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Suppressing entanglement sudden death by initial system-environment correlation*

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Abstract: This paper studies the entanglement dynamics of the system S composed of two non-interactional qubits A and B. The third qubit C is its environment, E, which only interacts with the S qubit B by the Dzyaloshinskii-Moriya spin-orbit coupling. Considering the following states as the whole (S+E): the initially S-E correlated state and the separable one, the entanglement of S has no sudden death for the former case. This result sheds some light on the control of quantum entanglement, which will be helpful for quantum information processing.

Key words: system-environment initial correlation, entanglement sudden death, Dzyaloshinskii-Moriya interaction

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1 Introduction

Entanglement is a key resource in quantum information processing [1], for example, quantum teleportation [2, 3], quantum dense coding [4], and quantum cryptography [5]. The solid-state system, especially spin chains, are one of the natural candidates for use as quantum information processors, as they are believed to have the best scalability. Bose [6] firstly noticed that spin chains can be used as a channel for short distance quantum communication, and the entanglement of spin chains has been extensively studied [7–9]. Recently, in addition to the spin-spin interaction, the spin-orbit DM interaction [10] between spins has excited wide interest. The anisotropic and antisymmetric DM interaction was introduced about fifty years ago to explain the weak ferromagnetism of antiferromagnetic crystals.

Entanglement dynamics is another important field. One interesting phenomenon, so-called entanglement sudden death (ESD), was reported by Yu et al. [11]. The special property of ESD is that entanglement disappears nonsmoothly for a finite time. Moreover, ESD has been observed in an optical system by Almeida et al. [12]. ESD also occurs in the two qubit mixed state with Ising-type interaction [13], and in the non-degenerate two photon Tavis-Cummings model [14]. Although sudden death is a universal feature of entanglement, it is crucial to find ways to suppress it if one uses entanglement as the resource for quantum information processing.

Recently, the influence of the initial correlation between S and E on the open-system dynamics has been extensively studied [15]. Previously, it is commonly accepted that the initially S-E factorized state $\rho_S \otimes \rho_E$ is suitable. This is a reasonable assumption if there is only weak coupling between S and E. However, our knowledge should not be limited to this case with the following considerations: (1) there is no basic rule to avoid the initial S-E correlation; (2) for the strong coupling between S and E, $\rho_S \otimes \rho_E$ is not a valid form [16]. In this paper, similar to Ref. [17], we study the entanglement dynamics of the whole system composed of three qubits A, B and C. The system S

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is composed of two non-interactional qubits, A and B. The third qubit C is considered as its environment, E, which only has DM interaction with the qubit B. To the best of our knowledge, there is no study on the influence of the initial S-E correlation on the dynamics of S under DM interaction. That is the aim of this paper.

2 Measure of entanglement

For a bipartite state, there are some equivalent measures of the degree of entanglement, such as concurrence, entanglement entropy, negativity, tangle, etc. In this paper, we adopt the Wootters concurrence [18]. For two qubits, the concurrence can be calculated by

$$C(\rho)=\max\left(0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\right), \quad (1)$$

where λ_i (i=1, 2, 3, 4) are the eigenvalues of ρ in decreasing order

$$R=\rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y). \quad (2)$$

Here ρ^* is the complex conjugation of ρ in the standard basis and σ_y is the Pauli matrix

$$\sigma_y=\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}. \quad (3)$$

The concurrence is monotone for the bipartite entanglement, $C=0$ corresponds to the separable state, and $C=1$ to the maximal entanglement one. For density matrix

$$\rho=\begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}, \quad (4)$$

the concurrence can be easily obtained [19]

$$C(\rho)=2\max\left\{0,|z|-\sqrt{ad},|w|-\sqrt{bc}\right\}. \quad (5)$$

3 Entanglement dynamics of S

We study S+E as a whole, with S composed of two qubits, A and B, and E is the third qubit C. There is no interaction between qubits A and B. The only interaction is the DM one between the qubit B of S and the qubit C of E. The Hamiltonian of the whole system is

$$H=H^{\mathrm{S}}+H^{\mathrm{SE}}, \quad (6)$$

where the interaction between S qubits A and B is

$$H^{\mathrm{S}}=\frac{1}{2}w(\sigma_{\mathrm{A}}^x\ \sigma_{\mathrm{B}}^x+\sigma_{\mathrm{A}}^y\ \sigma_{\mathrm{B}}^y), \quad (7)$$

with $w=0$ and that between qubit B of S and the qubit C of E is

$$H^{\mathrm{SE}}=\vec{D}\cdot(\vec{\sigma}_{\mathrm{B}}\times\vec{\sigma}_{\mathrm{C}}). \quad (8)$$

