Light meson spectrum and their leptonic decay widths in the frame work of constituent quark models

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Abstract The phenomenological non-relativistic quark model has been employed to obtain the ground state masses of light vector mesons and their radially excited states and their decay widths. The full hamiltonian used in the investigation has kinetic energy, the confinement potential and the one-gluon-exchange potential. A good agreement is obtained with the experimental masses and their leptonic decay widths.

Key words meson spectrum, leptonic decay width, constituent quark model, OGEP

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1 Introduction

The non-relativistic quark models (NRQM) have proven to be very successful in describing hadronic properties. Since the exact form of confinement from QCD is not known, one has to go for phenomenological models and the NRQM have proven to be very successful in describing hadronic properties [1–5]. The description of the mass spectrum is necessary, but is not a sufficient condition for aiming at a good explanation of non-perturbative QCD. In particular several different potentials reproduce the hadron spectrum and hence one needs other observables in order to test more precisely the resulting wave functions. Hence, a possibility is to study the transition between various states and their leptonic decay widths.

In our present investigation, we have employed the NRQM to explain the ground state mass spectrum of light vector mesons (VM). The total non-relativistic hamiltonian employed has kinetic energy, confinement potential and one-gluon-exchange potential (OGEP) [6]. One of the aims of the present study is to test whether the quark-gluon coupling constant (α_s)can be treated as a perturbative effect and to obtain a consistent set of parameters which reproduce both the mass spectrum and the leptonic decay widths of VM. The total energy or the mass of the meson is obtained by calculating the energy eigen values of the Hamiltonian in the harmonic oscillator basis spanned over a configuration space extending up to the radial quantum number $n_{\max}=11$. The details about the present employed model can be found in Refs. [7–9].

In the next section, we review briefly the NRQM and give the form of OGEP and the confinement potential. We also discuss the parameters involved in our model and the non-relativistic description of leptonic decay widths. The results of the calculation are presented in section 3 and the conclusions are given in section 4.

2 The model

In NRQM, quarks in a hadron are confined through the action of a linear/quadratic confinement potential. The full Hamiltonian is given by

$$H=K+V_{\text{conf}}(r_{ij})+V_{\text{OGEP}}(r_{ij}), \tag{1}$$

where

$$K=\sum_{i=1}^{2}M_i+\frac{P_i^2}{2M_i}-K_{\text{cm}}, \tag{2}$$

where M_i and P_i are the i^{th} quark mass and momentum. Thus K is the sum of the kinetic energies including the rest mass minus the kinetic energy of the center-of-mass motion of the total system. The potential energy part consists of confinement term V_{conf} the residual interaction V_{OGEP}. It may be noted that in calculation of mass spectrum, inclusion of only

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two-body central potentials as functions of relative position r_{ij} would suffice.

The confinement term is given by,

$$V_{\rm conf}(r)=-a_{\rm c}r_{ij}\lambda_i\cdot\lambda_j, \tag{3}$$

where $a_{\rm c}$ is the confinement strength and λ_i is the colour $SU(3)_{\rm c}$ generator for the $i^{\rm th}$ quark. Among the several versions of the one-gluon-exchange potential $V_{\rm OGEP}$, we have used the following one, first derived in Ref. [6] from the QCD Lagrangian in the non-relativistic limit.

$$V_{\rm OGEP}(r_{ij}) = \frac{\alpha_{\rm s}}{4}\ \lambda_i.\lambda_j\left[\frac{1}{r_{ij}}-\frac{\pi}{M_iM_j}\ \left(1+\frac{2}{3}\ \sigma_i.\sigma_j\right)\ \delta(r_{ij})\right], \tag{4}$$

where the first term is the residual Coulomb energy and the second-term the chromo-magnetic interaction leading to the hyperfine splittings.

The parameters of the model are the masses of the u,d and s quarks,$M_{\rm u}(=M_{\rm d})$ and $M_{\rm s}$,the confinement strength $a_{\rm c}$, the oscillator size parameter b,the strong coupling constant $\alpha_{\rm s}$. There are several papers in literature where size parameter b is defined [10–12]. The value of b is fixed by minimizing the expectation value of the Hamiltonian for pseudo-scalar and V mesons. To start with, we constructed the 11×11 Hamiltonian matrix for both pseudo-scalar and VM in the harmonic oscillator basis. The confinement strength $a_{\rm c}$ is fixed by the stability condition for variation of mass of the mesons against the size parameter b. To fit $\alpha_{\rm s}$ and $M_{\rm u}$, we started with a set of reasonable values of these parameters and diagonalise the matrix for ρ meson. Then we tuned the two parameters so as to obtain an agreement with the experimental value for the mass of the ρ meson. One can note that our value of $\alpha_{\rm s}=0.6$ is compatible with the perturbative treatment. The values of the parameters used in our calculations are listed in Table 1. To calculate the meson masses, the product of quark-antiquark oscillator wavefunctions is expressed in terms of oscillator wavefunctions corresponding to the relative and center-of-mass coordinates. The details can be found in reference [7]. Below we briefly give the theoretical overview of the leptonic decay widths.

