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Mixing between the 2^3S_1 and 1^3D_1 $\mathrm{D_s}$ *

YUAN Ling(袁玲) CHEN Bing(陈兵) ZHANG Ai-Lin(张爱林)1)

Department of Physics, Shanghai University, Shanghai 200444, China

Abstract: Mixing between the 2^3S_1 and 1^3D_1 $\mathrm{D_s}$ is studied within the $^3P_0$ model. If mixing between these two 1^- states exists, $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^\pm$ could be interpreted as the two orthogonal mixed states with mixing angle $\theta\approx-80^\circ$ in the case of a special β for each meson. However, in the case of a universal β for all mesons, $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$ could be interpreted as the mixed state of 2^3S_1 and 1^3D_1 with mixing angle $12^\circ<\theta<21^\circ$ but $\mathrm{D}^*_{\mathrm{sJ}}(2860)^\pm$ seems difficult to interpret as the orthogonal partner of $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$.

Key words: $^3P_0$ model, decay width, mixing, excited states

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1 Introduction

The properties of radially excited 2^3S_1 and orbitally excited 1^3D_1 heavy-light $\mathrm{D_s}$ mesons have been explored for a long time. Though no such higher excited $\mathrm{D_s}$ state has been definitely established, $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^\pm$ [1] are suggested as these two 1^- states [2–9] or as mixtures of them [7, 9–11].

As is well known, the study of strong decays is an important way to identify hadrons. The $^3P_0$ model has been employed successfully to evaluate the OZI-allowed strong decays of hadrons [12–18]. Thorough understanding of this model has also been investigated [19–25]. To identify $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^\pm$, some decay widths and branching ratios have been calculated in the $^3P_0$ model [3, 9–11].

The P-wave D and $\mathrm{D_s}$ mesons have been established [1]. For the P-wave multiplets of heavy-light mesons, the mixing between the $^3P_1$ and $^1P_1$ states has been studied and the mixing angle determined [5, 26, 27]. For the radially excited 2^3S_1 and the orbitally excited 1^3D_1, they may mix with each other through some mechanism. If the mixing exists, there is the "excited-vector-meson puzzle" [20] for light mesons. This puzzle is also expected for heavy-light mesons. Obviously, this mixing will complicate our understanding of the higher excited states.

As two higher excited 1^- resonances, $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^\pm$ have been explored as two mixed 1^- candidates. The mixing angle has been fixed [10, 11]. However, the determined mixing angles θ are different. In Ref. [11], $1.12\leqslant\theta\leqslant1.38$. In Ref. [10], $\theta\approx-0.5$. Accordingly, there are different assignments and conclusions to these two states. Therefore, it would be interesting to study the mixing and fix the mixing angle in detail.

The detail of the study of strong decays of $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^\pm$ was presented in Ref. [9]. In this paper, the mixing between the 2^3S_1 and 1^3D_1 $\mathrm{D_s}$ is studied. The paper is organized as follows. In Sec. 2, the strong decay widths and branching ratios $\Gamma(\mathrm{D^*K})/\Gamma(\mathrm{DK})$ of $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^\pm$ are evaluated within the $^3P_0$ model. Our conclusions and discussions are presented in the final section.

2 Study of $\mathrm{D}^*_{\mathrm{s1}}(2700)^\pm$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^\pm$ within the $^3P_0$ model

In the conventional nonrelativistic quark model, a resonance without mixing could be marked with $n^{2S+1}L_J$. For quarkonia, their quantum numbers parity P and charge parity C are determined by $P=(-1)^{L+1}$ and $C=(-1)^{L+S}$, respectively. For resonances with open flavor, they have no definite C. In experiment, physical resonance may be a mixture of some $n^{2S+1}L_J$ states with the same J^{PC} or J^P. If the 2^3S_1 and 1^3D_1 $\mathrm{D_s}$ mix with each other, the physically observed states [7, 10, 11] should be the mixed 1^- ones. Therefore, the two orthogonal partners can be denoted as [7]

$$\begin{aligned}|(SD)_1\rangle_{\mathrm{L}}&=\cos\theta|2^3S_1\rangle-\sin\theta|1^3D_1\rangle,\\|(SD)_1\rangle_{\mathrm{R}}&=\sin\theta|2^3S_1\rangle+\cos\theta|1^3D_1\rangle,\end{aligned}\tag{1}$$

where θ is the mixing angle.