Choosing $\vec{D}=D\vec{z}$, H^{SE} reduces to

$$H^{\mathrm{SE}}=D(\sigma_{\mathrm{B}}^x\ \sigma_{\mathrm{C}}^y-\sigma_{\mathrm{B}}^y\ \sigma_{\mathrm{C}}^x). \quad (9)$$

Note we are working in units where D is dimensionless.

Without loss of generality, we define $|g\rangle$ ($|e\rangle$) as the ground (excited) state of a qubit. And we mainly study the effect of the initial S-E correlation on the entanglement dynamics of S. We choose one specifically initial S-E correlational state of (S+E) as

$$\rho_1(0)=|\varphi^{\mathrm{ABC}}\rangle\langle\varphi^{\mathrm{ABC}}|, \quad (10)$$

with

$$|\varphi^{\mathrm{ABC}}\rangle = \sin(\alpha)\sin(\beta)|egg\rangle+\sin(\alpha)\cos(\beta)|geg\rangle$$

$$+\cos(\alpha)|gge\rangle, \quad (11)$$

which is the W state, and the first, second and third indexes represent the state of qubit A, B and C, respectively. In order to uncover the effect of an initially E-S correlational state on the entanglement of S, we also consider another initially E-S separable marginal state

$$\rho_2(0)=\rho_1^{\mathrm{S}}(0)\otimes\rho_1^{\mathrm{E}}(0), \quad (12)$$

with $\rho_1^{\mathrm{S}}(0)=\mathrm{Tr}_{\mathrm{E}}[\rho_1(0)]$ and $\rho_1^{\mathrm{E}}(0)=\mathrm{Tr}_{\mathrm{S}}[\rho_1(0)]$. Here $\mathrm{Tr}_{\mathrm{E(S)}}$ denotes tracing over the state of E(S). As $|\varphi^{\mathrm{ABC}}\rangle$ is an entanglement state, $\rho_1(0)\neq\rho_2(0)$.

The density matrix of the whole system at arbitrary time t is

$$\rho_i(t)=U(t)\rho_i(0)U(t)^{\dagger}, \quad (13)$$

with $i=1$, 2 and the evolution operator $U(t)=\exp(-\mathrm{i}Ht)$. The reduced density matrix of S is

$$\rho_i^{\mathrm{S}}=\mathrm{Tr}_{\mathrm{E}}[\rho_i(t)]. \quad (14)$$

After some straightforward calculations, we obtain the unitary operator $U(t)$

$$U(t)=\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}. \quad (15)$$

Here M is the (4×4) square matrix with the following non-zero elements

$$\begin{aligned} &M_{11}=M_{44}=1,\ M_{22}=M_{33}=\cos(2Dt),\\ &M_{23}=-M_{32}=\sin(2Dt). \end{aligned} \quad (16)$$

We first study one simple case $\beta=\dfrac{\pi}{2}$, which is possible to get the analytical result. For the initial state

$\rho_1(0)$, the reduced density matrix of S, i.e., qubits A and B, at arbitrary time t can be expressed as

$$\rho_1(t)^{\mathrm{S}}=\begin{pmatrix}0&0&0&0\\0&\rho_{22}&\rho_{23}&0\\0&\rho_{23}&\rho_{33}&0\\0&0&0&\rho_{44}\end{pmatrix}, \tag{17}$$

with $\rho_{22}=\cos(\alpha)^2$, $\rho_{23}=\rho_{23}=\dfrac{1}{2}\sin(2\alpha)\sin(2Dt)$, $\rho_{33}=\sin(\alpha)^2\sin(2Dt)^2$, and $\rho_{44}=\sin(\alpha)^2\cos(2Dt)^2$. It is easy to check that

