Charm structure functions and gluon shadowing effects with the AdS/CFT model^{*}

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Abstract: By means of the UGD function extracted from an AdS/CFT inspired saturation model, the charm and bottom structure functions are studied in fixed-order perturbation theory. It is shown that the theoretical results are in good agreement with the recent HERA data. Then, this UGD function is also used to investigate net-kaon rapidity distribution in Au+Au collisions at RHIC energies and the theoretical results fit well to the BRAHMS data. In the end of this paper, we give the predicted results for nuclear charm structure function at very small x where the popular shadowing parameterizations are invalid.

Key words: AdS/CFT inspired model, unintegrated gluon distribution, shadowing effect
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1 Introduction

Since perturbative quantum chromodynamics (pQCD) predicted that gluons in a hadron wavefunction at small Bjorken-x should form a color glass condensate (CGC), the search for signatures of the CGC has been a significat and on-going subject for many years (for recent reviews see Ref. [1]). The established saturation/CGC models can successfully interpret the phenomena of diffraction and F_2 data at HERA [2– 7]. Among the saturation models, the Golec-Biernat and Wüsthoff (GBW) model [2], which also has a simple form for the dipole-proton cross section, gave a first hint toward saturation effects at HERA. After that other saturation models such as the b-CGC model [3, 4], the Soyez model [5], and the Kowalski and Teaney (KT) model [6] were established. Unfortunately, the above-mentioned saturation models are based on perturbative QCD and their validity at very small Q^2 where one has to consider small-x evolution in the large coupling limit is questionable. Recently, Kovchegov Lu and Rezaeian (KLR) proposed a model inspired by the anti-de Sitter space/conformal field theory (AdS/CFT) correspondence [7]. This model can well describes the HERA data at small-x and Q^2 . In this paper, through the Fourier transform from the forward dipole scattering amplitude, an analytic formula for the unintegrated gluon distribution (UGD) function is extracted from this model and used to study the charm (bottom) structure function and netkaon production.

The UGD function extracted from the saturation model has been widely used to study hadron production in deuteron-gold (d+Au) [8] and nucleus-nucleus (A+A) [9, 10] collisions for many years. In d+Au or A+A collisions, the net-hadron number is essentially transported by the valence quarks that probe the saturation regime in the target. Since the valence quark parton distribution, which is at large x, can be clearly described by the well known parton distribution function, the above processes provide a clean probe of the UGD function in the saturation regime. In Refs. [9, 10], the UGD function from the GBW model is used to study net-baryon production in Au+Au collisions. Though the theoretical results can preferably explain the data from BRAHMS, the parameter value used

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in Refs. [9, 10] is not the one extracted from HERA data [2]. In this paper, without changing the parameter value of the AdS/CFT model extracted from HERA data, the UGD function from this model is first checked by the data of the charm and bottom structure functions in electron-proton (e+p) collisions. Then, it is used to study net-kaon rapidity distribution in Au+Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV.

During the investigation of net-kaon production in Au+Au collisions, the shadowing effect must be considered. Among the shadowing parameterizations, the Eskola, Paukkunen and Salgado (EPS) [11] and the de Florian and Sassot (DS) [12] are two popular used parameterizations. Based on the DGLAP evolution, both the EPS and the DS parameterizations give a best global fit to nuclear hard-process data from deep inelastic lepton-nucleus scattering (DIS) and the Drell-Yan (DY) process in proton-nucleus collisions. For the effective region of the above two parameterizations is $10^{-6} < x < 1$, they can not be used to study shadowing effect for $x < 10^{-6}$ that will be measured in the future Electron Ion Collider (EIC) and Large Hadron-electron Collider (LHeC) [13,14]. In the following, we will also give the predictive results for the charm quark structure function in e+Pb collisions at very small x where the popular shadowing effects are invalid.

