Cross section of reaction ^{181}Ta(p,nγ)^{181}W and the influence of the spin cut-off parameter on the cross section*

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Abstract: In this work, the program Cindy was modified to calculate the formation cross section of each energy level of residual nucleus ^{181}W resulting from the reaction ^{181}Ta(p,nγ)^{181}W. The concerned cross sections calculated at proton energy $E_{\rm p}$=4.5–8.5 MeV agreed well with experimental results. The influence of the spin cut-off parameter in the energy level density model on the cross section was studied. The obtained results show that the influence of spin cut-off is obvious for lower energy levels.

Key words: nuclear reaction, cross section, level density, spin cut-off parameter

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1 Introduction

The study of cross section is an important tool in finding out a nuclear reaction mechanism. Although experimental cross section data have been measured for many target nuclei at different energy ranges, due to limited experimental conditions, plenty of cross section data can't be obtained by an experimental approach. Therefore, an available theoretical calculation of cross section is very important. Meanwhile, the cross section, especially the formation cross section of each level of a residual nucleus could provide us with rich information on the level density. To get the level density information from a measured cross section, detailed and careful theoretical analysis is needed. However, most existing programs can only provide us with total cross section and cannot calculate the formation cross section of each level. In level density expressions, spin cut-off parameter σ is of importance. However, there is less work focused on the influence of the spin cut-off parameter on the total cross section, especially on the formation cross section of each level. In this work, we modified program Cindy [1], which is based on the optical potential model [1–3] and Hauser-Feshbach model [4], to calculate the (p,n) total cross section and formation cross section of each low energy level. At the same time, the various spin cut-off parameters for the calculation of the (p,nγ) cross section for ground states and excited states were used to test its influence on the cross section.

2 Theory

The optical model provides the basis for many theoretical analyses and evaluations of nuclear cross sections. It also offers a convenient tool for the calculation of reaction, elastic and total cross sections. The optical model potentials (OMPs) are widely used in quantum-mechanical pre-equilibrium and direct-reaction theory calculations, and in supplying particle transmission coefficients for Hauser-Feshbach statistical theory calculations as used in nuclear data evaluations.

As presently formulated, potentials of the form Ref. [1–3]

$$V(r)=V_{\rm V}f_{\rm V_v}(r)-{\rm i}W_{\rm V}f_{W_{\rm v}}(r)+V_{\rm s}\frac{\rm d}{{\rm d}r}f_{W_{\rm s}}+{\rm i}W_{\rm s}\frac{\rm d}{{\rm d}r}f_{W_{\rm s}} +V_{\rm C}+2\frac{\lambda_\pi^2}{r}\left[V_{\rm so}\frac{\rm d}{{\rm d}r}f_{V_{\rm so}}(r)+{\rm i}W_{\rm so}f_{W_{\rm so}}(r)\right](\boldsymbol{S}\cdot\boldsymbol{L}) \quad (1)$$

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are allowed, in which V_v and W_v are the real and imaginary volume potential well depths, V_s and W_s are the real and imaginary well depths for the surface derivative term, V_so and W_so are the real and imaginary well depths for the spin-orbit potential, V_C is the Coulomb potential for incident charged-particles, and λ_π^2 is the pion Compton wavelength squared

$$\lambda_\pi^2=\left(\frac{h}{m_\pi C}\right)^2\approx 2\ \mathrm{fm}^2.$$

The quantity $(\boldsymbol{S}\cdot\boldsymbol{L})$ is the scalar product of the intrinsic S and orbital L angular momentum operators, and for incident nucleons it is given by

$$2(\boldsymbol{S}\cdot\boldsymbol{L})=\begin{cases}1 \text{ for } j=1+\dfrac{1}{2}\\ -(1+1) \text{ for } j=1-\dfrac{1}{2}\end{cases}, \qquad (2)$$

where $f_\mathrm{i}(r)$ are the geometric form factors which are usually of Woods-Saxon form from Ref. [5]

$$f_\mathrm{i}(r)=\left[1+\exp\left(\frac{r-R_\mathrm{i}}{a_\mathrm{i}}\right)\right]^{-1}, \qquad (3)$$

but other functional forms are also accepted.

