Wigner function for the Dirac oscillator in spinor space*

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Abstract: The Wigner function for the Dirac oscillator in spinor space is studied in this paper. Firstly, since the Dirac equation is described as a matrix equation in phase space, it is necessary to define the Wigner function as a matrix function in spinor space. Secondly, the matrix form of the Wigner function is proven to support the Dirac equation. Thirdly, by solving the Dirac equation, energy levels and the Wigner function for the Dirac oscillator in spinor space are obtained.

Key words: Dirac oscillator, Wigner function, Dirac equation, spinor space

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1 Introduction

In recent years the Wigner function has enjoyed popularity in virtually all areas of physics. In fact, the Wigner function was first introduced in 1932 [1]. As a quasi-probability distribution function in phase-space as well as a special representation of the density matrix, it is of great value in quantum measurement [2, 3]. It has also been useful in describing quantum transport in quantum optics, nuclear physics, decoherence (e.g. quantum computing), quantum chaos, signal processing, etc. Moreover, the Wigner function is a highly semi-classical approximation [4–7]. Nevertheless, a remarkable aspect of the Wigner function was not pioneered until 1975 by Moyal according to the internal logic of Quantum Mechanics. With the Moyal $\star$-eigenvalue equation [8–12] as its general form, the Wigner function is as valuable as other formulations, such as the Schrodinger, the Hesneberge regularization operator, Feymen path integral quantization, etc. In this logically complete and self-standing formulation, one need not choose sides – coordinate or momentum space – because the function works in full phase-space, accommodating some uncertainty principles. What's more, as a time-independent function and a quasi-probability distribution function in phase space, the Wigner function is of significance in modern quantum measurement. Take Ref. [13] for example, the Wigner function of an ensemble of helium atoms was skillfully tested there and the result was the same as that obtained through theoretical calculation.

In this paper, we generalize the method of quantization in phase space into spinor space with a focus on the spin-1/2 particles. To be clear, this paper is organized as follows: in Section 2, we review the interpretation-quantization in a non-spinor space; In Section 3, we generalize the method into the spinor space with a detailed discussion of spin-1/2 space and the Wigner function in a spinor space; In Section 4, we calculate the energy level and the Wigner function for the Dirac oscillator. Conclusions are given in the last section.

2 Wigner function and $\star$-eigenvalue equation

In this part we begin to discuss the Wigner function in non-spinor space. As a quasi-probability distribution function in phase space, the Wigner function is a very good semi-classical approximation, with great importance in physics measurement. It is known that in phase space with the degree of freedom $s=n$, the general form of the Wigner function is described

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$$W(\vec{x},\vec{p},t)=\frac{1}{(2\pi\hbar)^n}\int_{-\infty}^{\infty}\mathrm{d}y\mathrm{e}^{-\mathrm{i}\vec{y}\vec{p}}\left\langle\vec{x}-\frac{\vec{y}}{2}|\hat{\rho}|\vec{x}+\frac{\vec{y}}{2}\right\rangle. \tag{1}$$

While in a stative situation the Wigner function reads

$$W(\vec{x},\vec{p})=\frac{1}{(2\pi\hbar)^n}\int_{-\infty}^{\infty}\mathrm{d}y\psi^*\left(\vec{x}-\frac{\vec{y}}{2}\right)\mathrm{e}^{-\mathrm{i}\vec{y}\vec{p}}\psi\left(\vec{x}+\frac{\vec{y}}{2}\right). \tag{2}$$

This is a special representation of the density matrix. According to the definition of Eq. (1), we can prove that the time-independent Wigner function has the following dynamic evolution equation

$$\frac{\partial W}{\partial t}=-\frac{\vec{p}}{m}\frac{\partial W}{\partial\vec{x}}+\frac{\partial V}{\partial\vec{x}}\frac{\partial W}{\partial\vec{p}}, \tag{3}$$

which is similar to Liouville theorem in classical mechanics. In fact, with the Hamiltonian $H(\vec{x},\vec{p})$, Eq. (3) can also be written as the following Moyal equation [9]

$$\frac{\partial W}{\partial t}=\frac{H\star W-W\star H}{\mathrm{i}\hbar} \tag{4}$$

where the $\star$-product is

$$\star\equiv\mathrm{e}^{\frac{\mathrm{i}\hbar}{2}(\overleftarrow{\partial}_x\overrightarrow{\partial}_p+\overleftarrow{\partial}_p\overrightarrow{\partial}_x)}. \tag{5}$$

