

Systematical law of (n, γ) reaction cross sections of even-even nuclei*

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Abstract: We have derived a formula for the neutron radiative capture cross section in the framework of a statistical model approach to nuclear reactions. Based on this formula, new systematics are established between the (n, γ) reaction cross section and the energy level density of a compound nucleus or a relative neutron excess of an even-even target nucleus for neutron incident energy above the resonance region to MeV. Good agreement with experimental data suggests that this new systematical law is helpful to analyze the experimental data.

Key words: neutron radiative capture cross section, statistical model, systematics

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1 Introduction

Neutron radiative capture reaction is an important electromagnetic probe for studying highly-excited nuclei. It provides data for studying collective excitations in a strongly interacting many-body system such as Giant Resonances [1, 2], as well as information on self-organization or the quantum chaos of a nucleus with a high excitation energy. Neutron radiative capture is also one of the most important processes in astrophysical nucleosynthesis [3–6]. Neutron capture cross sections of up to several MeV may meet astrophysical conditions in many cases [7]. The recent revival of interest in neutron cross section study is further driven by the needs of advanced reactor systems and fuel cycles as well as nuclear criticality safety [8–10]. Exact neutron cross section data, especially for reactions with unstable target nuclei are the prerequisite for the optimization and development of these systems. For unstable nuclei, the experimental measurement of a cross section is rather difficult ow-

ing to the rarity and radioactivity of target material [11–20]. Theoretical calculation based on the microscopic model plays an important role in predicting reaction cross section [4–7, 21–24]. The sophisticated model [4–7, 21–24] with reasonable parameters can reproduce (n, γ) cross sections satisfactorily for nuclei close to the β -stable line. However, theoretical calculation strongly depends on the mass and energy level density model and parameters which need to be constrained from experimental inputs. It remains a challenge to estimate model parameters in a reasonable way for nuclei far from the β -stable line in many cases [6, 24].

Besides these sophisticated numerical approaches, sometimes simple models can yield analytical results, which can help to clarify the underlying physics of a certain phenomenon in an approximate but simple and transparent way [25, 26]. It would be valuable to derive the analytical formula for neutron cross section calculations, which would be helpful for understanding the experimental data at least in a qualitative

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way. It is also useful and controllable for investigating the systematics of model parameters, such as level density parameter systematics from Ref. [24]. Up to date, we have not seen an analytical formula for nuclear (n, γ) reaction cross section calculations. In this paper, an attempt is made to derive an analytical formula for the systematical analysis of an (n, γ) cross section based on the statistical model. Based on the analytical formula, we will show that the empirical systematics between (n, γ) cross sections and level density parameters in Ref. [24] can be presented in an alternate way with a physics basis. Furthermore, the isotopic dependence of (n, γ) cross sections of even-even target nuclei is obtained.

This paper is organized as follows. In Section 2, an analytical formula is proposed for the (n, γ) cross section calculation. Based on this formula, we study the systematics of experimental data on the (n, γ) cross section of even-even nuclei in Section 3. A summary is given in the last section.

2 (n, γ) reaction cross section

For incident neutron energy below the pre-equilibrium region (around 14 MeV), from Bohr's postulate, nuclear reactions process can be divided into two well-separated independent stages [27]. The first is the formation of a compound nucleus in a well-defined state in which the incident energy is shared among all the constituent nucleons. Then the compound nucleus loses its memory of the formation process and goes to the second stage: decay happens through particle emission (neutron, γ ray, proton...) or the fission process. The (n, γ) cross section is the product of the formation cross section of the compound nucleus and the probability of its decay through γ emission, that is,

$$\sigma(n, \gamma) = \sigma_{\text{cn}} \frac{\Gamma_\gamma}{\Gamma} = \sigma_{\text{cn}} \frac{\Gamma_\gamma}{\sum_i \Gamma_i}, \quad (1)$$

where σ_{cn} is the formation cross section of the compound nucleus, Γ_γ is the γ -emission width of the formed compound nucleus, Γ represents the total decay width which is the summation over all possible decay channels Γ_i .