The energy dependence of the potential depths is expressed as [2, 3]

$$V_\mathrm{v}(E)=V_0+V_1E+V_2E^2+V_3\frac{N-Z}{A}+V_4\frac{Z}{A^{1/3}}, \qquad (4)$$

$$W_\mathrm{s}(E)=W_0+W_1E+W_2\frac{N-Z}{A}, \qquad (5)$$

$$W_\mathrm{v}(E)=U_0+U_1E+U_2E^2+U_3\frac{N-Z}{A}. \qquad (6)$$

The parameterization of the radius and diffuseness parameters (i=R, S, V, so, C) is: [2, 3]

$$R_\mathrm{i}\ =\ r_\mathrm{i}A^{1/3},\ r_\mathrm{i}=r_\mathrm{i}^{(0)}+r_\mathrm{i}^{(1)}A^{-1/3}, \qquad (7)$$

$$a_\mathrm{i}\ =\ a_\mathrm{i}^{(0)}+a_\mathrm{i}^{(1)}A^{1/3}+a_\mathrm{i}^{(2)}\frac{N-Z}{A}. \qquad (8)$$

The transmission coefficients are calculated in generalized form $T_1(E_\mathrm{p})$ from the real and imaginary parts of the elastic scattering amplitudes $C_1(E)$ Ref. [1]

$$T_1=(1-|\eta_1|^2)=1-\rho_1^2=4(\mathrm{lm}\ C_1-|C_1|^2). \qquad (9)$$

The underlying level-density formalism chosen for this procedure was that due to Gilbert and Cameron [6], wherein a low-energy and high-energy expression based upon the statistical Fermi-gas model and the superconductivity model with modifications to take empirical cognizance of shell-configuration and pairing-energy effects, were matched at an appropriate critical energy E_crit and specified quantitatively through requisite input parameters. In this, the Gilbert-Cameron composite level-density formula was designed to reconcile the conventional Bethe-type relationship for the density ρ at the excitation energy E^* as derived from Fermi-gas theory and as recognized as furnishing good agreement at fairly high energies.

Hence, we get the low-energy expression [1]

$$\begin{aligned}\rho_\mathrm{low}(E^*)\ &=\ \left(\frac{1}{kT}\right)\exp([E^*-E_0]/kT)\\ &=\ 0.0165A\exp[0.0165A(E^*-E_0)],\end{aligned} \qquad (10)$$

with an expression due to Gilbert et al. [6] that offers a better fit at low excitation energies, where the optimal value of $1/(kT)$ was deduced to be $1/(kT)$=0.0165 A MeV^{-1} and E_0 was inserted as an adjustable “energy-trimming” parameter.

The high-energy expression is [1]

$$\rho_\mathrm{high}(E^*)=\left(\frac{\sqrt{\pi}}{12}\right)[a(E^*_\mathrm{eff})^5]^{1/4}\exp\left(2\sqrt{aE^*_\mathrm{eff}}\right). \qquad (11)$$

Simple analytical expressions for the excited state density $\rho_\mathrm{high}(E^*)$ of a nucleus with a given excitation energy E^* and level density $\rho(E^*,J)$ of a nucleus with a given angular momentum J were obtained by Bethe on the basis of the Fermi-model [2, 3]

$$\rho_\mathrm{high}(E^*,J)=\left(\frac{2J+1}{2\sigma^2}\right)\exp\left[-\frac{J(J+1)}{2\sigma^2}\right]\rho_\mathrm{high}(E^*), \qquad (12)$$

where σ is the spin cut-off parameter of the level density.

The low-energy expression and the high-energy expression merge tangentially at the critical matching energy [1]

$$E_\mathrm{crit}=\left(2.5+\frac{150}{A}\right)+P(Z)+P(N), \qquad (13)$$

where $P(Z)$ and $P(N)$ are the respective pairing corrections for even-Z and even-N nuclides as a function of Z and $N.P(Z)+P(N)$ being the corresponding pairing energy, in this case in Refs. [2, 3]

$$P(Z)+P(N)=\begin{cases}\delta_Z+\delta_N \text{ for even-even}\\ \delta_Z \text{ for even } Z\\ \delta_N \text{ for even } N\\ 0 \quad \text{for odd-odd}\end{cases}, \qquad (14)$$

where δ_I is the corresponding phenomenological correction for even-odd differences of nuclear binding energies. Also the effective excitation energy is given by [1]

$$E^*_\mathrm{eff}=E^*-P(Z)-P(N), \qquad (15)$$

for insertion into the high-energy formula (10).