Since the $\star$-product involves exponential operators, which causes much difficulty in real calculations and $\hbar$ is a very small volume, $\star$-product, as a series expansion, can be expressed as [9]

$$f(x,p)\star g(x,p)=f\left(x+\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_p,\ p-\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_x\right)g(x,p) \tag{6}$$

and

$$f(x,p)\star g(x,p)=f(x,p)\ g\left(x-\frac{\mathrm{i}\hbar}{2}\overleftarrow{\partial}_p,\ p+\frac{\mathrm{i}\hbar}{2}\overleftarrow{\partial}_x\right). \tag{7}$$

In this way, the Wigner function meets the more binding $\star$-eigenvalue equations [9]

$$\begin{aligned}&H(x,p)\star W(x,p)\\&=H\left(x+\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_p,\ p-\frac{\mathrm{i}\hbar}{2}\overrightarrow{\partial}_x\right)\ W(x,p)=EW\end{aligned} \tag{8}$$

and

$$\begin{aligned}&H(x,p)\star W(x,p)\\&=H(x,p)\ W\left(x-\frac{\mathrm{i}\hbar}{2}\overleftarrow{\partial}_p,\ p+\frac{\mathrm{i}\hbar}{2}\overleftarrow{\partial}_x\right)=EW.\end{aligned} \tag{9}$$

Here E is the energy eigenvalue of $H\psi=E\psi$. Using Eqs. (8) and (9) the Wigner function and energy levels can be obtained.

3 Wigner function in spinor space

We are now in a position to describe the Wigner function and the Dirac equation in phase space for a particle with spin-$\frac{1}{2}$. As is known, the Dirac equation for a spin-$\frac{1}{2}$ particle is a first-order differential equation. In the case of potentials its form is

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t}=[c\vec{\alpha}\cdot\vec{p}+\beta mc^2+V(x)]\,\psi, \tag{10}$$

and the four-dimensional vector $j^\mu=\overline{\psi}\gamma^\mu\psi$ is conserved $\partial_\mu j^\mu=0$. Thus, the probability density j^0 is

$$\mathrm{tr}(\rho^{\mathrm{s}})=j^0=\overline{\psi}\gamma^0\psi=|\psi_1|^2+|\psi_2|^2+|\psi_3|^2+|\psi_4|^2, \tag{11}$$

which is clearly positive. j^0 can be the probability density. Therefore, for a particle with spin-$\frac{1}{2}$, the Wigner function is meaningful and the quantization in phase-space can be extended to spinor space with spin-$\frac{1}{2}$ particles.

We define the Wigner function W^{s} for a particle with spin-$\frac{1}{2}$ as

$$\begin{aligned}W^{\mathrm{s}}&=\frac{1}{2\pi\hbar}\int\mathrm{d}y\ \mathrm{e}^{-\mathrm{i}py}\left\langle x-\frac{y}{2}|\rho^{\mathrm{s}}|x+\frac{y}{2}\right\rangle\\&=\frac{1}{2\pi\hbar}\int\mathrm{d}y\ \mathrm{e}^{-\mathrm{i}py}\left\langle x-\frac{y}{2}|\psi\right\rangle(\gamma^0)^2\left\langle\psi|x+\frac{y}{2}\right\rangle\\&=\frac{1}{2\pi\hbar}\int\mathrm{d}y\ \mathrm{e}^{-\mathrm{i}py}\psi^\dagger\left(x-\frac{y}{2}\right)\psi\left(x+\frac{y}{2}\right),\end{aligned} \tag{12}$$

where the spinors $\psi^\dagger\left(x-\frac{y}{2}\right)$ and $\psi\left(x+\frac{y}{2}\right)$ each have four components. Therefore, the Wigner function for a particle with spin-$\frac{1}{2}$ is generally a four-order matrix function with sixteen components