$$C[\rho_1(t)^{\mathrm{S}}]=|\sin(2\alpha)\sin(2Dt)|. \tag{18}$$

This is an interesting result. The concurrence of S is in the separable form: one factor is determined by the initial state of S, and the other one by the S-E DM interaction. Moreover, only at some specific time, such as $t=\dfrac{\pi}{D}$, the concurrence of S equals zero. Making use of the same method, we get $C[\rho_2(t)^{\mathrm{S}}]=0$ all the time for the initially S-E separable state $\rho_2(0)$. Notice $C[\rho_1(0)^{\mathrm{S}}]=C[\rho_2(0)^{\mathrm{S}}]=0$. Therefore, Eq. (18) indicates that we can create entanglement for S with a suitably initial S-E correlation even without the direct interaction (the interaction only exists between S qubit B and E qubit C).

For the general parameters, α, β and D, the full solution is quite complicated and its form is not particularly instructive, hence it will not be reproduced here. We perform numerical simulation to obtain knowledge of the effect of these parameters on entanglement dynamics of S.

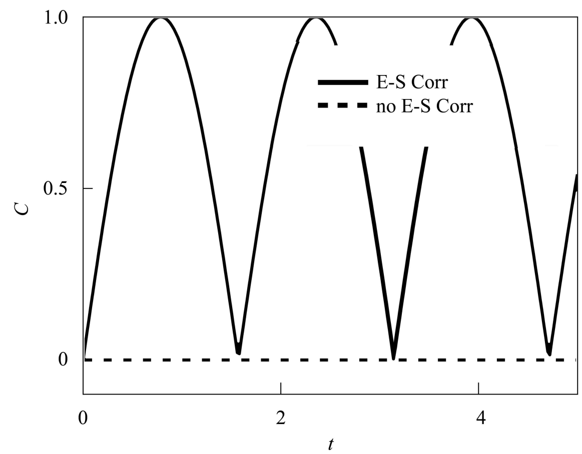

Fig. 1. The evolution of the concurrence of S with the different initial S-E states $\rho_1(0)$ and $\rho_2(0)$. The other parameters are $\beta=\pi/2$, $\alpha=\pi/4$ and $D=1$.