According to the detailed balance principle of an inverse reaction cross section [28, 29], the probability to evaporate a photon in the energy interval $(\epsilon_\gamma, \epsilon_\gamma + d\epsilon_\gamma)$ per unit of time is given by

$$W_\gamma(\epsilon_\gamma) = \frac{1}{\pi^2 \hbar^3 c^2} \sigma_\gamma(\epsilon_\gamma) \frac{\rho(E^* - \epsilon_\gamma)}{\rho(E^*)} \epsilon_\gamma^2, \quad (2)$$

where $\sigma_\gamma(\epsilon_\gamma)$ is the inverse (absorption of γ) reaction

cross section, ρ is the nuclear level density and E^* is the excited energy of the compound nucleus. Within the CT model [30], the level density dependence on excitation energy E^* can be written as

$$\rho(E^*) = \frac{1}{T} e^{(E^* - E_0)/T}. \quad (3)$$

Here the temperature T and backshift energy E_0 are two free parameters of this model. In our calculation, the parameters T and E_0 are taken from the fitting on an experimental level density by Egidy et al. [31].

In the first approximation we assume that dipole $E1$ -transitions are the main source of γ -emission from highly-excited nuclei. To describe the dipole photoabsorption cross section, we use the same parameterization as in Refs. [25] and [32] based on the Lorentzian representation of the giant dipole resonance (GDR), that is,

$$\sigma_\gamma(\epsilon_\gamma) = \frac{\sigma_0 \epsilon_\gamma^2 \Gamma_R^2}{(\epsilon_\gamma^2 - E_{\text{GDR}}^2)^2 + \epsilon_\gamma^2 \Gamma_R^2}, \quad (4)$$

where the empirical parameters of the GDR have the values $\sigma_0 = 2.5 A$ mb, $E_{\text{GDR}} = 40.3 A^{-1/5}$ MeV, $\Gamma_R = 0.3 E_{\text{GDR}}$, and A is the mass number of the compound nucleus.

When E^* is large, using a statistical hypothesis and the above parameterization for level density and photoabsorption cross section, the total photon emission rates can be written as

$$\begin{aligned} W_\gamma &= \int_0^{E^*} W_\gamma(\epsilon_\gamma) d\epsilon_\gamma \\ &= \frac{1}{\pi^2 \hbar^3 c^2} \int_0^{E^*} \sigma_\gamma(\epsilon_\gamma) \epsilon_\gamma^2 e^{-\epsilon_\gamma/T} d\epsilon_\gamma. \end{aligned} \quad (5)$$

If $\epsilon_\gamma \ll E_{\text{GDR}}$, $\sigma_\gamma(\epsilon_\gamma)$ can be expanded into the following series

$$\sigma_\gamma(\epsilon_\gamma) \approx \frac{\sigma_0 \epsilon_\gamma^2 \Gamma_R^2}{E_{\text{GDR}}^4} \sum_{n=0}^{\infty} \left(\frac{1.91 \epsilon_\gamma^2}{E_{\text{GDR}}^2} \right)^n. \quad (6)$$

Considering the first two terms of the series and $E^* \gg T$, one can carry out the integration [Eq. (5)] and obtain the total γ radiation probability per unit of time

$$W_\gamma \approx \frac{\sigma_0 \Gamma_R^2}{\pi^2 \hbar^3 c^2 E_{\text{GDR}}^4} \left(24T^5 + \frac{1375.2T^7}{E_{\text{GDR}}^2} \right). \quad (7)$$

Then we obtain the γ emission width of the compound nucleus

$$\Gamma_\gamma = \hbar W_\gamma = \frac{\sigma_0 \Gamma_R^2}{\pi^2 \hbar^2 c^2 E_{\text{GDR}}^4} \left(24T^5 + \frac{1375.2T^7}{E_{\text{GDR}}^2} \right). \quad (8)$$

For fast neutron projecting on a not heavy enough target nucleus, the total decay width of the compound nucleus can be approximately taken as $\Gamma \approx \Gamma_n$.

