Longitudinal instability caused by long drifts in the C-ADS injector- I*

LI Zhi-Hui(李智慧)[1)] TANG Jing-Yu(唐靖宇) YAN Fang(闫芳) GENG Hui-Ping(耿会平)
MENG Cai(孟才) SUN Biao(孙彪) CHENG Peng(程鹏) GUO Zhen(郭臻) SUN Ji-Lei(孙纪磊)
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract: The period length is usually much larger than the cavity effective length in a low energy superconducting linac.The long drifts between cavities will not only decrease the acceptance of the linac, but also lead to possible instability. The linac will be more sensitive to mismatch and other perturbations. From the longitudinal motion equation, the function which describes the parametric resonance is deduced and the relation between the instability region and the cavity filling factor is discussed. It indicates that if the zero current phase advance per period is kept below 90°, instability driven by parametric resonance will never occur. The space charge effect will enhance the instability, so that a stricter limitation on the phase advance per cell is required. From the numerical simulation results for two different schemes of Injector-I of the C-ADS driver linac, one can find that even with just three cells in the unstable region, significant emittance growth can be observed. Further investigations show that it is apt to produce halo particles under resonance, and the machine becomes more sensitive to errors and mismatches. Therefore, it is important to keep all cells in the stable region throughout the linac of very high beam power to minimize beam losses.

Key words: parametric resonance, instability, phase advance, low energy superconducting linac

PACS: 29.20.Ej **DOI:** 10.1088/1674-1137/37/3/037005

1 Introduction

Since the exploration of radio frequency (RF) superconductivity for particle accelerators began at Stanford University and the Institute of Nuclear Physics at Karlsruhe at 1960s, it has achieved a tremendous progress and become an important technology for particle accelerators today. The excellent properties of superconducting radio frequency cavities [1], such as low AC power consumption, large beam tubes, great potential in terms of reliability and flexibility thanks to its independently-powered structures, make it a good candidate for the acceleration of proton beams, especially the long pulse or CW (continue wave) machine, such as the driver linac for an accelerator driven system (ADS) [2]. However the existence of a static magnetic field will increase the surface resistance of the superconducting cavity and may cause it to quench, so the cavity needs to be well screened from any static magnetic field, which makes it impossible to integrate the transverse focusing lens with the cavity just as the normal conducting Alvarez DTL cavity does. As a consequence, the focusing period length will be much larger than the normal conducting one, especially at the low energy part, where the space charge effect is important and transverse focusing has to be applied between every cavity. The long period length, which is equivalent to a long drift for longitudinal motion, not only decreases the acceptance of the linac [3], but also causes beam instability by parametric resonance. In this paper, we start from the longitudinal motion equation to analyze the instability and try to find the way to avoid this instability by giving a limit to the phase advance per period for a low energy superconducting linac, where the period length is usually much greater than the field effective length.

2 Longitudinal motion

The longitudinal motion of particles in a linac is usually described by the following equation [4],

$$\frac{\mathrm{d}^2(\varphi-\varphi_\mathrm{s})}{\mathrm{d}s^2}+\frac{3}{\gamma_\mathrm{s}\beta_\mathrm{s}}\left[\frac{\mathrm{d}}{\mathrm{d}s}(\gamma_\mathrm{s}\beta_\mathrm{s})\right]\left[\frac{\mathrm{d}(\varphi-\varphi_\mathrm{s})}{\mathrm{d}s}\right]+\frac{2\pi qE_0T}{mc^2\lambda\gamma_\mathrm{s}^3\beta_\mathrm{s}^3}(\cos\varphi-\cos\varphi_\mathrm{s})=0, \tag{1}$$

Received 10 May 2012

∗ Supported by National Natural Science Foundation of China (10875099, 91126003), China ADS Project (XDA03020000) and Institute of High Energy Physics Special Funding (Y0515550U1)

1) E-mail: lizhihui@ihep.ac.cn

where E_0T is the effective acceleration gradient of the cavity. φ_s, γ_s and β_s are the synchronous phase, relativistic parameter and normalized velocity of the synchronous particle, respectively. For small oscillations

$$\cos\varphi \approx \cos\varphi_s - (\varphi-\varphi_s)\sin\varphi_s - \frac{(\varphi-\varphi_s)^2}{2}\cos\varphi_s + \cdots, \quad (2)$$

Substituting Eq. (2) into (1) and defining $\psi = \varphi - \varphi_s$, we can express Eq. (1) in a very simple form under the condition of adiabatic acceleration

$$\psi'' + k_{10}^2\psi = 0, \quad (3)$$

where $k_{10}^2 = \dfrac{2\pi qE_0T\sin(-\varphi_s)}{mc^2\beta_s^3\gamma_s^3\lambda}$ is the longitudinal wavelength of the phase oscillation.