The linking condition

$$\rho_{\text{low}}(E_{\text{crit}}) = \rho_{\text{high}}(E_{\text{crit}}) \text{ at } E^*_{\text{eff}} = E_{\text{crit}}, \tag{16}$$

serves to determine the values of parameters T (the nuclear temperature) and E_0 inasmuch as [1]

$$E_0 = E_{\text{crit}} - kT \ln\ [kT\rho_{\text{low}}(E_{\text{crit}})]. \tag{17}$$

The distribution function of the angular momentum of the compound nucleus J_c depends on the transmission coefficients of the protons. The function is given by [4]

$$\sigma(J_c, E_p) = \pi\lambda^2 \sum_{S=|I-S|}^{I+S} \sum_{I=|J_c-S|}^{J_c+S} \frac{2J_c+1}{(2s+1)(2I+1)} T_1(E_p), \tag{18}$$

where λ is the de Broglie wavelength of the incident projectile, s the spin of the projectile, I the spin of the target nucleus, S the spin of the compound nucleus and $T_1(E_p)$ the transmission coefficient of protons with orbital angular momentum I and energy E_p. This $\sigma(J_c,\ E_p)$ is distributed among the final state of residual nuclei by neutrons. A compound state with angular momentum J_c emits a neutron with orbital angular momentum I, which leads to a final state with angular momentum J_f.

Consider a transition from nuclear state a to nuclear state b with emission of a quantum of multipole radiation of angular momentum I (2^L-pole). For the transition probability of an electric 2^L-pole[7] we get

$$T_E(L) = \frac{4.4(L+1)}{L[(2L+1)!!]^2}\left(\frac{3}{L+3}\right)^2\left(\frac{h\omega}{197\text{ MeV}}\right)^{2L+1} \times (R\times 10^{-13}\text{ cm})^{2L} 10^{21}\text{s}^{-1}, \tag{19}$$

and for a magnetic 2^1-pole

$$T_M(L) = \frac{1.9(L+1)}{L[(2L+1)!!]^2}\left(\frac{3}{L+3}\right)^2\left(\frac{\hbar\omega}{197\text{ MeV}}\right)^{2L+1} \times (R\times 10^{-13}\text{ cm})^{2L-2} 10^{21}\text{s}^{-1}. \tag{20}$$

These expressions are approximations valid for $\kappa R \ll 1$, where R is the nuclear radius.

3 Calculation results and discussion

Calculations were performed in the framework of a statistical model. We obtained the particle transmission coefficients in the optical model by using the potential of [1–3] for the protons. The energy dependence of both the real and imaginary parts of the proton optical potentials were taken from Ref. [8–11]. In our calculation all probable γ-decays of each excited level of product nuclei were taken into account. In the product nuclei, the excited states were described by means of discrete level information as far as possible. Above the region of discrete levels, all excited states were treated as a continuum described by the Gilbert-Cameron level density model. According to the statistical model, the cross section depends not only on the spin of all excited states and the ground state, but also on the spin cut-off parameter in the level density model. The choice of level density parameters was guided by the compilation of Gilbert A et al. [6].The parameters were verified by checking the reproduction of cumulative level densities. The branching ratios of transitions E1, E2 and M1 were estimated using the Weisskopf model [7].

It is well known that the parameters of the level density have influence on the calculations of the cross section, so the spin cut-off parameter also influences the calculation results. In this work, we choose the various values of the spin cut-off parameters to calculate the formation cross section.

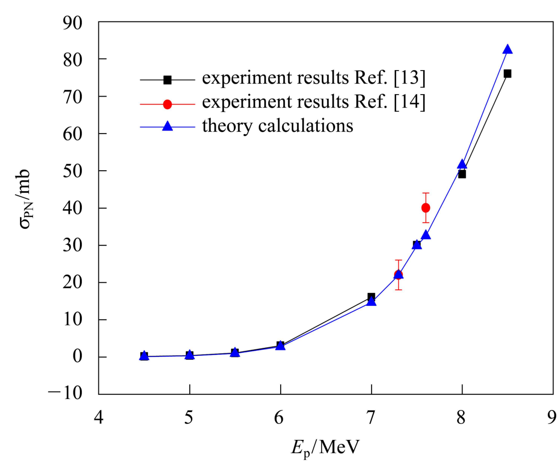

Fig. 1. The total cross section σ_{pn} of ^{181}Ta(p, nγ)^{181}W was calculated at proton energy E_p=4.5–8.5 MeV.

For the target ^{181}Ta, the values of optical parameters of the proton potential from Refs. [2, 3] have been used:

$$\begin{aligned} &V_v = 45.94 - 46.54\frac{N-Z}{A} + 0.00040E^2 - 0.28E, \\ &W_s = 7.72 + 0.026E + 11.84\frac{N-Z}{A}, \\ &W_v = 0.59 + 0.005E - 0.00076E^2, \\ &V_{so} = 7.5,\ r_{so} = 1.26,\ a_{so} = 0.65, \\ &r_v = 1.23,\ a_v = 0.65,\ r_w = 1.28,\ a_w = 0.60, \end{aligned} \tag{21}$$

where the incident proton energy E (in the laboratory system), the depth of the real potential $V_{\rm v}$, the surface absorption $W_{\rm s}$, the volume absorption $W_{\rm v}$ and the spin-orbit potential $V_{\rm so}$ are expressed in MeV and the parameterization of the radius, r and diffuseness parameters, a, in fermi.