$$\begin{pmatrix}W^{\mathrm{s}}_{11}&W^{\mathrm{s}}_{12}&W^{\mathrm{s}}_{13}&W^{\mathrm{s}}_{14}\\W^{\mathrm{s}}_{21}&W^{\mathrm{s}}_{22}&W^{\mathrm{s}}_{23}&W^{\mathrm{s}}_{24}\\W^{\mathrm{s}}_{31}&W^{\mathrm{s}}_{32}&W^{\mathrm{s}}_{33}&W^{\mathrm{s}}_{34}\\W^{\mathrm{s}}_{41}&W^{\mathrm{s}}_{42}&W^{\mathrm{s}}_{43}&W^{\mathrm{s}}_{44}\end{pmatrix} \tag{13}$$

where

$$W^{\mathrm{s}}_{ij}=\frac{1}{2\pi\hbar}\int\mathrm{d}y\ \mathrm{e}^{-\mathrm{i}py}\psi^*_i\left(x-\frac{y}{2}\right)\psi_j\left(x+\frac{y}{2}\right),$$

$$(i,j=1,2,3,4). \tag{14}$$

It is useful to split up the spinor ψ into two two-component spinors ϕ and χ so as to solve the Dirac equation. The Wigner function is then reduced to a diagonally partitioned matrix. However, with appropriate representation, the Wigner function can also be approximately transformed into a diagonal matrix, and its amount should be

$$W^{\rm s}_{ij} = \frac{\delta_{ij}}{2\pi\hbar}\int {\rm d}y\ {\rm e}^{-{\rm i}py}\psi_i^*\left(x-\frac{y}{2}\right)\psi_j\left(x+\frac{y}{2}\right),$$

$$(i,j = 1,2,3,4). \quad (15)$$

Now, generalizing the quantization in phase space to the Dirac equation, we can prove that in the phase space with spin, the general product should be replaced by the $\star$-product, and the Wigner function described as a four-order matrix function. This can be supported by the following.

$$\begin{aligned}H^{\rm s}\star W^{\rm s} &= (c\vec{\alpha}\cdot\vec{p}+\beta mc^2+V(x))\star W^{\rm s} = \frac{1}{(2\pi\hbar)^3}\left(c\vec{\alpha}\cdot\left(\vec{p}-\frac{{\rm i}\hbar}{2}\overrightarrow{\partial}_{\vec{x}}\right)+\beta mc^2\right.\\ &\left.+V(\vec{x})\right)\cdot\int{\rm d}y\ \exp\left[-{\rm i}\vec{y}\left(\vec{p}+\frac{{\rm i}\hbar}{2}\overleftarrow{\partial}_{\vec{x}}\right)\right]\psi^\dagger\left(x-\frac{y}{2}\right)\psi\left(x+\frac{y}{2}\right) = \frac{1}{(2\pi\hbar)^3}\int{\rm d}y\ \left(c\vec{\alpha}\cdot\left(\vec{p}-\frac{{\rm i}\hbar}{2}\overrightarrow{\partial}_{\vec{x}}\right)+\beta mc^2\right.\\ &\left.+V\left(\vec{x}+\frac{\hbar}{2}\vec{y}\right)\right)\exp(-{\rm i}\vec{y}\vec{p})\psi^\dagger\left(x-\frac{y}{2}\right)\psi\left(x+\frac{y}{2}\right) = \frac{1}{(2\pi\hbar)^3}\int{\rm d}y\ \exp(-{\rm i}\vec{y}\vec{p})\left(c\vec{\alpha}\cdot\left(-{\rm i}\overrightarrow{\partial}_{\vec{y}}-\frac{{\rm i}\hbar}{2}\overrightarrow{\partial}_{\vec{x}}\right)\right.\\ &\left.+\beta mc^2+V\left(\vec{x}+\frac{\hbar}{2}\vec{y}\right)\right)\psi\left(x+\frac{y}{2}\right)\psi^\dagger\left(x-\frac{y}{2}\right) = \frac{1}{(2\pi\hbar)^3}\int{\rm d}y\ \exp(-{\rm i}\vec{y}\vec{p})E\psi\left(x+\frac{y}{2}\right)\psi^\dagger\left(x-\frac{y}{2}\right)\\ &= EW^{\rm s}\end{aligned} \quad (16)$$