According to the detailed balance principle [28], the probability that the compound nuclei with excitation energy E^* emits a neutron with kinetic energy ϵ to energy level ν per unit of time is

$$W_n(\epsilon) = \frac{\sigma_{\text{inv}}(\epsilon) g m_n \epsilon}{\pi^2 \hbar^3 \rho_c(E^*)}, \quad (9)$$

where m_n is the neutron rest mass, $\sigma_{\text{inv}}(\epsilon)$ is the cross section where the daughter nucleus captures a neutron with energy ϵ to form a compound nucleus. g is the spin statistical factor given by $g = (2s_n + 1)(2s_d + 1)/(2s_c + 1)$, where s_n , s_d and s_c are the spins of neutron, residual and compound nuclei, respectively. The total neutron emission rate is the summation over all low-lying levels ν of residual nuclei below the neutron incident energy (E_n),

$$W_n = \sum_{\nu} \frac{\sigma_{\text{inv}}(\epsilon_{\nu}) g m_n \epsilon_{\nu}}{\pi^2 \hbar^3 \rho(E^*)}. \quad (10)$$

Here we consider that the projectile neutron energy E_n is smaller than the lowest excited energy level (E_1) of the target nucleus available for neutron emission. Therefore neutron emission occurs between the compound nucleus and the ground state of the daughter nucleus, and W_n can be written as

$$W_n = \frac{\sigma_{\text{inv}}(E_n) g m_n E_n}{\pi^2 \hbar^3 \rho(E^*)}, \quad (11)$$

where $\sigma_{\text{inv}}(E_n)$ is the formation cross section of compound nucleus σ_{cn} . Then the neutron decay width of the compound nucleus is

$$\Gamma_n = \hbar W_n = \frac{\sigma_{\text{cn}} g m_n E_n}{\pi^2 \hbar^2 \rho(E^*)}. \quad (12)$$

Combining Eqs. (8) and (12) with Eq. (1), we obtain the analytical expression of the neutron radiative capture cross section for $E_n < E_1$,

$$\begin{aligned} \sigma(n, \gamma) &= \sigma_{\text{cn}} \frac{W_{\gamma}}{W_n} \\ &= \frac{\sigma_0 \Gamma_R^2 \rho(E^*)}{g m_n c^2 E_{\text{GDR}}^4 E_n} \left(24T^5 + \frac{1375 \cdot 2T^7}{E_{\text{GDR}}^2} \right). \end{aligned} \quad (13)$$

3 Systematics of (n, γ) reaction cross sections of even-even nuclei

In this section, we study the systematics of (n, γ) reaction cross sections of even-even nuclei based on the newly derived formula [Eq. (13)]. From Eq. (13), the (n, γ) cross section is proportional to the level density of a compound nucleus with excitation energy E^* . The excitation energy is given by the mass-

energy relation

$$\begin{aligned} E^* &= [m(Z, A) + m_n - m(Z, A + 1)]c^2 + \frac{A}{A+1} E_n \\ &= S_n(Z, A + 1) + \frac{A}{A+1} E_n, \end{aligned} \quad (14)$$

where $S_n(Z, A + 1)$ is the neutron separation energy of the compound nucleus. For a non-magic target nucleus with $A \gg 1$, E^* can be calculated by Weizsäcker's mass formula [33],

$$\begin{aligned} E^* &\approx a_v - \frac{2}{3} a_s A^{-1/3} + \frac{1}{3} a_c \frac{Z^2}{A^{4/3}} + a_p \delta (A + 1)^{-1/2} \\ &\quad - a_p \delta A^{-1/2} - \frac{a_a}{4} \left[\frac{2(N - Z) + 1}{A} \right. \\ &\quad \left. - \frac{(N + 1 - Z)^2}{A^2} \right] + \frac{A}{A+1} E_n, \end{aligned} \quad (15)$$

where a_v , a_s , a_c , a_p , and a_a are the parameters of volume, surface, Coulomb, pairing, and asymmetry terms with the following values: $a_v = 15.835$ MeV, $a_s = 18.33$ MeV, $a_c = 0.714$ MeV, $a_p = 11.2$ MeV, and $a_a = 92.80$ MeV. For even-even nuclei, $\delta = 1$; for odd- A nuclei, $\delta = 0$; for odd-odd nuclei, $\delta = -1$.

