Pure annihilation type decays $B^0 \to D_s^- K_2^{*+}$ and $B_s \to \bar{D}a_2$ in the perturbative QCD approach *

ZOU Zhi-Tian(邹芝田) ZHOU Rui(周锐) LÜ Cai-Dian(吕才典)1)

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract: We calculate the branching ratios of pure annihilation type decays $B^0 \to D_s^- K_2^{*+}$ and $B_s \to \bar{D}a_2$ using the perturbative QCD approach based on k_T factorization. The branching ratios are predicted to be $(60.6^{+17.3+4.3+3.2}_{-16.5-10.4-2.1}) \times 10^{-6}$ for $B^0 \to D_s^- K_2^{*+}$, $(1.1^{+0.4+0.1+0.1}_{-0.4-0.2-0.1}) \times 10^{-6}$ for $B_s \to \bar{D}^0 a_2^0$ and $(2.3^{+0.8+0.2+0.1}_{-0.8-0.4-0.1}) \times 10^{-6}$ for $B_s \to D^- a_2^+$. They are large enough to be measured in the ongoing experiment. Due to the shortage of contributions from penguin operators, there are no direct CP asymmetries for these decays in the Standard Model. We also derive simple relations among these decay channels to reduce theoretical uncertainties for the experiments, to test the accuracy of theory, and to search for a signal of new physics.

Key words: B decay, PQCD, branching ratio

PACS: 13.25.Hw, 12.38.Bx **DOI:** 10.1088/1674-1137/37/1/013103

1 Introduction

Two body hadronic B decays have been a hot topic for many years, since they involve the perturbative QCD calculation and factorization study. They are also important for testing the standard model, the CKM angle measurements and the search for new physics phenomena. For many years, people did calculations based on the naive factorization assumption, later proved by the soft-collinear effective theory [1]. However, the so-called annihilation type diagrams have been argued to be helicity suppressed since no one knows how to calculate them. In the well developed collinear factorization, there is end-point singularity in the calculation of these diagrams. In fact, these kind of diagrams are essential for the strong phase of direct CP asymmetry in the $B \to K^+\pi^-$ decays [2], which is proved to be important.

Furthermore, there is one kind of B decay, which contains only annihilation type diagram contributions. One of the examples is the $B^0 \to D_s^- K^+$ decay, which is predicted in Refs.[3–5] and measured by the B factories later [6]. Recently, the CDF collaboration measured the first pure annihilation type decays in the B_s sector i.e. $B_s \to \pi^+\pi^-$ decay, which exactly confirms the perturbative QCD prediction for this decay [7, 8]. It is worth mentioning that the perturbative QCD (PQCD) approach is almost the only method that can do the quantitative calculations of the annihilation type diagrams [7, 8].

In this paper, we shall study the pure annihilation type charmed decays $B^0 \to D_s^- K_2^{*+}$ and $B_s \to \bar{D}a_2$ in the PQCD approach, which is based on the k_T factorization [9, 10]. These decays are predicted to have as large a branching ratio as 10^{-6} to 10^{-5}, which will be measurable in experiments in the near future. In the annihilation type diagrams, both the light quark and the heavy anti b quark in B meson annihilate into another quark anti-quark pair through the four quark operators, while another light quark pair in the final state mesons is produced by a gluon attaching to the four quark operator. Since the light quark in the final states is collinear, the gluon connection of them must be hard. So the hard part of the PQCD approach contains six quarks rather than four quarks. This is called six-quark effective theory or six-quark operator. In this approach, the quarks' intrinsic transverse momenta are kept to avoid the end-point divergence. Because of the additional energy scale introduced by the transverse momentum, double logarithms will appear in the QCD radiative corrections. We re-sum these double logarithms to give a Sudakov factor, which effectively suppresses the end-point region contribution. This makes the PQCD approach more reliable and consistent.

This paper is organized as follows. In Section 2, we present the formalism and perform the perturbative calculations for considered decay channels with the PQCD approach. The numerical results and phenomenological analysis are given in Section 3. Finally, Section 4 contains a short summary.