Within the $^3P_0$ model, the decay width for a process

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1) E-mail: zhangal@staff.shu.edu.cn

A→BC can be evaluated as that in Ref. [28]

$$\mathit{\Gamma}=\pi^2\frac{\left|\vec{K}\right|}{M_{\mathrm{A}}^2}\sum_{JL}\left|M^{JL}\right|^2, \tag{2}$$

where $J=J_{\mathrm{B}}+J_{\mathrm{C}}$ and M^{JL} is the partial wave amplitude

$$\begin{aligned} M^{JL}(\mathrm{A}\rightarrow\mathrm{BC}) = {} & \frac{\sqrt{2L+1}}{2J_{\mathrm{A}}+1}\sum_{M_{J_{\mathrm{B}}},M_{J_{\mathrm{C}}}}\langle L0JM_{J_{\mathrm{A}}}|\,J_{\mathrm{A}}M_{J_{\mathrm{A}}}\rangle \\ & \times\langle J_{\mathrm{B}}M_{J_{\mathrm{B}}}J_{\mathrm{C}}M_{J_{\mathrm{C}}}|\,JM_{J_{\mathrm{A}}}\rangle \\ & \times M^{M_{J_{\mathrm{A}}}M_{J_{\mathrm{B}}}M_{J_{\mathrm{C}}}}(\vec{K}). \end{aligned} \tag{3}$$

The simple harmonic oscillator (SHO) wave functions in the momentum-space are employed as

$$\begin{aligned} \mathit{\Psi}_{nLM_L} = {} & \frac{1}{\beta^{\frac{3}{2}}}\left[\frac{2^{l+2-n}(2l+2n+1)!!}{\sqrt{\pi}n![(2l+1)!!]^2}\right]^{\frac{1}{2}} \\ & \times\left(\frac{k}{\beta}\right)^l\exp\left[-\frac{1}{2}\left(\frac{k}{\beta}\right)^2\right] \\ & \times F\left(-n,l+3/2,\left(\frac{k}{\beta}\right)^2\right)Y_{LM_L}(\mathit{\Omega}_{\mathrm{p}}), \end{aligned} \tag{4}$$

where β is the harmonic oscillator strength parameter, $Y_{LM_L}(\mathit{\Omega}_{\mathrm{p}})$ is the spherical harmonic function, and

$$F\left(-n,l+3/2,\left(\frac{k}{\beta}\right)^2\right)$$

is the confluent hypergeometric function.

As pointed out in Ref. [9], the way of choice of β plays an important role in the evaluation. There are often two ways of choice for β. One way is to determine β individually for each meson [11, 27], and the other way is to choose β universally for all mesons [7, 24, 29]. Which way is more reasonable is still not clear, and we will present our results in both ways.

First, our results are obtained in the case of a special β for each meson. β is usually fixed to reproduce the realistic root mean square radius of the SHO wave function and located in the region 0.35–0.42 GeV. In our calculation, β of $\mathrm{D_{s1}}[2^3S_1]$, $\mathrm{D_{s1}}[1^3D_1]$, $\mathrm{D_{sJ}}[1^3D_3]$, D, K, D*, $\mathrm{D_s}$, $\mathrm{D_s^*}$, η, K* and ω are chosen as those in Ref. [17]. They are 0.35, 0.34, 0.34, 0.43, 0.46, 0.37, 0.52, 0.45, 0.48, 0.32 and 0.36 GeV, respectively.