Along one isotopic chain, the asymmetric term difference dominates the variation trend of neutron separation energy, since the differences of other terms only depend on A while the asymmetric term difference directly relates to relative neutron excess. Here, we take the neutron separation energy of odd- A Sn isotopes as an example. One can see from Fig. 1 that the sum over the differences of volume, surface, Coulomb and pairing terms in calculating neutron separation energy is slowly variational for odd- A Sn isotopes. Therefore, one can approximately write

$$\begin{aligned} E^* &\approx c_1 + \frac{a_a}{4} \left[\frac{2(N - Z) + 1}{A} - \frac{(N + 1 - Z)^2}{A^2} \right] \\ &\quad + \frac{A}{A+1} E_n, \end{aligned} \quad (16)$$

where c_1 is a constant along one isotopic chain. At the same incident energy E_n , approximately taking $A/(A+1) \sim 1$ and combining Eqs. (3), (13) and (16), one gets the isotopic dependence of $\sigma(n, \gamma)$ cross section, that is,

$$\sigma(n, \gamma) = C A^{1.4} e^{-K \left[\frac{2(N-Z)+1}{A} - \frac{(N+1-Z)^2}{A^2} \right]}. \quad (17)$$

In Fig. 2 we plot the variation of (n, γ) cross sections at fixed E_n divided by $A^{1.4}$ versus the relative neutron excess $\frac{2(N-Z)+1}{A} - \frac{(N+1-Z)^2}{A^2}$ for even-even nuclei along different isotopic chains. The (n, γ) cross section data are taken from an experimental

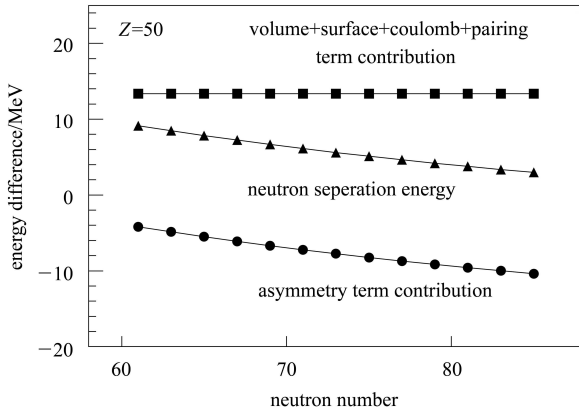


Fig. 1. Energy differences of various terms in neutron separation energy calculations for odd- A Sn ($Z=50$) isotopes.