Symmetrically,

$$\begin{aligned}W^{\rm s}\star H^{\rm s} &= W^{\rm s}\star(c\vec{\alpha}\cdot\vec{p}+\beta mc^2+V(x))\\ &= \frac{1}{(2\pi\hbar)^3}\left[\left[\int{\rm d}y\ \exp\left(-{\rm i}\vec{y}\left(\vec{p}-\frac{{\rm i}\hbar}{2}\overrightarrow{\partial}_{\vec{x}}\right)\right)\psi^\dagger\left(x-\frac{y}{2}\right)\psi\left(x+\frac{y}{2}\right)\right]\right]\left(c\vec{\alpha}\cdot\left(\vec{p}+\frac{{\rm i}\hbar}{2}\overleftarrow{\partial}_{\vec{x}}\right)+\beta mc^2+V(\vec{x})\right)\\ &= \frac{1}{(2\pi\hbar)^3}\int{\rm d}y\ \exp(-{\rm i}\vec{y}\vec{p})\psi\left(x+\frac{y}{2}\right)E\psi^\dagger\left(x-\frac{y}{2}\right) = EW^{\rm s}.\end{aligned} \quad (17)$$

Obviously, in phase space with spin the Wigner function is a matrix function and its Dirac equation becomes a $\star$ engenvalue equation.

4 Wigner function for the Dirac oscillator

In this section we investigate the enegy level and the Wigner function for the Dirac oscillator. It is known that in a stationary state the relativistic equation for the Dirac oscillator is described as

$$[c\vec{\alpha}\cdot(\vec{p}-{\rm i}m\omega\beta\vec{r})+\beta mc^2]\psi(\vec{r}) = E\psi(\vec{r}) \quad (18)$$

where

$$\psi(\vec{r}) = \begin{pmatrix}\psi_{\rm a}(\vec{r})\\ \psi_{\rm b}(\vec{r})\end{pmatrix},$$

$$\vec{\alpha} = \begin{pmatrix}0 & \alpha\\ \alpha & 0\end{pmatrix},$$

$$\vec{\beta} = \begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}. \quad (19)$$

Thus, the Wigner function has the following form

$$W^{\rm s} = \begin{pmatrix}W^{\rm s(a)} & 0\\ 0 & W^{\rm s(b)}\end{pmatrix} = \begin{pmatrix}W^{\rm s(a)}_{11} & W^{\rm s(a)}_{12} & 0 & 0\\ W^{\rm s(a)}_{21} & W^{\rm s(a)}_{22} & 0 & 0\\ 0 & 0 & W^{\rm s(b)}_{11} & W^{\rm s(b)}_{12}\\ 0 & 0 & W^{\rm s(b)}_{21} & W^{\rm s(b)}_{22}\end{pmatrix}. \quad (20)$$

Obviously, the Wigner function is transformed to a diagonal matrix.

In phase space the Dirac oscillator is defined by the following equation

$$[c\vec{\alpha}\cdot(\vec{p}-{\rm i}m\omega\beta\vec{r})+\beta mc^2]\star W^s = EW^{\rm s}, \quad (21)$$

where α and β are Pauli matrixes. If the eigne-state of the operator σ_z is chosen, the Wigner function is simplified as

$$W^{\rm s} = \begin{pmatrix}W^{\rm s(a)} & 0\\ 0 & W^{\rm s(b)}\end{pmatrix}. \quad (22)$$

With a straightforward calculation, the following two simultaneous equations appear

$$c\vec{\sigma}\cdot(\vec{p}-\mathrm{i}m\omega\beta\vec{r})\star W^{\mathrm{s(b)}}=(E-mc^2)W^{\mathrm{s(a)}}, \quad (23)$$

and

$$c\vec{\sigma}\cdot(\vec{p}-\mathrm{i}m\omega\beta\vec{r})\star W^{\mathrm{s(a)}}=(E+mc^2)W^{\mathrm{s(b)}}. \quad (24)$$