In the usual case where the cavity is composed by acceleration cells one by one along the longitudinal direction, this equation describes the particle motion perfectly. But for the low energy superconducting linac, the cavity length is several times longer than the effective field length because of the existence of the helium tank, tuner, and other necessary accessories. Furthermore, the cavity is separated by the transverse focusing elements and diagnostic elements which occupy even more longitudinal space than the cavity does, thus the period length is much larger than the effective field length. That means the particle will only get impact from the cavity field in a very limited time in one period. In order to describe the particle oscillation under such a situation, the wavenumber should be considered as discrete in the form

$$k_{10}^2(s+S) = k_{10}^2(s) = \begin{cases} k_{10}^2 \text{ in cavity} \\ 0 \text{ out cavity} \end{cases}, \quad (4)$$

where S is the period length. If all cavities are identical, the wave number can be expanded into Fourier series

$$k_{10}^2(s) = C_0 + C_1\cos(2\pi s/S) + C_2\cos(4\pi s/S) + \cdots, \quad (5)$$

and

$$C_0 = \frac{1}{S}\int_{-L/2}^{L/2} k_{10}^2 \mathrm{d}s = k_{10}^2 L/S = (\sigma_{10}/S)^2, \quad (6)$$

where σ_{10} is the zero current longitudinal phase advance per period, a very important parameter in linac design.

$$\sigma_{10} = \sqrt{LS}k_{10} = \sqrt{\frac{2\pi qE_0TL\sin(-\varphi_s)}{mc^2\beta_s^3\gamma_s^3}\frac{S}{\lambda}}, \quad (7)$$

$$C_1 = \frac{2}{S}\int_{-L/2}^{L/2} k_{10}^2\cos(2\pi s/S)\mathrm{d}s = (\sigma_{10}/S)^2\frac{2\cdot\sin(\pi L/S)}{\pi L/S}, \quad (8)$$

$$C_2 = \frac{2}{S}\int_{-L/2}^{L/2} k_{10}^2\cos(4\pi s/S)\mathrm{d}s = (\sigma_{10}/S)^2\frac{2\cdot\sin(2\pi L/S)}{2\pi L/S}. \quad (9)$$

Finally the longitudinal motion can be described by the following equation.

$$\psi'' + (\sigma_{10}/S)^2\left(1 + \frac{C_1}{C_0}\cos ks + \frac{C_2}{C_0}\cos 2ks + \cdots\right)\psi = 0, \quad (10)$$

where $k = 2\pi/S$ is the base frequency wave number. If we just consider up to the first harmonic, the motion equation is

$$\psi'' + (\sigma_{10}/S)^2(1 + C_1/C_0\cos ks)\psi = 0. \quad (11)$$

Let $ks = 2\theta + \pi$, $(2\sigma_{10}/kS)^2 = p$ and $(4\sigma_{10}^2C_1)/(C_0k^2S^2) = 2q$, then Eq. (11) is transformed to

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\theta^2} + [p - 2q\cos(2\theta)]\psi = 0, \quad (12)$$

and it is the Mathieu equation [5]. In accelerator physics applications, p and q are real with $q \ll 1$. The stable solutions of Mathieu's equation are obtained with the condition in which the parameter p is bounded by the characteristic roots $a_r(q)$ and $b_{r+1}(q)$, where r=0, 1, 2,$\cdots$. In other words, unstable solutions are in the region $b_r(q) \leqslant p \leqslant a_r(q)$, where r=1, 2,$\cdots$. The first unstable region is obtained from $b_1(q) \leqslant p \leqslant a_1(q)$ or, equivalently,

$$2\frac{\sigma_{10}}{S}(1 - C_1/4C_0) \leqslant k \leqslant 2\frac{\sigma_{10}}{S}(1 + C_1/4C_0). \quad (13)$$

Expressed with phase advance per period, we have

$$2\sigma_{10}(1 - C_1/4C_0) \leqslant 2\pi \leqslant 2\sigma_{10}(1 + C_1/4C_0). \quad (14)$$

And the center of the first unstable region is

$$\sigma_{10} = \pi, \quad (15)$$

the width of the unstable region is

$$\varepsilon = \frac{C_1\sigma_{10}}{4C_0} = \frac{\sigma_{10}}{2}\frac{\sin(\pi L/S)}{\pi L/S}. \quad (16)$$