Received 16 April 2012, Revised 28 April 2012

* Supported by National Natural Science Foundation of China (11075168)

1) E-mail: lucd@ihep.ac.cn

2 Formalism and perturbative calculation

The $B^0\to D_s^- K_2^{*+}$, $B_s\to \bar{D}^0 a_2^0$ and $B_s\to D^- a_2^+$ decays are pure annihilation type rare decays. At the quark level, these decays are described by the effective Hamiltonian $H_{\rm eff}$ [11]

$$H_{\rm eff}=\frac{G_{\rm F}}{\sqrt{2}}V_{\rm cb}^* V_{\rm uD}[C_1(\mu)O_1(\mu)+C_2(\mu)O_2(\mu)], \qquad (1)$$

where $V_{\rm cb}$ and $V_{\rm uD}$ are the CKM matrix elements, "D" denotes the light down quark d or s, and $C_{1,2}(\mu)$ are Wilson coefficients at the renormalization scale μ. $O_{1,2}(\mu)$ are the four quark operators.

$$\begin{aligned} O_1&=(\bar{b}_\alpha c_\beta)_{V-A}(\bar{u}_\beta D_\alpha)_{V-A},\\ O_2&=(\bar{b}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta D_\beta)_{V-A}, \end{aligned} \qquad (2)$$

where α and β are the color indices, $(\bar{b}_\alpha c_\beta)_{V-A}=\bar{b}_\alpha\gamma^\mu(1-\gamma^5)c_\beta$. Conventionally, we define the combined Wilson coefficients as

$$a_1=C_2+C_1/3, a_2=C_1+C_2/3. \qquad (3)$$

In the hadronic matrix element calculation, we factorize the decay amplitude into soft (Φ), hard (H), and harder (C) dynamics characterized by different scales [12, 13],

$$\begin{aligned} \mathcal{A}\sim&\int {\rm d}x_1{\rm d}x_2{\rm d}x_3 b_1{\rm d}b_1 b_2{\rm d}b_2 b_3{\rm d}b_3\\ &\times{\rm Tr}[C(t)\Phi_{\rm B}(x_1,b_1)\Phi_{\rm M_2}(x_2,b_2)\Phi_{\rm M_3}(x_3,b_3)\\ &\times H(x_i,b_i,t)S_{\rm t}(x_i){\rm e}^{-S(t)}], \end{aligned} \qquad (4)$$

where b_i is the conjugate variable of quark's transverse momentum $k_{i\rm T}$, x_i is the momentum fractions of valence quarks, and t is the largest energy scale in function $H(x_i,\ b_i,\ t)$ which is the hard part. $C(t)$ are the Wilson coefficients with resummation of the large logarithms $\ln(m_{\rm W}/t)$ produced by the QCD corrections of four quark operators. $S_{\rm t}(x_i)$ is the jet function, which is obtained by the threshold resummation and smears the end-point singularities on x_i [14]. The last term, ${\rm e}^{-S(t)}$, is the Sudakov form factor, from resummation of double logarithms, which suppresses the soft dynamics effectively and the long distance contributions in the large b region [15, 16]. Thus it makes the perturbative calculation of the hard part H applicable at an intermediate scale, i.e., $m_{\rm B}$ scale. The Φ_i, meson wave functions, are nonperturbative input parameters but universal for all decay modes.

The lowest order Feynman diagrams of the considered decays are shown in Fig. 1. The amplitude from factorizable diagrams (a) and (b) in Fig. 1 is

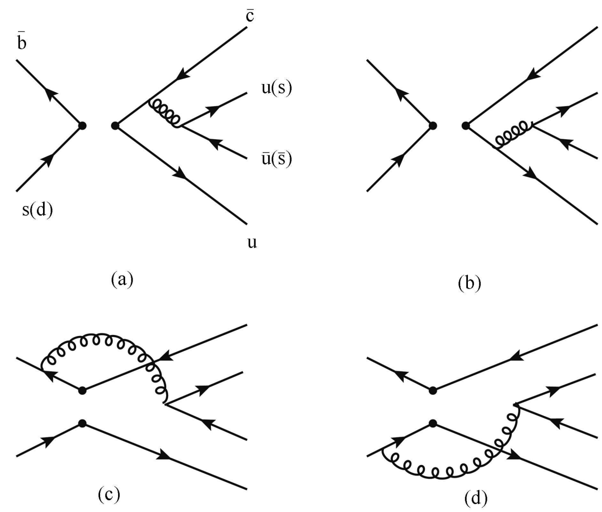

Fig. 1. The annihilation diagrams contributing to the $B\to\bar{D}T$ decays in PQCD.