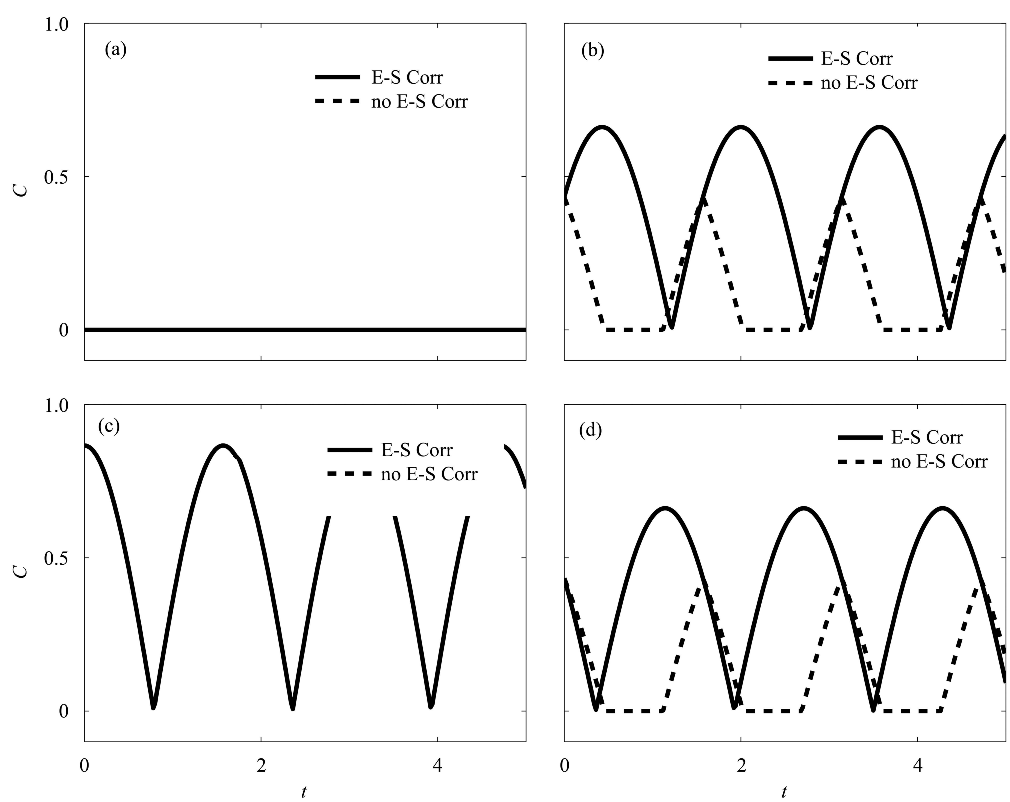

Fig. 2. The evolution of the concurrence of S with the different initial S-E states $\rho_1(0)$ and $\rho_2(0)$. (a), (b), (c) and (d) correspond to $s=0$, 1, 2 and 3 with $\alpha=s\dfrac{\pi}{4}$, respectively. The other parameters are $\beta=\pi/6$ and $D=1$.

Figure 1 is used to check the upper analytical result. It is easy to see that $C[\rho_2(t)^{\mathrm{S}}]=0$, and $C[\rho_1(t)^{\mathrm{S}}]$ is the periodic function with the period $T_0=\pi$ and amplitude $A_0=1$, which are consistent with the upper analytical results.

Figure 2 is the evolution of the concurrence of the reduced density matrix ρ^{S} of S with the different values of α. This figure shows that when $\alpha=k\dfrac{\pi}{2}$ with $k=0,\ 1,\ 2,\ 3,\cdots$, there is no difference of the concurrence of S with different S-E initial state $\rho_1(0)$ and $\rho_2(0)$. This is not surprising that whenever $\alpha=k\dfrac{\pi}{2}$, $|\varphi^{\mathrm{ABC}}\rangle$ reduces to a S-E initially separable state. When $\alpha\neq\dfrac{k\pi}{2}$, the concurrence of S shows very different behaviors, corresponding to the different S-E initially correlated state $\rho_1(0)$ and the separable one $\rho_2(0)$. With the same initial entanglement, the concurrence of S has ESD if the initial state of S-E is the separable one $\rho_2(0)$, while it doesn't have ESD if the initial state is the S-E correlation $\rho_1(0)$. Moreover, the extensive numerical simulations show that whether there is ESD or not is independent of the nonzero DM interaction, and all the behaviors have a period π for α. This means one may avoid the ESD for S if we can suitably choose an initial S-E correlational state.

The influence of D on the entanglement dynamics of S is shown in Fig. 3, which implies that with the increase of D, the maximum entanglement of S does not increase and it only modifies the period of the concurrence of S. This point can be intuitively understood from Eq. (18): the DM interaction only contributes an oscillating factor and the maximum entanglement of S is determined by the S-E initial state.

The upper results are limited to the case, $w=0$, i.e., there is no direct interaction between the qubits A and B of S. It is interesting to explore the role of this interaction on the entanglement of S. In Fig. 4, we study two cases $w=2$ and 4. It shows that whether the sudden death of the entanglement dynamics of S occurs or not is still only determined