In both Eq. (24) and Eq. (25) $W^{\mathrm{s(b)}}$ is a small component, which tends to be zero in the non-relativistic limit. Inserting Eq. (24) into Eq. (23), we can have

$$[c^2(p^2+m^2\omega^2r^2)-2\hbar\omega mc^2 -4mc^2(\omega/\hbar)\vec{L}\cdot\vec{S})]\star W^{\mathrm{s(a)}}=(E^2-m^2c^4)W^{\mathrm{s(a)}} \quad (25)$$

where $\vec{L}$ is the angular momentum operator, and $\vec{S}$ is the spinor operator. If the eigne-state of the operator σ_z is chosen, the Wigner function also reduces to

$$W^{\mathrm{s}}=\begin{pmatrix} W_{11}^{\mathrm{s(a)}} & 0 \\ 0 & W_{22}^{\mathrm{s(a)}}\end{pmatrix}. \quad (26)$$

In the two dimensions, we deduce by using the matrix-equation that the Wigner function satisfies a Dirac oscillator in a non-relativistic state as the following

$$c^2\{(p_1^2+p_2^2)+m^2\omega^2(x_1^2+x_2^2)-\frac{\hbar^2}{4}m^2\omega^2(\partial_{p_1}^2+\partial_{p_2}^2) -\frac{\hbar^2}{4}(\partial_{x_1}^2+\partial_{x_2}^2)+2m\omega(x_1p_2-x_2p_1)+\frac{\hbar^2}{2}m\omega(\partial_{x_2}\partial_{p_1} -\partial_{x_1}\partial_{p_2})\}\begin{pmatrix} W_{11}^{\mathrm{s(a)}} & 0 \\ 0 & W_{22}^{\mathrm{s(a)}}\end{pmatrix}=\begin{pmatrix} \epsilon W_{11}^{\mathrm{s(a)}} & 0 \\ 0 & \epsilon W_{22}^{\mathrm{s(a)}}\end{pmatrix} \quad (27)$$

where $\epsilon = E^2-m^2c^4$. With further calculation we arrive at the following equations

$$c^2\{(p_1^2+p_2^2)+m^2\omega^2(x_1^2+x_2^2)-\frac{\hbar^2}{4}m^2\omega^2(\partial_{p_1}^2+\partial_{p_2}^2) -\frac{\hbar^2}{4}(\partial_{x_1}^2+\partial_{x_2}^2)+2m\omega(x_1p_2-x_2p_1)+\frac{\hbar^2}{2}m\omega(\partial_{x_2}\partial_{p_1} -\partial_{x_1}\partial_{p_2})\}W_{11}^{\mathrm{s(a)}}=\epsilon_1 W_{11}^{\mathrm{s(a)}} \quad (28)$$

and

$$c^2\{(p_1^2+p_2^2)+m^2\omega^2(x_1^2+x_2^2)-\frac{\hbar^2}{4}m^2\omega^2(\partial_{p_1}^2+\partial_{p_2}^2) -\frac{\hbar^2}{4}(\partial_{x_1}^2+\partial_{x_2}^2)+2m\omega(x_1p_2-x_2p_1) +\frac{\hbar^2}{2}m\omega(\partial_{x_2}\partial_{p_1}-\partial_{x_1}\partial_{p_2})\}W_{22}^{\mathrm{s(a)}}=\epsilon_1 W_{22}^{\mathrm{s(a)}}. \quad (29)$$

This equation is similar to the Landau problem and is equivalent to the movement of a relativistic charged particle in an external magnetic field. For Eq. (29), we introduce four new variables X_i $(i=1,2,3,4)$,

$$\begin{aligned} X_1 &= \left(\sqrt{\frac{1}{2m\omega}}p_1+\sqrt{\frac{m\omega}{2}}x_2\right), \\ X_2 &= \left(\sqrt{\frac{1}{2m\omega}}p_2+\sqrt{\frac{m\omega}{2}}x_1\right), \\ X_3 &= \left(\sqrt{\frac{1}{2m\omega}}p_2-\sqrt{\frac{m\omega}{2}}x_1\right), \\ X_4 &= \left(\sqrt{\frac{1}{2m\omega}}p_1-\sqrt{\frac{m\omega}{2}}x_2\right). \end{aligned} \quad (30)$$

By straightforward calculation we can derive

$$c^2\{3m\omega(X_2^2+X_4^2)-m\omega(X_1^2+X_3^2) -\frac{3\hbar^2}{8}m\omega(\partial_{X_2}^2+\partial_{X_4}^2)+\frac{\hbar^2}{8}m\omega(\partial_{X_1}^2 +\partial_{X_3}^2)\}W_{22}^{\mathrm{s(a)}}=\epsilon_1 W_{22}^{\mathrm{s(a)}}. \quad (31)$$

