fig. 3. The cross sections for the  ${}^7\text{Li}(n, \gamma){}^8\text{Li}$  as a function of neutron energy.

energy. The results of this analysis, assuming only an  $s$ -wave capture, are shown in Fig. 3.

From the figures described above, one can see that nuclear deformation largely affects the shape of the potential, hence WF of the valence part and even the cross sections. This supports the conclusion of Y. Nagai et al. [9] that the cross sections are sensitive to the interaction potential. From the analysis, the difference between cross sections with two kinds of potential up to 6.2% will greatly change the components in the inhomogeneous big bang nucleosynthesis. Thus, nuclear deformation should be taken into account for potential model in extrapolations. In this work, we wanted to

give the cross sections for  ${}^7\text{Li}(n, \gamma){}^8\text{Li}$  reaction exactly with nuclear deformation effect. Unfortunately, it is suspended because there is no precise range  $R_c$  and diffuseness parameter  $a_0$  for the  $q$ -deformed WS potential, since different choices of  $R_c$  and  $a_0$  can be made to reproduce the same binding energy, which can cause an irreducible uncertainty in the calculation. For example, when  $R_c=2.296$  fm ( $r_0=1.20$  fm) and  $a_0=0.60$  fm are adopted with the same deformation, the largest difference of cross sections reaches to 11%. Therefore, we did an experiment in HIAMC to measure the reaction cross section of  ${}^8\text{Li}+{}^{12}\text{C}$ ,  ${}^9\text{Be}$  and  ${}^{27}\text{Al}$  to remove the irreducible uncertainty, which will be discussed in next article.

## 4 Summary

The authors have analyzed the effect of nuclear deformation in direct radiative capture reactions through calculating the cross sections of  ${}^7\text{Li}(n, \gamma){}^8\text{Li}$ . The analyzed results show that nuclear deformation visibly affects cross sections, and should be taken into account when discussing direct radiative capture reactions.

*The authors downloaded the code of potential model from Computer Physics Community (<http://cpc.cs.qub.ac.uk/summaries/ADSH>), the analysis is based on this code. We would like to acknowledge the financial support provided by Anhui University of Science and Technology (11130, QN201315).*

## References

- 1 Kawano L H et al. *Astrophys. J.*, 1991, **372**: 1
- 2 Bures S et al. *Phys. Rev. Lett.*, 1999, **82**: 4176
- 3 Rolfs C et al. *Cauldrons in the Cosmos*. University of Chicago Press, Chicago, 1988
- 4 Guimarães V et al. *Phys. Rev. C*, 2007, **75**: 054602
- 5 Descouvemont P, Baye D. *Nucl. Phys. A*, 1988, **487**: 420
- 6 Kremer R M et al. *Phys. Rev. Lett.*, 1988, **60**: 1475, and references therein
- 7 Tombrello T. *Nucl. Phys.*, 1965, **71**: 459
- 8 Davids B, Typel S. *Phys. Rev. C*, 2003, **68**: 045802
- 9 Nagai Y et al. *Phys. Rev. C*, 2005, **71**: 055803
- 10 Gautam Rupak, Renato Higa. *Phys. Rev. Lett.*, 2011, **106**: 222501
- 11 Rolfs C. *Nucl. Phys. A*, 1973, **217**: 29
- 12 Bertulani C A. *Computer Physics Commun.*, 2003, **156**: 123
- 13 Ikhdair S M, Falaye B J, Hamzavi M. *Chin. Phys. Lett.*, 2013, **30**: 020305