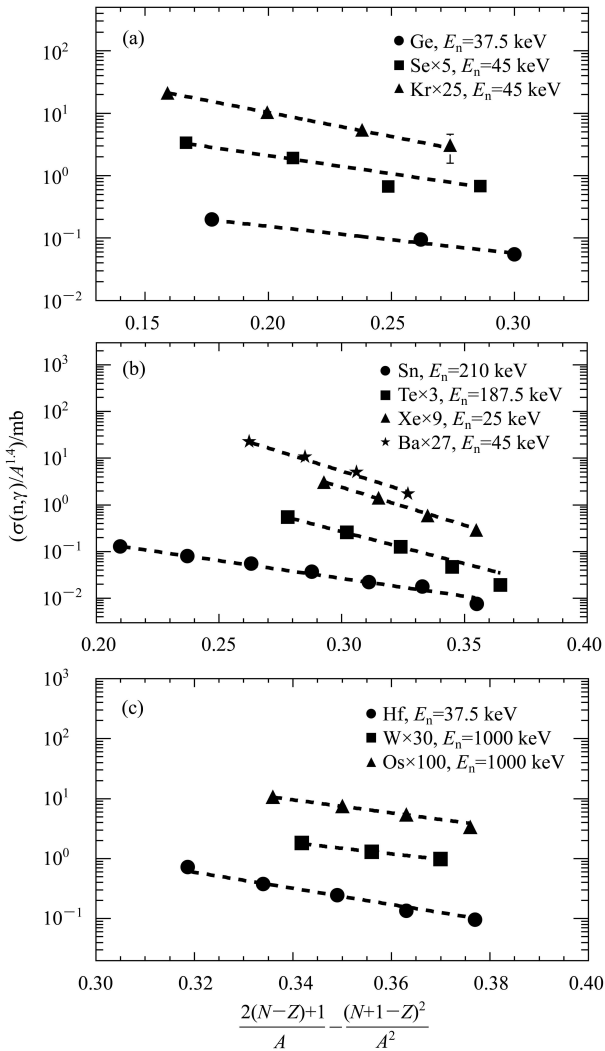


Fig. 2. The dependence of (n, γ) cross sections divided by $A^{1.4}$ on the relative neutron excess $\frac{2(N-Z)+1}{A} - \frac{(N+1-Z)^2}{A^2}$ for even-even nuclei along different isotopic chains.

nuclear reaction database (EXFOR/CSISRS) [34]. It is seen from Fig. 2 that the logarithms of $A^{1.4}$ scaled

(n, γ) cross sections at fixed E_n linearly depend on the relative neutron excess. The dashed lines in Fig. 2 are plotted according to the least- χ^2 fits of experimental (n, γ) cross sections (in lg-scale) using Eq. (17).

All fitting parameters C and K and the χ^2 per degree of freedom (χ^2/ndf) are listed in Table 1. The first column of Table 1 denotes the proton number of the isotopic chain. The second column shows the incident neutron energy. The fitting parameters C , K and the χ^2/ndf of the fit are listed in Column 3, 4 and 5, respectively. It should be noted that the nuclei with the neutron magic number is not included in the fits because the shell effect is not taken into account in Weizsäcker's mass formula and Eq. (17). In Fig. 3 we plot the ratios between the calculated (n, γ) cross sections from fitting and the experimental ones versus the mass numbers of target nuclei. One can see from Fig. 3 that the ratio is close to unity for most of the nuclei, indicating good agreement between calculation and experiment. This suggests a firm physics basis of Eq. (17) in describing the (n, γ) cross sections along different isotopic chains. It would be helpful in estimating the (n, γ) cross sections of neighboring even-even isotopes for which no experimental data are available.

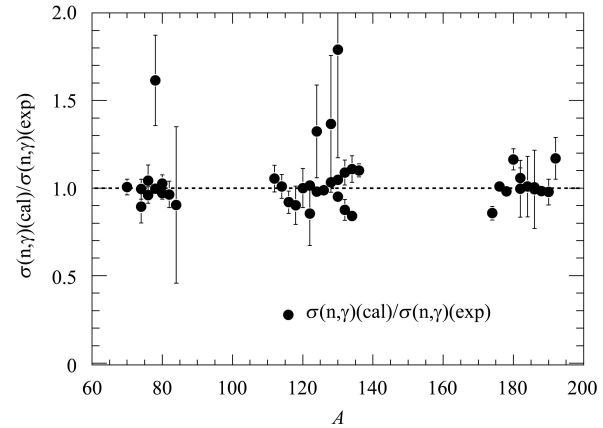


Fig. 3. The ratios between the calculated (n, γ) cross sections from Eq. (17) and the experimental ones versus the mass numbers of target nuclei.