If $\mathrm{D_{s1}^*}(2700)^{\pm}$ and $\mathrm{D_{sJ}^*}(2860)^{\pm}$ are the higher excited states 2^3S_1 and 1^3D_1 $\mathrm{D_s}$, respectively, their partial widths and branching ratios have been evaluated [3, 8]. If $\mathrm{D_{s1}^*}(2700)^{\pm}$ and $\mathrm{D_{sJ}^*}(2860)^{\pm}$ are mixtures of the higher excited states, their partial widths and branching ratios have also been evaluated and compared with experiments [10, 11]. However, for lack of data, only the strong decay features have been compared. Furthermore, theoretical predictions are different in the same $^3P_0$ model. In Ref. [9], the strong decays of $\mathrm{D_{s1}^*}(2700)^{\pm}$ and $\mathrm{D_{sJ}^*}(2860)^{\pm}$ are explored in detail. In the following, the mixing is studied and parameters are chosen as those in Ref. [9] to obtain our numerical results.

In the mixing scheme indicated by Eq. (1), both partial widths and branching ratio $\mathit{\Gamma}(\mathrm{D^*K})/\mathit{\Gamma}(\mathrm{DK})$ depend on the mixing angle θ. In Fig. 1(a), the dependence of the branching ratio on the mixing angle θ is shown. To get a comparable branching ratio with experiment

$$\left(\frac{\mathcal{B}(\mathrm{D_{s1}}(2700)^+\rightarrow\mathrm{D^*K})}{\mathcal{B}(\mathrm{D_{s1}}(2700)^+\rightarrow\mathrm{DK})}=0.91\pm0.13_{\mathrm{stat}}\pm0.12_{\mathrm{syst}}\right)$$

[1], the mixing angle is fixed in the figure. In the same figure, the dependence of the decay widths on the mixing angle θ is also shown. Similarly, the mixing angle can be fixed through the comparison of theoretical results with experiments. Obviously, the fixed mixing angle θ in the figure through two different ways are almost the same. The fact indicates that it is reasonable to identify $\mathrm{D_{s1}^*}(2700)^{\pm}$ with the state $|(SD)_1\rangle_{\mathrm{L}}$ in Eq. (1) with a mixing angle θ at $-88^\circ\leqslant\theta\leqslant-76^\circ$. At this mixing angle, the total width of $\mathrm{D_{s1}^*}(2700)^{\pm}$ is determined around $\mathit{\Gamma}\simeq(111\pm1)$ MeV. The fixed mixing angle θ is different from those determined in Ref. [11] ($1.12\leqslant\theta\leqslant1.38$) and Ref. [10] ($\theta\approx-0.5$).

Following the same process, we evaluate the partial decay widths and branching ratio $\mathit{\Gamma}(\mathrm{D^*K})/\mathit{\Gamma}(\mathrm{DK})$ of $|(SD)_1\rangle_{\mathrm{R}}$. The branching ratio and decay widths dependence on the mixing angle θ are shown in Fig. 1(b). From this figure, it is found that the experimental decay widths and branching ratio

$$\frac{\mathcal{B}(\mathrm{D_{sJ}}(2860)^+\rightarrow\mathrm{D^*K})}{\mathcal{B}(\mathrm{D_{sJ}}(2860)^+\rightarrow\mathrm{DK})}=1.10\pm0.15_{\mathrm{stat}}\pm0.19_{\mathrm{syst}}$$

of $\mathrm{D_{sJ}^*}(2860)^{\pm}$ are well reproduced by the $|(SD)_1\rangle_{\mathrm{R}}$ with $-80^\circ\leqslant\theta\leqslant-73^\circ$.

The mixing angle determined by $\mathrm{D_{sJ}^*}(2860)^{\pm}$ is almost the same as that determined by $\mathrm{D_{s1}^*}(2700)^{\pm}$. That is to say, the experimental data supports the assignment of $\mathrm{D_{s1}^*}(2700)^{\pm}$ and $\mathrm{D_{sJ}^*}(2860)^{\pm}$ with the excited mixture of 2^3S_1 and 1^3D_1. $\mathrm{D_{s1}^*}(2700)^{\pm}$ is very possibly the mixed $|(SD)_1\rangle_{\mathrm{L}}$, and $\mathrm{D_{sJ}^*}(2860)^{\pm}$ is very possibly its orthogonal partner $|(SD)_1\rangle_{\mathrm{R}}$. The existing experiments are interpreted quite well with a large mixing angle $\theta\approx-80^\circ$.