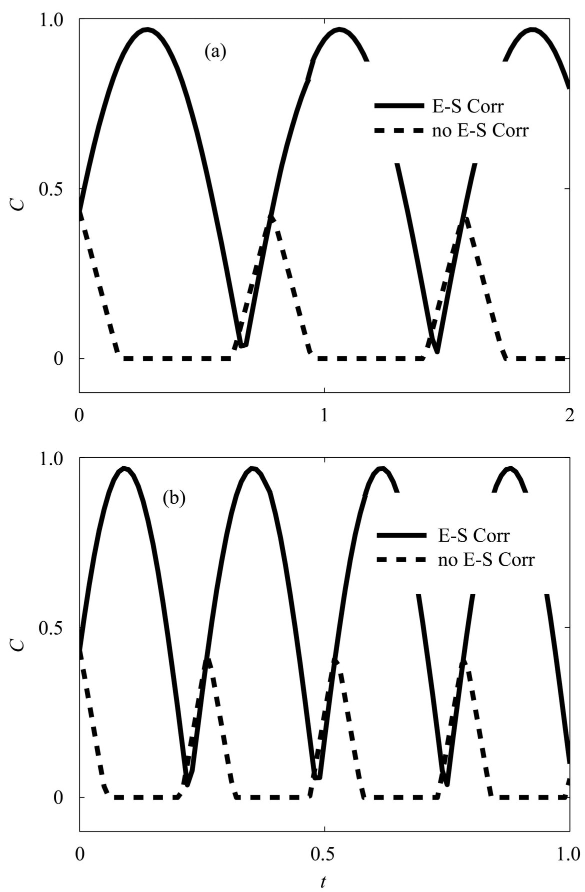

Fig. 3. The evolution of the concurrence of S with the different initial S-E states $\rho_1(0)$ and $\rho_2(0)$. (a) and (b) correspond to $D=2$ and $D=6$, respectively. The other parameters are $\beta=\pi/3$ and $\alpha=\pi/4$.

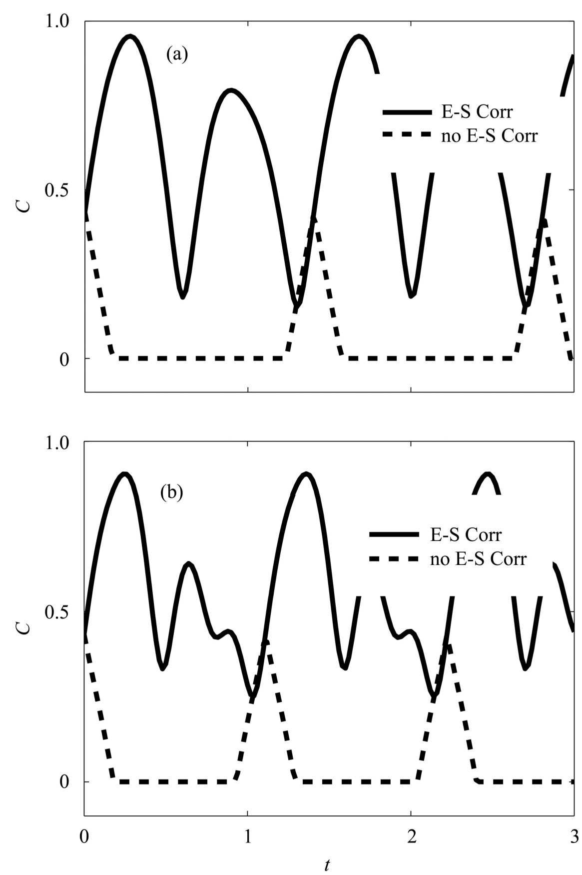

Fig. 4. The evolution of the concurrence of S with the different initial S-E states $\rho_1(0)$ and $\rho_2(0)$. (a) and (b) correspond to $w=2$ and $w=4$, respectively. The other parameters are $\beta=\pi/3$, $\alpha=\pi/4$ and $D=2$.

by the initial S-E state, even if there is interaction in S. When this state is correlational, there is no ESD. The interaction in S only changes its evolution of concurrence from periodic to quasi-periodic.

4 Conclusion

In this paper, with the entanglement degree of concurrence, we study the entanglement dynamics of a system S composed of two qubits A and B. The third qubit C is the environment E. There is no interaction between qubits A and B, and the E qubit C only has Dzyaloshinskii-Moriya spin-orbit interaction with the S qubit B. In order to identify the influence of the initial S-E correlation on the entanglement dynamics of S, we compare the results obtained without this kind of correlation by the product state of the S-E marginals. For a simple case, we get the analytical results and find there is no ESD if the initial state of the whole system (S+E) is a correlational one. For the general parameters, numerical experiments show that the entanglement dynamics of S is mainly determined by the initial S-E state, and with the initial S-E correlated state, there is no ESD of S, too. This implies one can always suppress ESD of S with a suitably chosen initial S-E correlated state. These results shed some light on the control of quantum entanglement, which will be helpful for quantum information processing.

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