With two more new variables ξ and η,

$$\xi:=\frac{2}{\hbar}(X_1^2+X_3^2), \quad \eta:=\frac{2}{\hbar}(X_2^2+X_4^2), \quad (32)$$

Eq.(32) may be rewritten as follows,

$$c^2m\omega\left[6\left(\frac{\eta}{4}-\eta\partial_\eta^2-\partial_\eta\right) -2\left(\frac{\xi}{4}-\xi\partial_\xi^2-\partial_\xi\right)\right]W_{11}^{\mathrm{s(a)}}=\epsilon_1 W_{11}^{\mathrm{s(a)}} \quad (33)$$

With the separation of variables, $W_{11}^{\mathrm{s(a)}}(\xi,\eta) = W_{11}^{\mathrm{s(a)}}(\xi)^1 W_{11}^{\mathrm{s(a)}}(\eta)^2$, $\epsilon=6\epsilon^2-2\epsilon^1$, we have

$$c^2m\omega\left[\frac{\xi}{4}-\xi\partial_\xi^2-\partial_\xi-\epsilon^1\right]W_{11}^{\mathrm{s(a)}}(\xi)^1=0 \quad (34)$$

and

$$c^2m\omega\left[\frac{\eta}{4}-\eta\partial_\eta^2-\partial_\eta-\epsilon^2\right]W_{11}^{\mathrm{s(a)}}(\eta)^2=0. \quad (35)$$

Finally, we can find the solutions for Eq. (34) and Eq. (35)

$$\begin{aligned} W_{11}^{\mathrm{s(a)}}(\xi)_m^1 &= \frac{(-1)^m}{\pi\hbar}\mathrm{e}^{-\xi/2}L_m(\xi), \\ \epsilon^1 &= \left(m+\frac{1}{2}\right)c^2m\hbar\omega,\ m=0,1,\cdots \end{aligned} \quad (36)$$

and

$$\begin{aligned} W_{11}^{\mathrm{s(a)}}(\eta)_n^2 &= \frac{(-1)^n}{\pi\hbar}\mathrm{e}^{-\eta/2}L_n(\eta), \\ \epsilon^2 &= \left(n+\frac{1}{2}\right)c^2m\hbar\omega,\ n=0,1,\cdots \end{aligned} \quad (37)$$

Thus, we have

$$W_{11}^{\mathrm{s(a)}}(\xi,\eta)_{nm} = \frac{(-1)^{m+n}}{(\pi\hbar)^2}\,\mathrm{e}^{-(\xi+\eta)/2}\,L_m(\xi)L_n(\eta),$$

$$\epsilon = 6\left(n+\frac{1}{2}\right)c^2m\hbar\omega$$

$$-2\left(m+\frac{1}{2}\right)c^2m\hbar\omega. \qquad (38)$$

Symmetrically, we can find the solutions for $W_{22}^{\mathrm{s(a)}}$,

$$W_{22}^{\mathrm{s(a)}}(\mu,\nu)_{nm} = \frac{(-1)^{m+n}}{(\pi\hbar)^2}\,\mathrm{e}^{-(\mu+\nu)/2}\,L_m(\mu)L_n(\nu),$$

$$\epsilon = 6\left(n+\frac{1}{2}\right)c^2m\hbar\omega$$

$$-2\left(m+\frac{1}{2}\right)c^2m\hbar\omega. \qquad (39)$$

This is the very Wigner function and energy level for an oscillator in phase space.

5 Conclusion

In summary, by defining a matrix Wigner function in spinor space, this paper first provides the Dirac equation which a four-order matrix Wigner function obeys in a phase space with spin. From this, one knows that in phase space the Dirac equation should be described as a matrix equation with a $\star$-product, and in spinor space the Wigner function can be described as a matrix function. As a result, by solving the Dirac equation in phase space, the energy level and the Wigner function for a Dirac oscillator in spinor space are obtained. In addition, in recent years increasing attention has been paid to the noncommutative feature of the Wigner function [14-22]. This is to be reported elsewhere.

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