β can be chosen as a universal parameter for all mesons [9, 24, 29]. In the following, β=0.38 GeV is fixed as in [9]. Under the mixing scheme Eq. (1), the decay widths and branching ratios $\mathit{\Gamma}(\mathrm{D^*K})/\mathit{\Gamma}(\mathrm{DK})$ of $|(SD)_1\rangle_{\mathrm{L}}$ and $|(SD)_1\rangle_{\mathrm{R}}$ are calculated. To determine the mixing angle θ, their dependence on the mixing angle θ is shown in Fig. 2.

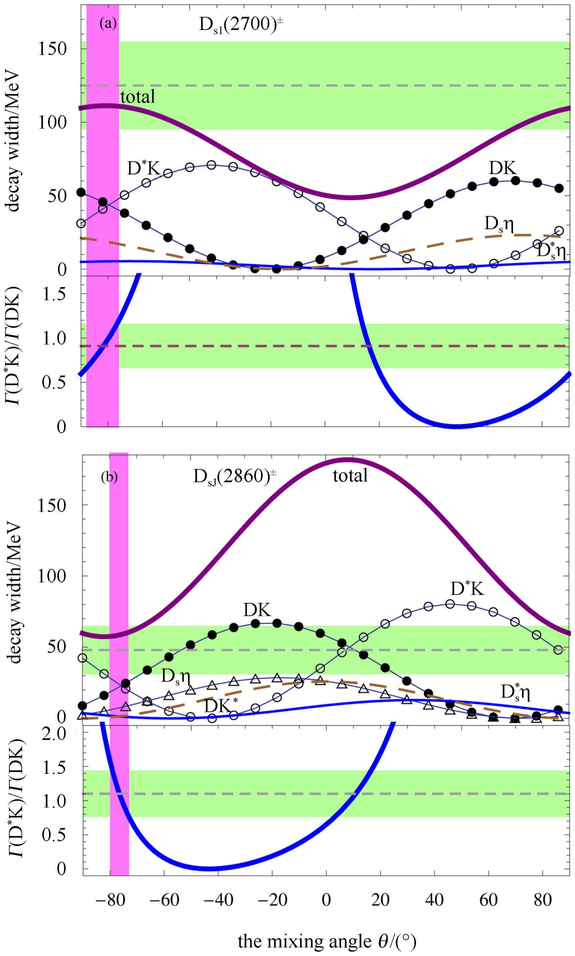

Fig. 1. Decay widths and $\Gamma(\mathrm{D^*K})/\Gamma(\mathrm{DK})$ of $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ versus θ. The horizontal dashed lines indicate the central values and the green regions indicate the upper and lower limits of the PDG data.

If $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ is regarded as the $|(SD)_1\rangle_{\mathrm{L}}$, the mixing angle is fixed at two different places in Fig. 2(a): $-90^\circ \leqslant \theta \leqslant -85^\circ$ and $12^\circ \leqslant \theta \leqslant 21^\circ$ by comparing the decay widths and branching ratio $\Gamma(\mathrm{D^*K})/\Gamma(\mathrm{DK})$ with experiments. The $-90^\circ \leqslant \theta \leqslant -85^\circ$ was excluded by the mass spectrum of these two states [7]. That is to say, $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ can be identified with the $|(SD)_1\rangle_{\mathrm{L}}$ with a small mixing angle $12^\circ \leqslant \theta \leqslant 21^\circ$. This fixed angle is similar to that in Ref. [7].

However, $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ is difficult to be identified with the $|(SD)_1\rangle_{\mathrm{R}}$ with a universal β parameter for all mesons. From Fig. 2(b), the predicted branching ratio $\Gamma(\mathrm{D^*K})/\Gamma(\mathrm{DK})$ of $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ is larger than the observed one at the mixing angles θ determined by the decay widths. Furthermore, the predicted total decay width (>150 MeV) of $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ is much broader than the observed $\Gamma = 48\pm3\pm6$ MeV. In short, $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ is hard to interpret as the orthogonal partner $|(SD)_1\rangle_{\mathrm{R}}$.