We can see that the unstable region width is directly determined by the ratio of the effective field length and the period, we call it the filling factor

$$F = L/S. \quad (17)$$

If the cavity is very short compared with the period length, the unstable range width will be

$$\varepsilon = \lim_{L/S\to 0}\frac{\sigma_{10}}{2}\left(\sin\left(\frac{\pi L}{S}\right)\Big/\left(\frac{\pi L}{S}\right)\right) = \sigma_{10}/2, \quad (18)$$

since the unstable center is $\sigma_{10} = \pi$, so $\varepsilon = \pi/2$. In this case, the unstable region is the widest one. On the other hand, if the effective field length is equal to the period length, as in the case of Alvarez DTL, the unstable width will be zero, which means there is no such instability. The stability chart is shown in Fig. 1.

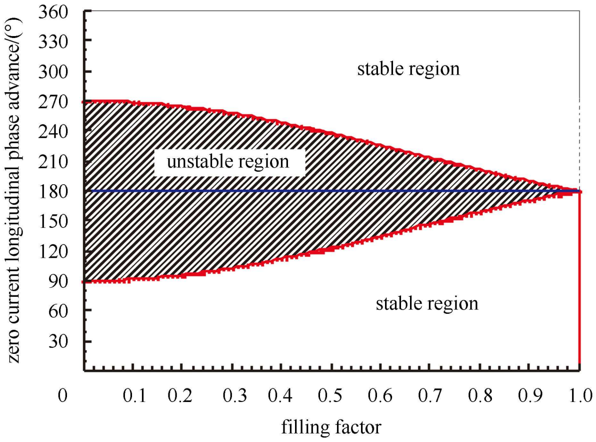

Fig. 1. The longitudinal stability chart.

From Fig. 1 we can see that the width of the first stable region is a function of filling factor, if the field is uniformly distributed along the linac, the longitudinal motion is stable even the phase advance per period is almost 180°. If the field is confined in a limited region in one period, the stable region will decrease. But if we confine the zero current longitudinal phase advance per period within 90°, no matter how much the filling factor is,there will be no chance for the instability to happen.

There are also some other unstable regions corresponding to r=2, 3 and so on. We can find the phase advance per period for other unstable regions is greater than 180. For linacs the phase advance per period is seldom greater than 180°, so they are out of our concern.

3 Zero current phase advance per period

In practice, the focusing lattice of the linear accelerator is a quasi-periodic structure because of the acceleration, it is difficult to achieve perfect matching of the beam envelopes, so the parametric resonance may cause serious problems if the working points fall in the unstable regions, such as emittance growth, halo formation and particle loss. If the space charge effect is involved for high current machine, the situation will be even worse. The nonlinear space charge force may cause emittance exchange between different planes and envelope instability in case the phase advance per period is greater than 90° with sufficiently high intensity. It is widely accepted that in high current linac design the zero current phase advances in both transverse and longitudinal phase planes should be kept below 90° to avoid possible instabilities.

According to the definition of the longitudinal zero current phase advance per period in Eq. (7), there are three ways to decrease it: (1) decrease the acceleration gradient; (2) decrease the absolute synchronous phase and (3) decrease the ratio of period length to RF wavelength. For Case (1), it means lower acceleration efficiency and should be the last consideration. The acceleration gradient is determined by the obtainable maximum electric field and the effective transient time factor (TTF). The RF cavity cannot work stably below the multipacting barrier, which is about 4 MV/m for a low beta spoke cavity. If the phase advance has to be decreased further, then the geometry beta of the cavity can be optimized to decrease the TTF at low energy. For Case (2), it is usually limited by the required longitudinal acceptance. At low energy both the phase advance and the required acceptance are larger, so its contribution to the decrease of the phase advance is very limited. If both (1) and (2) are not capable of decreasing the phase advance to the required value, then we have to apply a lower RF frequency.

4 Simulations with the C-ADS Injector-Ⅰ

4.1 C-ADS linac and Injector- Ⅰ

The China-ADS Project (abbr. as C-ADS) is aimed at building an accelerator-driven system demonstration facility with 1000 MW thermal power. The driver linac is composed of two parallel 10 mA, 10 MeV injectors and a 10 mA 1.5 GeV CW superconducting main linac [2]. Injector I, which is composed of a 3.2 MeV normal conducting 4-vane RFQ and a superconducting section composed of beta=0.12 spoke cavities and solenoids, will be designed and constructed by the Institute of High Energy Physics (IHEP). The main beam parameters at the superconducting section entrance are listed in Table 1, and we will use this injector to demonstrate the dangers of the parametric resonance discussed in the previous sections.