$$\begin{aligned} \mathcal{A}_{\rm af}\ =\ & 8\sqrt{\frac{2}{3}}C_{\rm F}f_{\rm B}\pi m_{\rm B}^4\\ &\times\int_0^1{\rm d}x_2{\rm d}x_3\int_0^{1/\Lambda}b_2{\rm d}b_2b_3{\rm d}b_3\phi_{\rm D}(x_2,b_2)\\ &\times\{[-\phi_{\rm T}(x_3)x_2+2r_{\rm D}r_{\rm T}\phi_{\rm T}^{\rm s}(x_3)(x_2+1)]\\ &\times h_{\rm af}((1-x_3),x_2(1-r_{\rm D}^2),b_2,b_3)E_{\rm af}(t_{\rm e})\\ &-[\phi_{\rm T}(x_3)(x_3-1)-r_{\rm D}r_{\rm T}(\phi_{\rm T}^{\rm t}(x_3)(1-2x_3)\\ &+\phi_{\rm T}^{\rm s}(x_3)(2x_3-3))]\\ &\times h_{\rm af}(x_2,(1-x_3)(1-r_{\rm D}^2),b_3,b_2)E_{\rm af}(t_{\rm f})\}. \end{aligned} \qquad (5)$$

In this function, $C_{\rm F}=4/3$ is the group factor of $SU(3)_{\rm c}$, and $r_{\rm D(T)}=m_{\rm D(T)}/m_{\rm B}$. The hard scale $t_{\rm e,f}$ and the functions $E_{\rm af}$ and $h_{\rm af}$ are given by

$$\begin{aligned} t_{\rm e}\ &=\ \max\left\{\sqrt{x_2(1-r_{\rm D}^2)}m_{\rm B},1/b_2,1/b_3\right\},\\ t_{\rm f}\ &=\ \max\{\sqrt{(1-x_3)(1-r_{\rm D}^2)}m_{\rm B},1/b_2,1/b_3\}, \end{aligned} \qquad (6)$$

$$E_{\rm af}(t)\ =\ \alpha_{\rm s}(t)\cdot\exp[-S_{\rm T}(t)-S_{\rm D}(t)], \qquad (7)$$

$$\begin{aligned} h_{\rm af}(x_2,x_3,b_2,b_3)\ =\ & \left(\frac{{\rm i}\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_2x_3}m_{\rm B}b_2)S_{\rm t}(x_3)\\ &\times\Big[\theta(b_2-b_3)H_0^{(1)}(\sqrt{x_3}m_{\rm B}b_2)\\ &\times J_0(\sqrt{x_3}m_{\rm B}b_3)\\ &+\theta(b_3-b_2)H_0^{(1)}(\sqrt{x_3}m_{\rm B}b_3)\\ &\times J_0(\sqrt{x_3}m_{\rm B}b_2)\Big]. \end{aligned} \qquad (8)$$

The amplitude for nonfactorizable diagrams (c) and (d) in Fig. 1 is

$$\begin{aligned}\mathcal{M}_{\rm anf} =& \frac{32}{3}C_{\rm F}\pi m_{\rm B}^4\int_0^1 {\rm d}x_1{\rm d}x_2{\rm d}x_3\int_0^{1/\Lambda} b_1{\rm d}b_1 b_2{\rm d}b_2\\ &\times\phi_{\rm B}(x_1,b_1)\phi_{\rm D}(x_2,b_2)\\ &\times\{[\phi_{\rm T}(x_3)x_2+r_{\rm D}r_{\rm T}(\phi_{\rm T}^s(x_3)(x_3-x_2-3)\\ &+\phi_{\rm T}^{\rm t}(x_3)(x_2+x_3-1))]\\ &\times h_{\rm anf1}(x_1,x_2,x_3,b_1,b_2)E_{\rm anf}(t_{\rm g})\\ &+[\phi_{\rm T}(x_3)(x_3-1)\\ &+r_{\rm D}r_{\rm T}(\phi_{\rm T}^{\rm s}(x_3)(x_2-x_3+1)\\ &+\phi_{\rm T}^{\rm t}(x_3)(x_2+x_3-1))]\\ &\times h_{\rm anf2}(x_1,x_2,x_3,b_1,b_2)E_{\rm anf}(t_{\rm h})\}. \qquad (9)\end{aligned}$$

