# Transverse radius dependence for transverse velocity and elliptic flow in intermediate energy HIC＊ 

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#### Abstract

The mean transverse velocity and elliptic flow of light fragments $(A \leqslant 2)$ as a function of transverse radius are studied for $25 \mathrm{MeV} /$ nucleon ${ }^{64} \mathrm{Cu}+{ }^{64} \mathrm{Cu}$ collisions with impact parameters $3-5 \mathrm{fm}$ by the isospin－ dependent quantum molecular dynamics model．By comparison between the in－plane and the out－of－plane transverse velocities，the elliptic flow dependence on the transverse radius can be understood qualitatively，and variation of the direction of the resultant force on the fragments can be investigated qualitatively．


Key words：mean transverse velocity，elliptic flow，intermediate energy HIC
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## 1 Introduction

Anisotropic flows are very useful for exploring heavy－ion collision dynamics，and many studies of directed flow $v_{1}$ and elliptic flow $v_{2}$ have been car－ ried out to explore the properties and origin of the collective motion in both nucleonic and partonic lev－ els［1－12］．The elliptic flow at intermediate energy HIC is complex because it is determined by the in－ terplay among fireball expansion，collective rotation， the shadowing of spectators，coulomb repulsion，and so on．Both the mean field and two－body collision parts play important roles：the mean field plays a dominant role at low energies，and then gradually the two－body collisions become dominant with en－ ergy increase．The transverse radius dependent trans－ verse velocity can reflect the correlation between spa－ cial and momentum coordinates，and reveal the force change on fragments along the transverse radius．In this paper，elliptic flow dependence on transverse ra－ dius is studied by comparing the variation trend of the mean in－plane transverse velocity and the mean out－ plane transverse velocity for light fragments $(A \leqslant 2)$ from $25 \mathrm{MeV} /$ nucleon ${ }^{64} \mathrm{Cu}+{ }^{64} \mathrm{Cu}$ collisions with im－ pact parameters $3-5 \mathrm{fm}$ simulated by an isospin－ dependent quantum molecular dynamics（IDQMD） model．

Anisotropic flows are defined as different $n$th har－ monic coefficients $v_{n}$ of the Fourier expansion for the particle invariant azimuthal distribution，

$$
\begin{equation*}
\frac{\mathrm{d} N}{\mathrm{~d} \phi} \propto 1+2 \sum_{n=1}^{\infty} v_{n} \cos (n \phi) \tag{1}
\end{equation*}
$$

where $\phi$ is the azimuthal angle between the transverse momentum of the particle and the reaction plane． Anisotropic flows generally depend on both particle transverse momentum and rapidity，and for a given rapidity the anisotropic flows at transverse momen－ tum $p_{\mathrm{t}}\left(p_{\mathrm{t}}=\sqrt{p_{x}^{2}+p_{y}^{2}}\right)$ can be evaluated according to

$$
\begin{equation*}
v_{n}\left(p_{\mathrm{t}}\right)=\langle\cos (n \phi)\rangle, \tag{2}
\end{equation*}
$$

where $\langle\cdot\rangle$ denotes the average over the azimuthal dis－ tribution of particles with transverse momentum $p_{\mathrm{t}}$ ． The anisotropic flows $v_{n}$ can further be expressed in terms of single－particle averages，

$$
\begin{align*}
& v_{1}=\langle\cos \phi\rangle=\left\langle\frac{p_{x}}{p_{\mathrm{t}}}\right\rangle  \tag{3}\\
& v_{2}=\langle\cos (2 \phi)\rangle=\left\langle\frac{p_{x}^{2}-p_{y}^{2}}{p_{\mathrm{t}}^{2}}\right\rangle \tag{4}
\end{align*}
$$

where $p_{x}$ and $p_{y}$ are，respectively，the projections of particle transverse momentum parallel and perpen－ dicular to the reaction plane．