Table 1. The general input beam parameters of the Injector Ⅰ superconducting section.

input energy /MeV	3.2
beam current /mA	10
longitudinal Norm. RMS emittance/(mm·mrad)	0.159081
transverse Norm. RMS emittance/(mm·mrad)	X: 0197765 Y: 0.199213

Figure 2 shows the schematic layout of one period of the superconducting section and the geometric length of each element. It is composed of a spoke cavity with geometry beta 0.12, one solenoid with the total length of 300 mm and a BPM of 100 mm. The filling factor is

$$F=L/S=\beta_{\rm g}\lambda/S=0.135, \tag{19}$$

the corresponding stable region is $\sigma_{l0}<92.7°$. The maximum voltage provided by the spoke cavity is about 0.82 MV at β=0.14, the effective TTF at the first cavity is about 0.5 compared with which at optimum beta. Even so, if we set the field level at maximum for the whole cavities in Injector I, the longitudinal phase advance of the first three cells will be greater than the border of the stable region. In order to investigate the effects of the parametric resonance on beams, we constructed two linacs with different field level settings. The main parameters of Scheme 1, with a derated field level for maintaining a longitudinal phase advance per period of less than 90°, and Scheme 2, with a constant field level, are listed in Table 2. The transverse phase advances for both schemes are set to satisfy the equipartition condition,

$$\varepsilon_{\mathrm{l}}/\varepsilon_{\mathrm{t}}=\sigma_{\mathrm{t}}/\sigma_{\mathrm{l}}. \tag{20}$$

By assuming that

$$\sigma_{\mathrm{t}}/\sigma_{\mathrm{l}}\approx\sigma_{\mathrm{t0}}/\sigma_{\mathrm{l0}}. \tag{21}$$

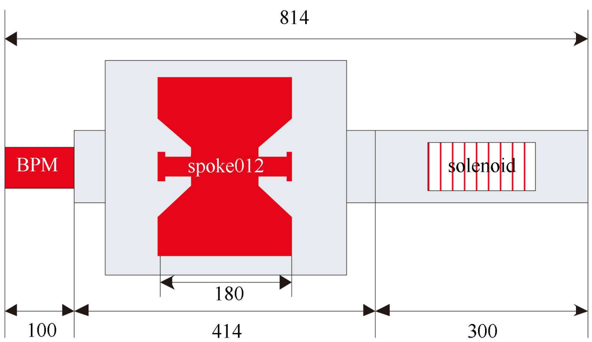

Fig. 2. The layout of one period of the Injector Ⅰ superconducting section.

Table 2. Main parameters of the cavity in Injector Ⅰ.

period No.	field level		synchronous phase/(°)
	Scheme1	Scheme2	
1	0.615	1	−37
2	0.692	1	−33
3	0.769	1	−31
4	0.846	1	−30
5	0.908	1	−30
6	0.962	1	−30
7	1.000	1	−30
8	1.000	1	−30
9	1.000	1	−30
10	1.000	1	−30
11	1.000	1	−30
12	1.000	1	−30

The longitudinal and transverse phase advances per cell of Scheme 1 and Scheme 2 are shown in Fig. 3. We can see that there are three cells with a longitudinal phase advance per cell greater than the border of the stable region. Detailed simulations for the two schemes are performed with 100000 macro particles, the initial particles distributions are approximated with 5σ Gaussian and 3σ Gaussian in longitudinal and transverse planes, respectively, which is a good approximation of the RFQ output distribution by numerical simulation. The results will be presented in the following section.

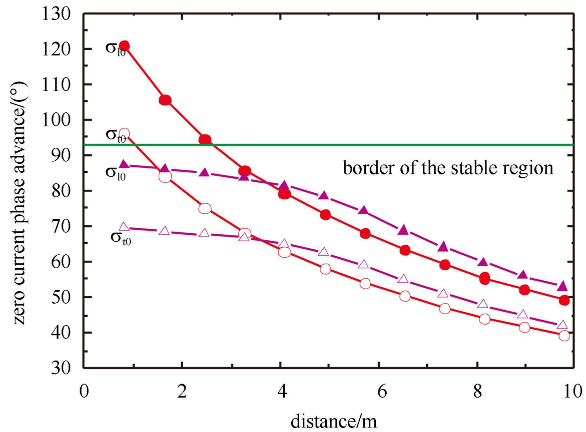

Fig. 3. The longitudinal and transverse zero current phase advances per cell along the Injector Ⅰ superconducting section (red: Scheme 2, pink: Scheme 1).

