A tentative method for assigning the configuration of a triaxial nuclei in TRS*

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Abstract: A tentative method based on the principle of minimum energy is put forward for assigning the reasonable configuration of a triaxial nucleus in TRS. This method is proved by the TSD of ^{167}Lu nucleus that has been calculated previously by TRS.

Key words: configuration, triaxial, quasiparticle, signature

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1 Introduction

The investigation of the shape of atomic nuclei has been an interesting topic in nuclear structure physics for many years. Only nuclei with magic proton or neutron numbers, corresponding to the closed shells, are spherical at low excitation energy, while the majority of nuclei are deformed and many of them can be described based on the axial symmetry. The triaxial motion is a fundamental problem in nuclear physics and plays an important role in many nuclear phenomena [1]. For example, the triaxiality has been used to explain the signature splitting in rotational bands in the A=130 region [2], the possible chiral band doublets in some odd-odd nuclei [3, 4] and the unusually fast decay from isomers, e.g. in ^{176}W [5]. The best direct evidence for the triaxiality is the discovery of the wobbling mode in a nuclear system at high spin. The crucial experiments providing clear evidence for the triaxial rotation are the discoveries of the first- and second-phonon wobbling bands in ^{163}Lu [6, 7]. The wobbling mode provides unique evidence for a stable triaxial shape, and thus these wobbling superdeformed bands were identified as the triaxial superdeformed bands (TSD). The confirmations for the TSD bands in the mass region around ^{163}Lu are the discoveries of the wobbling bands in ^{161}Lu [8], ^{165}Lu [9, 10] and ^{167}Lu [11].

In theory, the potential energy surface was calculated by some different methods, e.g. the ultimate cranker (UC) code [12, 13] and the total routhian surface (TRS). In this paper, the configuration-dependent TRS approach is employed to calculate the total potential energy consisting of the core energy and quasiparticles (qp) energies. However, we will take a newly tentative method based on the principle of minimum energy to assign the configuration corresponding to the triaxial deformation of an atomic nucleus.

2 Brief description of the mode

The total energy in the rotating frame, as a function of deformations ε_2, γ and ε_4, for a given quasiparticle configuration (cf) may be calculated in the one-dimensional cranking approximation,

$$E_{\rm total}(\omega)=E_{\rm LD}+E_{\rm corr}(\omega=0)+E_{\rm rot}(\omega)+\sum_{i\in{\rm cf}}e_i^{\omega}, \quad (1)$$

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where $E_{\rm LD}$ is the liquid-drop model energy [14], $E_{\rm corr}$ is the quantal effect correction to the energy, which includes both the shell correction and the pairing correction [15]. The collective rotational energy $E_{\rm rot}$ is calculated microscopically as the difference between the expectation values of the cranking Hamiltonian H^{ω} with and without rotation by using the wave function of the qusiparticle vacuum [16, 17]

$$E_{\rm rot}(\omega) = \langle\psi_{\rm vac}(\omega\neq 0)\,|H^{\omega}|\,\psi_{\rm vac}(\omega\neq 0)\rangle - \langle\psi_{\rm vac}(\omega=0)\,|H^{\omega}|\,\psi_{\rm vac}(\omega=0)\rangle\,, \quad (2)$$

where the cranking Hamiltonian H^{ω} describes quasiparticles moving in a quadrupolely deformed potential rotating around a principle axis, with a frequency ω and is written as

$$H^{\omega}=H_{\rm sp}(\varepsilon_2,\gamma,\varepsilon_4)-\lambda N+\Delta(P^{+}+P^{-})-\omega J_x, \quad (3)$$

where $H_{\rm sp}$ is the deformed single particle Nilsson Hamiltonian [18] depending on the deformation parameters, and having the modified harmonic oscillator(MHO) potential containing a proper spin-orbit force, and the Nilsson parameter set is taken from Ref. [19]. The parameters ε_2, γ and ε_4, describe the elongation deformation, triaxial deformation and hexadecapole deformation, respectively. The second term in Eq. (3) is the chemical potential, the third term is the pairing interaction, and the last term stands for the Coriolis and centrifugal forces.

The Fermi level λ is determined by the particle number in the BCS approximation. By considering the anti-pairing effect of rotation, the pairing gap parameters for both protons and neutrons are evaluated empirically with Δ=0.9$\Delta_{\rm o.e}$ where $\Delta_{\rm o.e}$ is the experimental odd–even mass difference taken from Ref. [20] For an approximation, the full dependence of the pairing gap on rotation is not considered in the TRS calculations. According to the checking calculation, the location of the TSD minimum in the TRS is not sensitive to the reasonable choice of the reduction factor of the pairing gap parameters.