[^0]Elliptic flow is considered to arise from the anisotropic pressure gradient in the overlap region at relativistic energies, and carry the correlation between spacial and momentum coordinates. The mean transverse velocity also includes spacial and momentum coordinates, and is defined as

$$
\begin{equation*}
\beta_{\mathrm{t}}=\frac{1}{N} \sum_{i=1}^{N} \frac{\boldsymbol{p}_{\mathrm{t}_{i}} \cdot \boldsymbol{r}_{\mathrm{t}_{i}}}{p_{0_{i}} r_{\mathrm{t}_{i}}} \tag{5}
\end{equation*}
$$

where $r_{\mathrm{t}}$ is the transverse location of the final state fragment, $p_{0}$ is the energy of the fragment. $N$ is the number of fragments whose transverse radiuses are near a certain $\boldsymbol{r}_{\mathrm{t}}$. To explore the relation between elliptic flow and mean transverse velocity is the main motivation of this work.

## 2 The IDQMD model

The intermediate energy heavy-ion collision dynamics are complex since both the mean field and the nucleon-nucleon collisions are playing the competition role, and the effect of Pauli blocking is remarkable. Furthermore, the isospin-dependent role should also be incorporated for asymmetric reaction systems. The isospin-dependent quantum molecular dynamics model (IDQMD) has been affiliated with isospin degrees of freedom with the mean field, nucleon-nucleon collisions [13-18]. The IDQMD model can explicitly represent the many-body state of the system and principally contains correlation effects to all orders and all fluctuations, and can well describe the time evolution of the colliding system. When the spatial distance $\Delta r$ is smaller than 3.5 fm and the momentum difference $\Delta p$ between two nucleons is smaller than $300 \mathrm{MeV} / c$, two nucleons can coalesce into a cluster [13]. With this simple coalescence mechanism, which has been extensively applied in transport theory, different size clusters can be recognized.

In the model, the nuclear mean-field potential is parameterized as

$$
\begin{align*}
U\left(\rho, \tau_{z}\right)= & \alpha\left(\frac{\rho}{\rho_{0}}\right)+\beta\left(\frac{\rho}{\rho_{0}}\right)^{\gamma}+\frac{1}{2}\left(1-\tau_{z}\right) V_{\mathrm{c}} \\
& +C_{\mathrm{sym}} \frac{\left(\rho_{\mathrm{n}}-\rho_{\mathrm{p}}\right)}{\rho_{0}} \tau_{z}+U^{\mathrm{Yuk}} \tag{6}
\end{align*}
$$

where $\rho_{0}$ is the normal nuclear matter density $\left(0.16 \mathrm{fm}^{-3}\right), \rho_{\mathrm{n}}, \rho_{\mathrm{p}}$ and $\rho$ are the neutron, proton and total densities, respectively; $\tau_{z}$ is $z$ th component of the isospin degree of freedom, which equals 1 or -1 for neutrons or protons, respectively. The coefficients $\alpha$, $\beta$ and $\gamma$ are parameters for nuclear equation of state. $C_{\text {sym }}$ is the symmetry energy strength due to the den-
sity difference of neutrons and protons in a nuclear medium, which is important for asymmetry nuclear matter $\left(C_{\text {sym }}=32 \mathrm{MeV}\right.$ is used). $V_{c}$ is the Coulomb potential and $U_{\text {Yuk }}$ is the Yukawa (surface) potential. In the present work, we take $\alpha=124 \mathrm{MeV}, \beta=70.5$ MeV and $\gamma=2$, which corresponds to the so-called hard EOS with an incompressibility of $K=380 \mathrm{MeV}$.

The nucleon-nucleon (NN) cross section is the experimental parametrization, which is isospin dependent. The neutron-proton cross section is about three times larger than the neutron-neutron or protonproton cross section below $300 \mathrm{MeV} /$ nucleon.

## 3 Results and discussion

Now we move to the calculations. About 100000 events have been simulated for the collision system ${ }^{64} \mathrm{Cu}+{ }^{64} \mathrm{Cu}$ at $25 \mathrm{MeV} /$ nucleon and impact parameter from 3 fm to 5 fm with hard EOS. In this study, we extract the physical results at $200 \mathrm{fm} / c$ for $A \leqslant 2$ fragments at rapidity of -0.5 to 0.5 .