2 Method

In Ref. [15], charm and bottom quarks are not considered as intrinsic partons in a nucleon, but are treated in fixed-order perturbation theory. In leading order (LO), a cc̄ pair can be created by boson-gluon fusion, $\gamma^* g \rightarrow cc̄$, when the squared invariant mass of the hadronic final state W^2 is larger than $4m_c^2$. For $W^2 \equiv \frac{Q^2(1-x)}{x}$, the cc̄ pair production at small x can occur well below the Q^2 threshold, $Q^2 = 4m_c^2$. Thus, the charm structure function in LO can be given by [16]

$$\frac{1}{x} F_2^{\rm c}(x, Q^2, m_{\rm c}^2)
= 2e_{\rm c}^2 \frac{\alpha_{\rm s}(\mu^2)}{2\pi} \int_{ax}^1 \frac{\mathrm{d}y}{y} C_{\rm g,2}^{\rm c}\left(\frac{x}{y}, \frac{m_{\rm c}^2}{Q^2}\right) g(y, \mu^2), \quad (1)$$

where $a = 1 + 4m_c^2/Q^2$ and the factorization scale μ is taken as $\mu^2 = 4m_c^2 + Q^2$ with the charm quark mass $m_c = 1.5$ GeV. For the LO fusion process $\gamma^* g \to c\bar{c}$, the coefficient function $C_{g,2}^c$ is given by

$$C^{\rm c}_{\rm g,2}(z,\frac{m_{\rm c}^2}{Q^2}) \,=\, \frac{1}{2} \left\{ \left[z^2 + (1-z)^2 + z(1-3z) \frac{4m_{\rm c}^2}{Q^2} \right. \right. \right. \label{eq:cg_g2}$$

$$-z^{2}\frac{8m_{c}^{4}}{Q^{4}}\left[\ln\frac{1+\beta}{1-\beta}+\beta\left[-1+8z(1-z)\right]\right]$$
$$-z(1-z)\frac{4m_{c}^{2}}{Q^{2}}\right],$$
(2)

where $\beta^2 = 1 - (4m_c^2/Q^2)z(1-z)^{-1}$.

In Eq. (1), $g(y,\mu^2)$, which is the gluon distribution function, is usually given by the parameterizations evolved with DGLAP dynamics, such as CTEQ [17], MRST (Martin, Roberts, Stirling and Thorne) [18] and GRV (Glück, Reya and Vogt) [16]. In this paper, in order to study the gluon shadowing effect at very small-x, we will use the gluon distribution functions extracted from the AdS/CFT inspired model [7]. In this model, the UGD function can be given by a Fourier transform from the dipole-proton (dp) cross section σ_{dp}^{AdS} [19]

$$\varphi_{\text{AdS}}(x,k^2) = -\frac{N_{\text{c}}}{(2\pi)^2} k^2 \int \frac{\mathrm{d}^2 \boldsymbol{r}}{2\pi} \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}) \sigma_{\text{dp}}^{\text{AdS}}(x,r)$$
$$= \frac{4\sigma_0^{\text{AdS}}}{\pi^2} \frac{k^2}{Q_{\text{s}}^2(x)} \frac{1}{(1+16k^2/Q_{\text{s}}^2)^{3/2}}, \quad (3)$$

where the saturation scale is defined as [7, 20, 21]

$$Q_{\rm s}(x) = \frac{2A_0 x}{M_0^2 (1-x)\pi} \left(\frac{1}{\rho_m^3} + \frac{2}{\rho_m} - 2M_0 \sqrt{\frac{1-x}{x}} \right),\tag{4}$$

with

$$\begin{split} \rho_m = \begin{cases} \left(\frac{1}{3m}\right)^{1/4} \sqrt{2\cos\frac{\theta}{3}} : \ m \leqslant \frac{4}{27} \\ \sqrt{\frac{1}{3m\Delta} + \Delta} : \ m > \frac{4}{27} \end{cases}, \\ \Delta = \begin{bmatrix} \frac{1}{2m} - \sqrt{\frac{1}{4m^2} - \frac{1}{27m^3}} \end{bmatrix}^{1/3}, \\ m = \frac{M_0^4(1-x)^2}{x^2}, \\ \cos\theta = \sqrt{\frac{27m}{4}}. \end{split}$$

The other parameters $A_0 = \sqrt{20}$ GeV, $M_0 = 6.16 \times 10^{-3}$ and $\sigma_0^{\text{AdS}} = 21.31$ mb are obtained from a fit to HERA data [7]. By comparison with an analogous formula obtained in the double logarithmic limit (DLL) of the DGLAP evolution equations, the gluon distribution can be obtained from the UGD function by the following relation [22]

$$xg(x,Q^2) = \frac{1}{\alpha_{\rm s}(Q^2)} \int^{Q^2} \frac{\mathrm{d}^2 \boldsymbol{k}}{\pi k^2} \varphi(x,k^2), \qquad (5)$$

where the running coupling constant $\alpha_{\rm s}(Q^2) = 12\pi/[25\ln(Q^2/\Lambda^2)]$ with $\Lambda = 0.224$ GeV.