The equilibrium deformation of a nucleus can be obtained by minimizing the total routhian energy $E_{\rm total}$, calculated by Eq. (1), with respect to the deformation parameters ε_2, γ and ε_4. In the three-dimensional TRS calculation, the minimization procedure includes two steps. First, the three-dimensional total routhian energy $E_{\rm total}$ is minimized with respect to the hexadecapole deformation ε_4 for each deformation point in the (ε_2, γ) plane. As a result, one obtains the equilibrium hexadecapole deformation surface, $\varepsilon_{4\rm min}(\varepsilon_2,\ \gamma)$, and the corresponding new total routhian energy surface,

$$E'_{\rm total}(\varepsilon_2,\gamma)=E_{\rm total}(\varepsilon_2,\gamma,\varepsilon_{4\rm min}).$$

Second, we minimize $E'_{\rm total}(\varepsilon_2,\gamma)$ with respect to the ε_2 and γ to obtain the equilibrium quadrupole deformations, and then it is straightforward to read out the corresponding equilibrium hexadecapole deformation by sprawling the $\varepsilon_{4\rm min}(\varepsilon_2,\gamma)$ surface.

3 The tentative method for assigning the configuration

In TRS, the total energy of a triaxial nucleus is calculated under an assured configuration. Usually, the configuration is chosen according to results of experiments. The configuration is only described by L and j quantum numbers of proton or neutron, whereas this shell contains $2j$+1 nucleons for degeneracy. It is necessary to know more information about the configuration. The configuration of a nucleus is denoted by Nilsson levels, namely $[nl_zs_z]\Omega^{\pm}$ where the "±" symbol stands for the signature and the parity is $(-1)^n$. However, the method that assigns the configuration of the nucleus is very rough, which supposes quasiparticles take up orbits near or above the Fermi surface in Nilsson levels.

As is known, the total energy of a stable nucleus is minimal. The principle of minimum energy is universal in nuclear regions. For example, nucleons make up nuclei from the lowest energy level to higher energy levels under Pauli's exclusion principle. However, the pairing protons or neutrons first take up the higher energy levels which have a greater pairing gap than lower energy levels according to the shell model of nuclei [21]. So we can make a conclusion that the total energy surface of a metastable nucleus should have a local minimum. This is the reason that we can get the quadrupole deformation and the trixial deformation in TRS.

In this paper, we will consider the relationship between the minimum energy and the steady state of the nucleus from an opposite side. If what we got first is the energy of a number of quasi-particles of different configurations, one of these configurations whose total potential (namely the sum of the energy of its corresponding quasi-particle and the core energy) is more prone to be of the local minimum, should be the optimal configuration option corresponding to the stable deformation.

We will prove the feasibility of our method by the configuration of [660]1/2 proton calculated by TRS accounting for the TSD in ^{167}Lu nucleus [22].

Each grid point corresponds to the unique ε_2, γ and ε_4 in the total potential energy surface. We fix y-axis variable where $y=\varepsilon_2\sin(\gamma+30^\circ)$, and then the horizontal abscissa variable where $x=\varepsilon_2\cos(\gamma+30^\circ)$ is changed continuously. The fixed y-axis variable is chosen at y=0.3 point, which corresponds to large deformation points. In order to accurately trace the configuration we seek, the horizontal abscissa variable is divided into 100 parts, namely 100 times calculations are done from 0 to 0.5 in x-axis for 0.05 interval.

In Fig. 1, several dotted-curves show the relation between the energy of proton configuration and the deformation of the nucleus for different signatures in n=6 main shell. The majority energies of proton quasiparticles for n=6 gradually decrease with increasing the horizontal abscissa variable x. We can see a local minimum energy at "A" position in the routhian energy surface, hence only configurations that have a minimum near the horizontal abscissa corresponding "A" position in Fig. 2 contribute to this local minimum energy based on the principle of minimum energy. It is shown that the configuration of $[660]1/2^-$ does it in Fig. 1. It is coincident with the conclusion of previous calculations. In addition, we have also found two other interesting phenomena. First, the curve of the energy of $[633]7/2^-$ proton quasiparticle has the same shape as $[660]1/2^-$ proton quasiparticle, and moreover its minimum is deeper. We think that $[633]7/2^-$ may be a candidate configuration for the TSD in ^{167}Lu though the position of its minimum in x-axis is less than $[660]1/2^-$. Second, $[642]5/2^-$ and $[624]9/2^+$ quasi-protons have very similar curves of energies where the horizontal abscissa variable is greater than 0.25. For explanation of this phenomenon more studies need to be carried out.