$$\begin{aligned}t_{\rm g} =& \max\Big\{1/b_1,1/b_2,\sqrt{x_2(1-x_3)(1-r_{\rm D}^2)}m_{\rm B},\\ &\sqrt{1-(1-(1-x_3)(1-r_{\rm D}^2))(1-x_1-x_2)}m_{\rm B},\Big\},\\ t_{\rm h} =& \max\Big\{1/b_1,1/b_2,\sqrt{x_2(1-x_3)(1-r_{\rm D}^2)}m_{\rm B},\\ &\sqrt{(1-x_3)(1-r_{\rm D}^2)|x_1-x_2|}m_{\rm B},\Big\}, \qquad (10)\end{aligned}$$

$$E_{\rm anf}=\alpha_{\rm s}(t)\cdot\exp[-S_{\rm B}(t)-S_{\rm T}(t)-S_{\rm D}(t)]|_{b_2=b_3}, \qquad (11)$$

$$\begin{aligned}&h_{{\rm anf}j}(x_1,x_2,x_3,b_1,b_2)=\frac{{\rm i}\pi}{2}\\ &\times\Big[\theta(b_1-b_2)H_0^{(1)}(Fm_{\rm B}b_1)J_0(Fm_{\rm B}b_2)\\ &+\theta(b_2-b_1)H_0^{(1)}(Fm_{\rm B}b_2)J_0(Fm_{\rm B}b_1)\Big]\\ &\times\begin{cases}\frac{{\rm i}\pi}{2}H_0^{(1)}\left(\sqrt{|F_j^2|}m_{\rm B}b_1\right), & F_j^2<0,\\ K_0(F_jm_{\rm B}b_1), & F_j^2>0,\end{cases} \qquad (12)\end{aligned}$$

with j=1,2.

$$\begin{aligned}F^2 =& x_2(1-x_3)(1-r_{\rm D}^2),\\ F_1^2 =& 1-(1-(1-x_3)(1-r_D^2))(1-x_1-x_2),\\ F_2^2 =& (1-x_3)(1-r_{\rm D}^2)(x_1-x_2). \qquad (13)\end{aligned}$$

The expressions of $S_{\rm B}(t)$, $S_{\rm T}(t)$, $S_{\rm D}(t)$ and $S_{\rm t}$ can be found in Refs. [10, 14, 16, 17]. The wave functions of initial and final states can be found in Refs. [4, 5, 8, 18–23].

With the functions obtained above, the amplitudes of these pure annihilation type decay channels can be given by

$$\mathcal{A}({\rm B}^0\to{\rm D}_{\rm s}^-{\rm K}_2^{*+})=\frac{G_{\rm F}}{\sqrt{2}}V_{\rm cb}^*V_{\rm ud}[a_2\mathcal{A}_{\rm af}+C_2\mathcal{M}_{\rm anf}], \qquad (14)$$

$$\mathcal{A}({\rm B}_{\rm s}^0\to\bar{\rm D}^0{\rm a}_2^0)=\frac{G_{\rm F}}{\sqrt{2}}\frac{1}{\sqrt{2}}V_{\rm cb}^*V_{\rm us}[a_2\mathcal{A}_{\rm af}+C_2\mathcal{M}_{\rm anf}], \qquad (15)$$

$$\mathcal{A}({\rm B}_{\rm s}^0\to{\rm D}^-{\rm a}_2^+)=\frac{G_{\rm F}}{\sqrt{2}}V_{\rm cb}^*V_{\rm us}[a_2\mathcal{A}_{\rm af}+C_2\mathcal{M}_{\rm anf}]. \qquad (16)$$