Having checked the UGD function from the AdS/CFT model with HERA data, we will use it to investigate net-kaon production in Au+Au collisions. The net-kaon rapidity distribution is due to the interaction of fast valence quarks with soft gluons in the target, and the differential cross section reads [9, 10]

$$\frac{\mathrm{d}N_{\Delta K}}{\mathrm{d}y} = \frac{C}{(2\pi)^2} \int \frac{\mathrm{d}^2 p_{\mathrm{T}}}{p_{\mathrm{T}}^2} x_1 q_{\mathrm{v}}(x_1, Q_{\mathrm{f}}) \varphi(x_2, p_{\mathrm{T}}), \quad (6)$$

where $x_1 = p_T/\sqrt{s} \cdot \exp(y)$, $x_2 = p_T/\sqrt{s} \cdot \exp(-y)$ are the longitudinal momentum fractions carried by the valence quark in the projectile and the soft gluon in the target, respectively. p_T is the transverse momentum of the produced quark and y is its rapidity. Cis the overall constant, which depends on the nature of the produced hadron, and q_v is the valence quark parton distribution function of a nucleus. Here, the shadowing effect must be considered in the gluon distribution. With the AdS/CFT saturation model, $g^A(x, Q^2)$ can be simply obtained from Eq. (3) with the following basic transformations [23, 24]:

$$\sigma_{0} \to \sigma_{0}^{A} = \frac{\pi R_{A}^{2}}{\pi R_{p}^{2}} \sigma_{0} \approx A^{2/3} \sigma_{0},$$

$$Q_{s}^{2}(x) \to Q_{s}^{2A}(x) = A^{1/3} Q_{s}^{2}(x),$$
(7)

where $R_{\rm p}$ and $R_{\rm A}$ are the radius of proton and nucleus, respectively.

3 Results and discussion

In Fig. 1, we show the charm structure function of proton $F_2^{c(p)}$ as a function of x obtained with the above method. For comparison, the results of the popular GBW model are also shown, and the UGD function φ_{GBW} can be given by [2]

$$\varphi_{\rm GBW}(x,k^2) = \frac{3\sigma_0^{\rm GBW}}{4\pi^2} R_0^2(x) k^4 \exp(-R_0^2(x)k^2), \quad (8)$$

where the x-dependent radius $R_0(x) = (x/x_0)^{\lambda/2} \text{ GeV}^{-1}$. The parameters $\sigma_0^{\text{GBW}} = 29.12 \text{ mb}$,



Fig. 1. The charm structure function of a proton, $F_2^{c(p)}$, computed with the AdS/CFT (the solid curves), GBW (the dashed curves), and CTEQ (the dash-dot curves) models. The experimental data are from Ref. [25] (the solid points) and [26] (the triangular points).

 $x_0 = 0.41 \times 10^{-4}$ and $\lambda = 0.277$. The solid and dashed curves correspond to the results of the AdS/CFT inspired model and the GBW model, respectively. In our calculation, the factor $(1-x)^4$ is introduced to describe the fact that the gluon density is small at $x \to 1$ as described by the quark counting rules [8], and a constant parameter C is adjusted by requiring that the gluon distribution satisfies the momentum sum rule $\int_{0}^{1} dxxg(x,Q^2) = p$ [17]. Here, p is the value obtained with the CTEQ gluon density. The experimental data come from Refs. [25, 26]. In Fig. 2, we also calculate the bottom structure function of proton $F_2^{\rm b(p)}$ by a simple modification: $m_{\rm c} \rightarrow m_{\rm b} = 4.5 \text{ GeV}.$ The figure captions are the same as in Fig. 1 and the experimental data are from Ref. [27]. As shown in Figs. 1 and 2, the theoretical results of the AdS/CFT inspired model are in good agreement with the experimental data, especially at low Q^2 . It is also shown that the results of the GBW and the AdS/CFT models are almost the same at large-x, but drastically different at small-x. For comparison, the results used in the CTEQ parameterizations are also given by the

dash-dot curves in Figs. 1 and 2, and the theoretical results fit well to the experimental data too.