4.2 Simulation results and discussions

The normalized RMS emittance growths along Injector I for Scheme 1 and Scheme 2 are shown in Fig. 4. Even when just three cells fall in the unstable region, the emittance growths for both cases are quite different. We can see that the emittance growth for Scheme 1 with zero current is only negligibly 2%, as compared with nearly 12.5% for Scheme 2. The difference originates from the difference of the focusing lattice. It clearly shows the danger of the parametric resonance. When the beam current is increased to 10 mA, the total emittance can be regarded as a superposition of zero current emittance growth and an additional 5% growth contributed by the nonlinear space charge force, which should be the reason for the charge redistribution at the first several cells [4]. If 10% mismatch is set to the input beams, for Scheme 1, only a very small RMS emittance growth can be observed, but for Scheme 2, two particles are lost, and it can be naturally concluded that Scheme 1 is more stable.

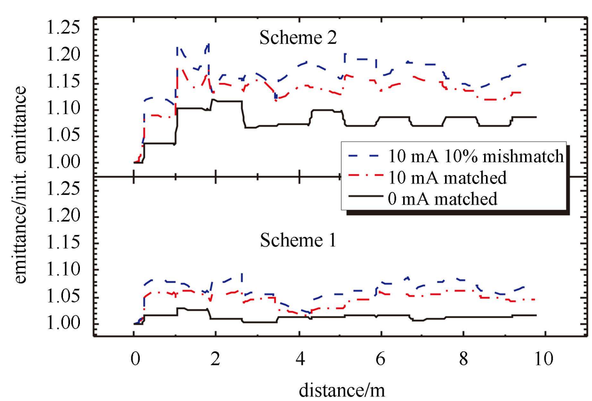

Fig. 4. The normalized RMS emittance growths for Scheme 1 and Scheme 2 with different currents and mismatching factors.

Another consequence of the parametric resonance is the formation of the halo particles. The particles may obtain energy under resonance. Fig. 5 shows the emittance with different ratios of particles growth along the linac for both schemes. We can see that the maximum 100% emittance growth is about 3 times for Scheme 1, but for Scheme 2, the maximum 100% emittance is about 8 times the initial one; that means some particles obtain more energy and have more chances to excurse far away from the bunch cente. If more mismatches or errors are presented, there would be more chances of particle losses.

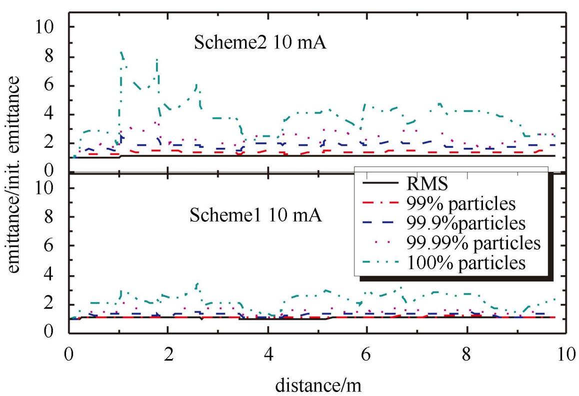

Fig. 5. The normalized emittance growth along linac with different particle ratios.

5 Conclusions

For a low-energy superconducting linac with low filling factors, it is apt to suffer instabilities driven by parametric resonance. In order to avoid it, the zero current phase advance per period should be kept within the stable region. At low energy, the zero current phase advance per period can be decreased in several ways, such as a lower acceleration gradient, a smaller absolute synchronous phase, and a smaller ratio of period length to RF wavelength. The parametric resonance is very dangerous even if just a few periods fall in the unstable region. It may cause additional emittance growth, and halo particle formation. The machine is more sensitive to errors and mismatches. When taking into account the possible instabilities caused by space charge force, it is better to keep the zero current phase advance per cell in every plane below 90° for a high current machine.

References

1 PADAMSEE H et al. RF Superconductivity for Accelerators. New York: John Wiley & Sons, 1998. 1–30

2 TANG J Y et al. Conceptual Physics Design on the C-ADS Accelerator. IHEP-CADS-Report/2011-1E

3 FATEEV A P et al. Nuclear Instruments and Methods in Physics Research, 1984, **222**: 420–427

4 WANGLER T. RF Linear Accelerators. New York: John Wiley & Sons, 1998

5 LEE S Y. Accelerator Physics. 2nd Edition. Beijing: World Scientific, 2004