3 Numerical results and discussions

For numerical analysis, we use the following input parameters:

$$\begin{aligned}&f_{\rm B/B_s}=0.21/0.23\ {\rm GeV},\ f_{\rm D/D_s}=0.205/0.241\ {\rm GeV},\\ &f_{{\rm K}_2^*}^{\rm (T)}=118(77)\ {\rm MeV},\ f_{{\rm a}_2}^{\rm (T)}=102(117)\ {\rm MeV},\\ &M_{\rm D/D_s}=1.869/1.968\ {\rm GeV},\\ &M_{\rm B/B_s}=5.279/5.366\ {\rm GeV},\\ &|V_{\rm cb}|=0.0415\pm0.0011,\ |V_{\rm ud}|=0.9742\pm0.0002,\\ &|V_{\rm us}|=0.2257\pm0.0012,\ \Lambda_{\rm QCD}^{\rm f=4}=0.25\ {\rm GeV}. \qquad (17)\end{aligned}$$

After numerical calculation, the branching ratios of these decays are:

$$\begin{aligned}&Br({\rm B}^0\to{\rm D}_{\rm s}^-{\rm K}_2^{*+})=(60.6^{+17.3+4.3+3.2}_{-16.5-10.4-2.1})\times10^{-6},\\ &Br({\rm B}_{\rm s}\to\bar{\rm D}^0{\rm a}_2^0)=(1.1^{+0.4+0.1+0.1}_{-0.4-0.2-0.1})\times10^{-6},\\ &Br({\rm B}_{\rm s}\to{\rm D}^-{\rm a}_2^+)=(2.3^{+0.8+0.2+0.1}_{-0.8-0.4-0.1})\times10^{-6}. \qquad (18)\end{aligned}$$

The branching ratio obtained from the analytic formulas may be sensitive to many parameters especially those in the meson wave function. We estimated three kinds of theoretical uncertainties in our calculations. The first errors in our calculations are caused by the hadronic parameters, such as the decay constants and the shape parameters in wave functions of the charmed meson and the $\rm B_{(s)}$ meson, and the decay constants of tensor mesons. The second errors are estimated from the unknown next-to-leading order QCD corrections with respect to $\alpha_{\rm s}$ and nonperturbative power corrections with respect to scales in Sudakov exponents, characterized by the choice of the $\Lambda_{\rm QCD}=(0.25\pm0.05)$ GeV and the variations of the factorization scales defined in Eq. (6) and Eq. (10). The third errors are from the uncertainties of the CKM matrix elements. It is easy to see that the most important theoretical uncertainty is caused by the non-perturbative hadronic parameters, which are universal and can be improved by experiments.

These pure annihilation type decays considered in this work are dominant by W exchange diagram. None of these decays have contributions from the penguin operators. Since the direct CP asymmetry is caused by the interference between the contributions of tree operators and those of penguin operators, it does not appear in these modes. Although the annihilation type diagrams are power suppressed in the PQCD approach, the branching ratio of these considered Cabibbo-Kobayashi-Maskawa-favored decays is sizable and large enough to be measured in experiment. Through the study of these pure annihilation type decay modes, we can understand the annihilation mechanism in B physics well.

It is easy to find that there are large theoretical uncertainties in any of the individual decay mode calculations. However, we can reduce the uncertainties by ratios of decay channels. For example, simple relations among these decay channels are derived from Eqs. (14)–(16)

$$\frac{Br(\mathrm{B}^0\to \mathrm{D}_\mathrm{s}^-\mathrm{K}_2^{*+})}{Br(\mathrm{B_s}\to \bar{\mathrm{D}}^0\mathrm{a}_2^0)}\sim\frac{2f_{\mathrm{D_s}}^2V_{\mathrm{ud}}^2}{f_{\mathrm{D}}^2V_{\mathrm{us}}^2}\sim 60\sim\frac{60.6}{1.1},$$

$$\frac{Br(\mathrm{B_s}\to \bar{\mathrm{D}}^0\mathrm{a}_2^0)}{Br(\mathrm{B_s}\to \mathrm{D}^-\mathrm{a}_2^+)}\sim\frac{1}{2}\sim\frac{1.1}{2.3}. \tag{19}$$

It is obvious that any significant deviation from the above relations will be a signal of new physics.