One can directly utilize Eq. (13) to investigate the dependence of an (n, γ) cross section on the level density of compound nuclei. From Eq. (13) we can see that the (n, γ) cross section is proportional to the energy level density of the compound nucleus with excitation energy E^* . Since the dominant term in the bracket on the right side of Eq. (13) is the T^5 term, in Fig. 4 we plot the variations of $\sigma(n, \gamma)E_n/(A^{1.4}T^5)$ versus the energy level density $\rho(E^*)$. Here, E^* is

Table 1. The parameters C and K in Eq. (17) from the fits of experimental (n, γ) cross sections of even-even nuclei and the corresponding χ^2/ndf values of the fits.

Z	E_n/keV	C/mb	K	χ^2/ndf
32	37.5	1.13 ± 0.17	9.96 ± 0.70	1.34/1
34	45	6.04 ± 0.80	13.34 ± 0.54	16.14/2
36	45	14.26 ± 2.21	17.73 ± 0.88	0.59/2
50	210	5.15 ± 1.11	17.56 ± 0.83	5.69/5
52	187.5	$(1.17 \pm 0.27) \times 10^3$	31.61 ± 0.76	8.31/3
54	25	$(1.94 \pm 0.51) \times 10^4$	37.35 ± 0.83	12.15/2
56	45	$(1.72 \pm 0.31) \times 10^4$	37.94 ± 0.61	39.88/2
72	37.5	$(1.23 \pm 0.25) \times 10^4$	31.05 ± 0.59	27.67/3
74	1000	85.7 ± 295.1	21.30 ± 9.87	0.01/1
76	1000	490.8 ± 211.3	25.13 ± 1.22	3.82/2

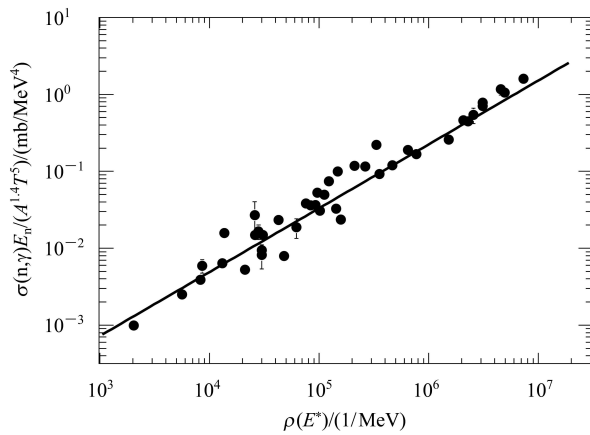


Fig. 4. The dependence of $\sigma(n, \gamma)E_n/(A^{1.4}T^5)$ (in lg scale) on the energy level density (in lg scale) of a compound nucleus with excitation energy E^* for even-even target nuclei. The line is there to guide your eye.

calculated by Eq. (14) with the experimental atomic mass taken from the Nubase table of nuclear and decay properties by Audi et al. [35]. Then $\rho(E^*)$ is calculated from Eq. (3) with parameters taken from Ref. [31]. One can clearly see a linear dependence of the logarithm of $\sigma(n, \gamma)E_n/(A^{1.4}T^5)$ on the logarithm of $\rho(E^*)$, which verifies the validity of compound nu-

clei and statistical hypotheses in calculating nuclear (n, γ) cross section. This is an analytical relationship between (n, γ) cross sections and the energy level density of a compound nucleus, which is helpful for understanding the experimental data in a transparent and systematical way. It is also useful for checking the goodness of various input level density parameters in sophisticated cross section calculations [24].

4 Summary

In summary, a formula for neutron radiative capture cross section is derived within the framework of statistical and compound nucleus hypothesis for incident neutron energy above the resonance region up to MeV. Based on this formula, we find a linear relationship between the logarithm of an (n, γ) cross section and that of the energy level density of a compound nucleus or the relative neutron excess of the target nucleus. The calculated (n, γ) cross sections from this relationship are in good agreement with the experimental data for even-even target nuclei. More experimental data on (n, γ) cross sections are needed to further test this systematical law in the region of unstable nuclei.

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