The net-kaon rapidity distribution in central Au+Au collisions at RHIC energies of $\sqrt{s_{\rm NN}} = 200$ GeV is shown in Fig. 3 and the data come from BRAHMS [28]. Since the contribution of quarks in the other beam nucleus can be obtained from Eq. (6) by changing $y \sim -y$, the total theoretical results is given by

$$\frac{\mathrm{d}N_{\Delta K}}{\mathrm{d}y}|_{\mathrm{total}} = \frac{\mathrm{d}N_{\Delta K}}{\mathrm{d}y}(y) + \frac{\mathrm{d}N_{\Delta K}}{\mathrm{d}y}(-y). \tag{9}$$

For the valence quark parton distribution function of Au, the HKN07 (Hirai, Kumano, Nagai) nuclear parameterizations is used [29]. The UGD function is given by Eq. (3) with a basic transformation shown in Eq. (7). Here, for interpreting the experimental data at $y \sim 0$, the gluon distribution at large x is replaced by $\propto x^{-0.2}(1-x)^4$ as given in Ref. [30]. The overall normalization C is decided by the net-charge content (=44) for kaons. It is shown that the theoretical results are in good agreement with the experimental data.



Fig. 2. The bottom structure function of proton, $F_2^{b(p)}$, and the figure captions are the same as Fig. 1. The experimental data come from Ref. [27].



Fig. 3. The rapidity distributions of net-Kaon distribution for central (0–5%) Au+Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV. The experimental data come from Ref. [28].



Fig. 4. The nuclear charm structure function, $F_2^{c(A)}/A$, versus x at $Q^2 = 2 \text{ GeV}^2$ (a), 11 GeV² (b) and 30 GeV² (c). The curves are the results computed with the AdS/CFT (the solid curves), EPS08 (the dashed curves) and HKN07 (the dotted curves) models.

In Fig. 4, we give the predictive results of the nuclear charm structure function $F_2^{c(A)}$ versus x at different Q^2 that will be measured in the future EIC and LHeC experiments [13, 14]. The curves are the theoretical results with the AdS/CFT inspired model (the solid curves), the EPS08 (the dashed curves) and

the HKN07 [29] (the dotted curves) parameterizations. With the same figure captions as in Fig. 4, Fig. 5 shows the results versus Q^2 at $x = 10^{-5}$ (a), 10^{-6} (b) and 10^{-7} (c). It is shown that the theoretical results of the AdS/CFT model are similar to those of the EPS08 parameterizations at $x = 10^{-5}$ and 10^{-6} . At $x = 10^{-7}$, where the EPS08 parameterizations are invalid, we only give the theoretical results of the AdS/CFT model and the HKN07 parameterizations. As shown in Fig. 5, the theoretical results of the HKN07 parameterizations are larger than those of the AdS/CFT model. The reason is that an abnormal larger gluon shadowing effect is shown at large x for the HKN07 parameterizations [11, 29], and the corresponding integral results with Eq. (1) will become large.



Fig. 5. The nuclear charm structure function, $F_2^{c(A)}/A$, versus Q^2 at $x = 10^{-5}$ (a), 10^{-6} (b) and 10^{-7} (c), and the figure captions are the same as in Fig. 4.

In summary, we have extracted the UGD function of the AdS/CFT model and used it to study the charm and bottom structure functions in e+p collisions. Having checked that it can give a good description of the HERA data, we use it to investigate net-kaon rapidity distribution in Au+Au collisions. It is shown that the theoretical results are in good agreement with the experimental data from

BRAHMS. Then, by considering the gluon shadowing effect in the saturation model, the predictive results of the nuclear charm structure function for Pb are also given at very small x where the popular gluon shadowing models are invalid, and the theoretical results

will be examined by future EIC and LHeC experiments [13, 14]. Since the UGD function is an ideal tool to study the nature of gluon saturation, a further investigation of the UGD function will be given in the near future.

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