4 Summary

We calculate the branching ratios of three pure annihilation type decays in the perturbative QCD approach. The predicted branching ratios are $Br(\mathrm{B}^0\to \mathrm{D}_\mathrm{s}^-\mathrm{K}_2^{*+})\sim 6\times10^{-5}$, $Br(\mathrm{B_s}\to \bar{\mathrm{D}}^0\mathrm{a}_2^0)\sim 1\times10^{-6}$ and $Br(\mathrm{B_s}\to \mathrm{D}^-\mathrm{a}_2^+)\sim 2\times10^{-6}$. They are sizable and large enough to be measured in the forthcoming experiments. The study of pure annihilation type decays can help us understand the annihilation mechanism in B physics. There are no direct CP asymmetries, because these decays have no contributions from penguin operators in the Standard Model.

We are very grateful to Xin Yu and Dr. Run-Hui Li for helpful discussions.

References

1 Bauer C W, Pirjol D, Rothstein I Z et al. Phys. Rev. D, 2004, **70**: 054015
2 Keum Y Y, LI H N, Sand A I. Phys. Rev. D, 2001, **63**: 054008; HONG Bi-Hai, LÜ Cai-Dian. Sci. China G, 2006, **49**: 357
3 LÜ Cai-Dian, Ukai K. Eur. Phys. J. C, 2003, **28**: 305
4 LI Run-Hui, LÜ Cai-Dian, ZOU Hao. Phys. Rev. D, 2008, **78**: 014018
5 ZOU Hao, LI Run-Hui, WANG Xiao-Xia et al. J. Phys. G, 2010, **37**: 015002
6 Nakamura K et al. (Particle Data Group). J Phys. G, 2010, **37**: 075021; Krokovny P et al. (Belle collaboration). Phys. Rev. Lett., 2002, **89**: 231804; Das A et al. (Belle collaboration). Phys. Rev. D, 2010, **82**: 051103; Aubert B et al. (BABAR collaboration). arXiv:hep-ex/0207053
7 LI Ying, LÜ Cai-Dian, XIAO Zhen-Jun et al. Phys. Rev. D, 2004, **70**: 034009
8 Ali A, Kuamer G, LI Ying et al. Phys. Rev. D, 2007, **76**: 074018
9 Keum Y Y, LI H N, Sanda A I. Phys. Lett. B, 2001,**504**: 6; Phys. Rev. D, 2001, **63**: 054008
10 LÜ Cai-Dian, Ukai K, YANG Mao-Zhi. Phys. Rev. D, 2001, **63**: 074009
11 Buchalla G, Buras A J, Lautenbacher M E. Rev. Mod. Phys., 1996, **68**: 1125; Buras A J. arXiv:hep-ph/9806471
12 DU Dong-Sheng, LI Ying, LÜ Cai-Dian. Chin. Phys. Lett., 2006, **23**: 2038
13 LIU Xin, XIAO Zhen-Jun, LÜ Cai-Dian. Phys. Rev. D, 2010, **81**: 014002
14 LI H N. Phys. Rev. D, 2002, **66**: 094010
15 LI H N, Tseng B. Phys. Rev. D, 1998, **57**: 443
16 LÜ Cai-Dian, YANG Mao-Zhi. Eur. Phys. J. C, 2002, **23**: 275
17 LÜ Cai-Dian, YANG Mao-Zhi. Eur. Phys. J. C, 2003, **28**: 515
18 LI H N. Phys. Lett. B, 2005, **622**: 63
19 LI H N, Mishima S. Phys. Rev. D, 2005, **71**: 054025
20 LI Run-Hui, LÜ Cai-Dian, Sanda A I et al. Phys. Rev. D, 2010, **81**: 034006
21 WANG Wei. Phys. Rev. D, 2011, **83**: 014008
22 CHENG H Y, Koike Y, YANG Kwei-Chou. Phys. Rev. D, 2010, **82**: 054019
23 CHENG H Y, YANG Kwei-Chou. Phys. Rev. D, 2011, **83**: 034001