^{167}Lu is an odd-even nucleus, so it is considered one quasiproton configuration for the triaxial deformation nucleus with a low spin state. Some nuclei have a more complicated configuration with three or five quasiparticles as a result of breaking of proton or/and neutron pairs if they populate high spin states in experiments.

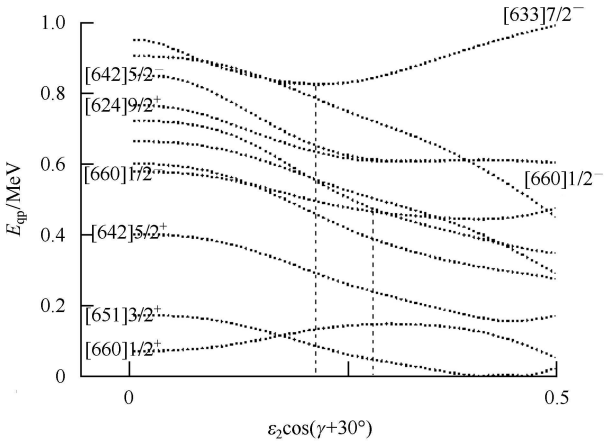

Fig. 1. Curves of energies of proton quasiparticles for n=6.

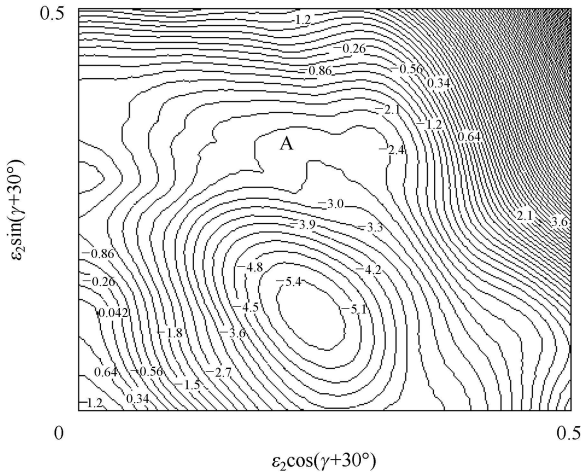

Fig. 2. Routhian for ^{167}Lu.

References

1 CHEN Y S, GAO Z C. Chinese. Physics. C (HEP & NP), 2009, **33**(SI): 1-4
2 CHEN Y S, Frauendorf S, Leander G A. Phys. Rev. C, 1983, **28**(6): 2437–2445
3 Frauendorf S, MENG J. Nucl. Phys. A, 1997, **617**(2): 131–147
4 XIAO X W, CHEN H. C. Chinese. Physics. C (HEP & NP), 2009, **33**(SI): 105–108
5 Crowell B, Chowdhury P, Freeman S J et al. Phys. Rev. Lett., 1994, **72**(8): 1164–1167
6 Ødegard S W, Hagemann G B Jensen D R et al. Phys. Rev. Lett., 2001, **86**(26): 5866–5869
7 Jensen D R, Hagemann G B, Hamamoto I et al. Nucl. Phys. A, 2002, **703**(1-2): 3–44
8 Bringel P, Hübel H Amro H et al. Eur. Phys. J. A, 2003, **16**: 155–158
9 Schönwaßer G, Hübel H, Hagemann G B et al. Phys. Lett. B, 2003, **552**(1-2): 9–16
10 Schönwaßer G, Nenoff N Hübel H et al. Nucl. Phys. A 2004, **735**(3-4): 393–424
11 Amro H, MA W C, Hagemann G B et al. Phys. Lett. B, 2003, **553**(3-4): 197–203
12 Bengtsson R. Nucl. Phys. A, 1990, **512**(1): 124–148
13 Bengtsson R. Nucl. Phys. A, 1989, **496**(1): 56–92
14 Wapstra A H Audi G. Nucl. Phys. A, 1985, **432**(1): 1–54
15 Wyss R, Nyberg J, Johnson A et al. Phys. Lett. B, 1988, **215**(2): 211–217
16 CHEN Y S Frauendorf S, Riedinger L L. Phys. Lett. B, 1986, **171**(1): 7–12
17 LUO W D, CHEN Y S. Nucl. Phys. A, 1993, **564**(3): 413–424
18 Nilsson S G, Tsang C F, Sobiczewski A et al. Nucl. Phys. A, 1969, **131**(1): 1–66
19 Bengtsson T, Ragnarsson I. Nucl. Phys. A, 1985, **436**(1): 14–82
20 Ragnarsson I, Sheline R K. Phys. Scr., 1985, **385**: 29
21 Ring P, Schuck P. The Nuclear Many-body Problem. Berlin: Spring Verlag, 1980. 217–243
22 YANG C X, WU X G, ZHENG H et al. Eur. Phys. J. A, 1998, **1**: 237