The results of the calculation are presented in Figs. 1, 2. For ^{181}Ta the total cross section of (p, n) was studied and the experimental results are consistent with the present calculated results, especially for the high incident energy (see Fig. 1). It shows that the total cross section of (p, n) increased with the increase of the incident proton energy. Meanwhile, the formation cross section of each level is also calculated with the various values of the spin cut-off parameter. The information of the energy level for ^{181}W is presented in Table 1. For the incident energy $E_{\rm p}$=6.5 MeV and $E_{\rm p}$=8.5 MeV, as seen in Fig. 2, the formation cross section of each level changed with the spin cut-off parameter, especially for the ground state and the lower excited states. Also it was found that when, the spin of the level is greater or equal to the

Table 1. Energy level information for ^{181}W [12].

elevel/keV	J^{π}	E_{γ}/keV	intensity(%)	γ multipolarity
0.0	9/2+			
113.40	11/2+	113.30	100	M1
250.72	13/2+	137.28	100	M1
		251.2	40	E2
365.55	5/2−	252.3	1.7	E3
		365.5	100	M2
385.19	1/2−	19.7	100	E2
409.23	7/2−	43.5	100	M1+E2
		409.0	53	
414.3	15/2+	163.5	95	D
		301.0	100	E2
450.12	3/2−	65.0	100	M1+E2
457.84	1/2−	72.7	100	M1
475.60	7/2−	109.89	100	M1+E2
		475.6	16	
488.43	5/2−	38.1	35	M1
		103.1	100	E2
528.6	9/2−	119.4	100	[M1]
529.42	3/2−	71.7	30	M1+E2
		144.3	100	M1
		163.9	34	M1+E2

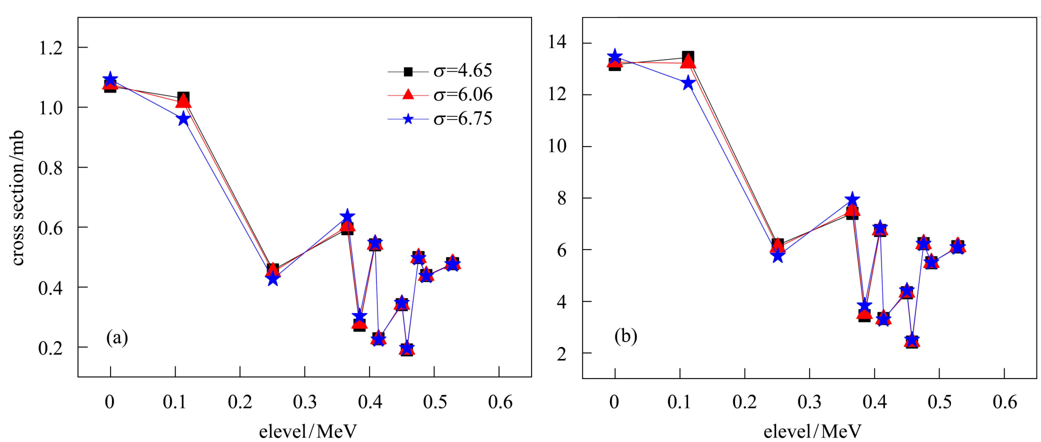

Fig. 2. The formation cross section of each energy level was calculated by using various spin cut-off parameters σ: 4.65[9], 6.06[15] and 6.75[15] for target ^{181}Ta at $E_{\rm p}$=6.5 MeV, 8.5 MeV.

spin of the ground state J=4.5, the formation cross section of the level increased slightly with increasing value of the spin cut-off parameter σ. However, the formation cross section of the level whose spin is less than J=4.5 has the opposite trend. It can be explained that the spin cut-off parameter characterizes the angular momentum distribution of the level density. With the increase in the value of the spin cut-off parameter, more excited states were distributed on the high spin. Therefore most of the transition occurred on the high spin. So we get the different trends with the increase of the spin cut-off parameter. Furthermore, as seen in Fig. 2, it indicates that the ground state and the lower excitation energy are more sensitive to the spin cut-off parameter.

To conclude, program Cindy was modified to calculate the formation cross section of residual nuclear levels. To check the modified program, the total cross section of the reaction ^{181}Ta(p,nγ)^{181}W was calculated and compared with experimental results. Furthermore, the influence of the spin cut-off parameter on the formation cross section of the residual nuclear level was studied.

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