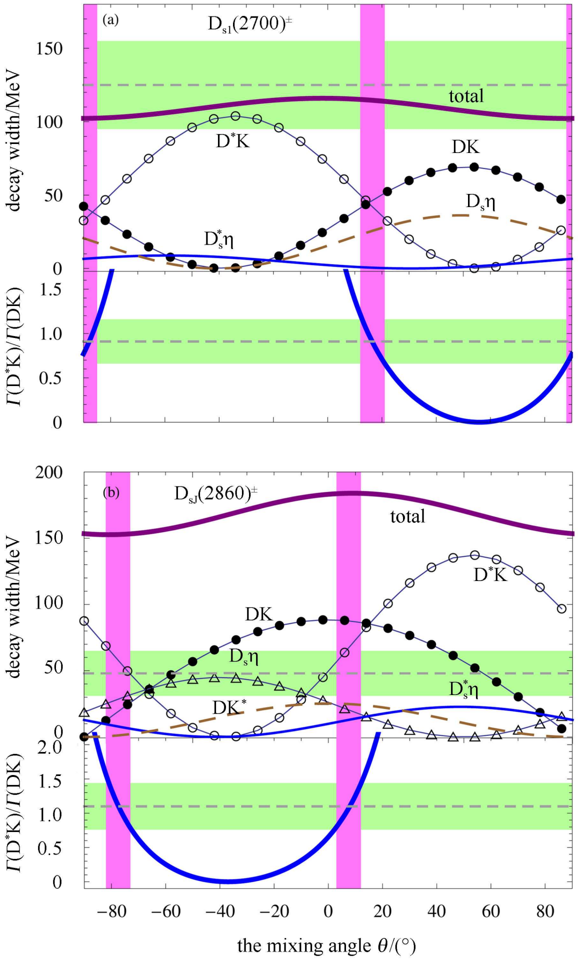

Fig. 2. Partial widths and $\Gamma(\mathrm{D^*K})/\Gamma(\mathrm{DK})$ for the $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ versus θ at $\beta = 0.38$ GeV. The horizontal dashed lines indicate the central values and the green regions indicate the upper and lower limits of the PDG data.

3 Conclusions and discussions

In this work, we have studied the mixing of $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ within the $^3P_0$ model. The strong decay widths are evaluated with two different ways of choice of the harmonic oscillator parameter β. The way of choice of β plays an important role in the interpretation of these two states. $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ are interpreted in two different ways of choice of β.

In the case of a special β for each meson, $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ is identified with the mixed 1^- state of $2\,^3S_1$ and $1\,^3D_1$, and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ is identified with the orthogonal partner of $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ with a large mixing angle θ, which implies that $1\,^3D_1$ is dominant.

In the case of a universal β for all mesons, $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ is interpreted as the mixed state of $2\,^3S_1$ and $1\,^3D_3$ with a small mixing angle θ ($2\,^3S_1$ is dominant). However, it is hard to interpret $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ as the orthogonal partner of $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$.

The interpretations of $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ are the same for different ways of choice of β when there is no mixing [9], but the interpretations of $\mathrm{D}^*_{\mathrm{s1}}(2700)^{\pm}$ and $\mathrm{D}^*_{\mathrm{sJ}}(2860)^{\pm}$ are different in different ways of choice of β when mixing exists. The origin of the difference is not clear, it may result from some inherent uncertainty within the $^3P_0$ model. If this uncertainty is accidental when there is mixing, the exploration of many other branching ratios may give more information about this uncertainty. Otherwise, the origin has to be explored in future study. Anyway, one must be careful to draw conclusions for these mixed states through the study of their strong decays within the $^3P_0$ model. To find the mixing detail of some higher excited states, the exploration of their strong decays in other models is necessary. In the meantime, the exploration of their other kinds of decays is required.

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