The mean transverse velocity dependence on transverse radius for fragments $A \leqslant 2$ at rapidity from -0.5 to 0.5 at the time of $200 \mathrm{fm} / c$ is shown in Fig. 1. This shows that the mean transverse velocity increases monotonically with transverse radius. At RHIC energies, it also increases monotonically with transverse radius but for tending saturation at large $r_{\mathrm{t}}$ [19]. This may be because the spectators go off the participants rapidly at RHIC energies and leave no effect on the overlap region, and just the pressure gradient in the overlap region causes the transverse flow. So the fragments that are far away from the overlap region feel a smaller effect of the pressure gradient, and their transverse velocity increases more and more gently till arriving at saturation. But at intermediate energies, the fragments can feel the density dependent nuclear forces, which are attractive, the long-range coulomb forces, which are repulsive, and the effect of the relatively smaller pressure gradient, while the resultant effect is repulsive. So the mean transverse velocity increases with the increase in $r_{\mathrm{t}}$ under the action of repulsive resultant force at $r_{\mathrm{t}}$ direction. And it also shows a trend of saturation at large $r_{\mathrm{t}}$, where the resultant forces are too small. But what we are more concerned about is not the variation trend of the resultant force value but the variation trend of the resultant force direction, which may tend to in-plane or out-of-plane and cause the emission of fragments in-plane or out-of-plane. So the mean transverse velocities of in-plane and out-ofplane are studied respectively as shown in Fig. 2.


Fig. 1. The mean transverse velocity dependence on transverse radius for fragments $A \leqslant 2$ at $200 \mathrm{fm} / c$.


Fig. 2. The mean transverse velocities for inplane (squares) and out-of-plane (circles) dependence on transverse radius for fragments $A \leqslant 2$ at $200 \mathrm{fm} / c$.

Figure 2 shows the transverse radius dependence of mean transverse velocity for in-plane ( $\beta_{\text {t-in }}$ ) and out-of-plane ( $\beta_{\text {t-out }}$ ) for fragments $A \leqslant 2$ at rapidity from -0.5 to 0.5 at $200 \mathrm{fm} / \mathrm{c}$. The squares are for the in-plane transverse velocity, and the circles are for that of the out-of-plane. This shows that the trends of mean transverse velocity are consistent with each other for that of in-plane and out-of-plane at small and medium transverse radius, i.e., increasing monotonically with transverse radius. But at large transverse radius, the two depart from each other, the in-plane transverse velocity ascending but the out-ofplane transverse velocity descending with transverse radius. By detailed comparison, the velocity of out-of-plane is a little larger than that of in-plane at $r_{\mathrm{t}}<9$ fm , and equal to each other at about $r_{t}=9 \mathrm{fm}$, and then becomes smaller than in-plane with increasing $r_{\mathrm{t}}$. But the change speeds of the two are different from what is mentioned above. For example, the out-
of-plane transverse velocity increases rapidly firstly and then slower than the in-plane transverse velocity at $r_{\mathrm{t}}$ from 1 to 9 fm where the out-of-plane transverse velocity is bigger than the in-plane transverse velocity all the way. So $\delta \beta_{\mathrm{t}}=\beta_{\mathrm{t} \text {-in }}-\beta_{\mathrm{t} \text { tout }}$ is inducted to investigate the value and variation differences between $\beta_{\mathrm{t} \text {-in }}$ and $\beta_{\mathrm{t} \text {-out }}$, as shown in Fig. 3.


Fig. 3. The difference between the in-plane and out-of-plane transverse velocity dependence on transverse radius for fragments $A \leqslant 2$ at $200 \mathrm{fm} / c$.

Figure 3 shows $\delta \beta_{\mathrm{t}}$ dependence on transverse radius for fragments $\mathrm{A} \leqslant 2$ at $200 \mathrm{fm} / c$. And Fig. 4 shows the elliptic flow dependence on transverse radius at $200 \mathrm{fm} / c$, whose variation trend is similar and can be understood by $\delta \beta_{\mathrm{t}}$ variation dependence on transverse radius, which also reflects the transverse anisotropy for fragments. In the $r_{\mathrm{t}}<7 \mathrm{fm}$ range, $\delta \beta_{\mathrm{t}}$ descends with $r_{\mathrm{t}}$ increasing, that is to say, $\beta_{\mathrm{t} \text {-in }}$ increases slower than $\beta_{\mathrm{t} \text {-out }}$, as shown in Fig. 2. This reflects that the resultant force on fragment at $r_{\mathrm{t}}<7 \mathrm{fm}$ tends to out-of-plane, so fragments are emitted more and more tending to out-of-plane, and so $v_{2}$ descends with increasing $r_{\mathrm{t}}$ from positive value to negative. And then $\delta \beta_{\mathrm{t}}$ changes its trend to go up with $r_{\mathrm{t}}$ increasing, i.e., $\beta_{\mathrm{t} \text {-in }}$ increases faster than $\beta_{\mathrm{t} \text {-out }}$ and the resultant force on the fragments tends to in-plane. But $\beta_{\mathrm{t} \text {-in }}$ is still less than $\beta_{\mathrm{t} \text {-out }}$, so $v_{2}$ still descends till to the minimum about at $r_{\mathrm{t}}=9 \mathrm{fm}$ where $\beta_{\mathrm{t} \text {-in }} \approx \beta_{\mathrm{t} \text {-out }}$, i.e., $\delta \beta_{\mathrm{t}} \approx 0$. And then $v_{2}$ changes its trend to increase with $r_{\mathrm{t}}$ increasing as the fragments are emitted more and more tending to in-plane from a little negative value to positive. Till to the $r_{\mathrm{t}}$ range from 19 fm to 35 fm where $\delta \beta_{\mathrm{t}}$ is nearly a constant, namely, $\beta_{\text {t-out }}$ increases nearly as fast as $\beta_{\mathrm{t} \text {-in }}$, which reflects that the resultant force on fragments in this $r_{\mathrm{t}}$ range tends to neither in-plane nor out-of-plane, $v_{2}$
also keeps a constant, as shown in Fig. 3. When going to large $r_{\mathrm{t}}, \beta_{\mathrm{t} \text {-in }}$ increases faster than $\beta_{\mathrm{t} \text {-out }}$ again, and the fragments emitted would be tending to inplane much more. And so $v_{2}$ goes up correspondingly at large $r_{\mathrm{t}}$. That is to say, elliptic flow is strongly sensitive to $\delta \beta_{\mathrm{t}}$.


Fig. 4. Elliptic flow dependence on transverse radius for fragments $A \leqslant 2$ at $200 \mathrm{fm} / c$.

## 4 Conclusion

In summary, we have investigated the behavior of elliptic flow as a function of transverse radius by studying the mean transverse velocities of the inplane and out-of-plane dependence on transverse radius for $A \leqslant 2$ fragments at rapidity from -0.5 to 0.5 for the simulations of $25 \mathrm{MeV} /$ nucleon ${ }^{64} \mathrm{Cu}+{ }^{64} \mathrm{Cu}$ noncentral collisions by the IDQMD model. The difference between in-plane and out-of-plane transverse velocity, i.e., $\delta \beta_{\mathrm{t}}=\beta_{\mathrm{t} \text {-in }}-\beta_{\mathrm{t} \text {-out }}$ is introduced to investigate the anisotropic feature of fragment emission, and can reveal the direction variation of resultant force on fragment qualitatively. It is shown that the trend of $\delta \beta_{\mathrm{t}}$ dependence on transverse radius is similar to that of elliptic flow, and they can be well understood with each other qualitatively. That is to say, the effects of factors that contribute to $v_{2}$ during the expansion process of a nucleon system, including the density dependent potential, coulomb repulsion, pressure gradient, and so on, can be studied with this method.

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