Table 1. Values of parameters used in the present model.

b	$M_{\rm u,d}$	$M_{\rm s}$	$\alpha_{\rm s}$	$a_{\rm c}$
0.9 fm	352 MeV	545 MeV	0.6	20 MeV·fm^{-1}

In non-relativistic treatment of ground state mesons as q$\bar{\rm q}$ bound system,the Schrodinger wave equation with suitable boundary condition satisfies,

$$|\psi(0)|^2=\frac{\mu}{2\pi\hbar^2}\langle\frac{{\rm d}v}{{\rm d}r}\rangle, \tag{5}$$

here,

$$\langle\frac{{\rm d}v}{{\rm d}r}\rangle=\int_0^\infty\chi(r)\frac{{\rm d}v}{{\rm d}r}\chi(r){\rm d}r$$

is the expectation value of the potential where $\chi(r)$is the radial wavefunction and μ is the reduced mass of the q$\bar{\rm q}$ system. The decays of $^3S_1$ state into charged leptons proceeds through the virtual photon. The scattering cross section for q$\bar{\rm q}\rightarrow{\rm l}\bar{\rm l}$ is given by,

$$\sigma = \frac{4\pi\alpha^2}{3}\langle Q^2\rangle\frac{\beta_l}{s\beta_{\rm Q}}\left[1+\frac{2-\beta_{\rm Q}^2-\beta_{\rm l}^2}{2}+\frac{(1-\beta_{\rm Q}^2)(1-\beta_{\rm l}^2)}{4}\right], \tag{6}$$

where

$$\beta_{\rm l}=\frac{\sqrt{s-4m_{\rm l}^2}}{\sqrt{s}},\quad \beta_{\rm Q}=\frac{\sqrt{s-4m_{\rm Q}^2}}{\sqrt{s}}$$

and $s=E_{\rm CM}^2$, Q is the charge of the quark.

Decay rate in the limit $\beta_{\rm l}\rightarrow 1$ is given by

$$\Gamma=(\text{incident flux})\sigma=2\beta_{\rm Q}|\psi(0)|^2\frac{4}{3}\sigma. \tag{7}$$

Taking into account the colour

$$|V\rangle=\frac{1}{\sqrt{3}}\sum\nolimits_{\rm a}|\bar{Q}_{\rm a}Q_{\rm a}\rangle$$

and using Eq. (6) and Eq. (7)the non-relativistic ($\beta_{\rm Q}=0$) for leptonic decay width is given by [13–15],

$$\Gamma_{\rm l^+l^-}=16\pi\alpha^2\langle Q^2\rangle\frac{|\psi(0)|^2}{m_{\rm V}^2}. \tag{8}$$

3 Results and discussions

The masses of the VM after diagonalisation for successive values of $n_{\max}$ are listed in Table 2. We get a very good agreement with the experimental masses. Table 3 gives the predicted radially excited states of VM in comparison with the experimental masses [16]. The results predict decreasing spacing between radially excited states in the meson spectra which are in agrement with the experiment [16] and with the prediction of the other models [17, 18].

As has been discussed in previous sections, the leptonic decay width is proportional to the average value of the squared charge, squared wavefunction at the origin and the mass of the vector mesons. The average value of the squared charge for different vector mesons are calculated using the respective wavefunctions of the vector mesons. For example, for ρ meson

Table 2. The VM masses (in MeV) for successive value of $n_{\max}$.

$n_{\max}$	ρ	K*	ϕ
1	841.89	990.92	1142.35
2	809.71	951.60	1095.75
3	797.96	935.67	1074.45
4	784.62	918.18	1051.38
5	780.46	912.07	1042.26
6	777.86	908.15	1036.20
7	774.70	903.30	1031.80
8	773.83	901.96	1028.41
9	773.63	900.01	1026.22
10	772.63	898.17	1022.81
11	771.53	893.17	1019.47
Expt.	771.1	893.14	1019.42

Table 3. The dominant spectral composition and predicted masses of VM in MeV. The experiment masses are taken from the particle data group. The spectroscopic notation is $N^{2s+1}L_{\mathrm{J}}$, where the symbols have their usual meaning.

meson	mass	1^3S_1	2^3S_1	3^3S_1	4^3S_1
ρ	Theory	769.49	1454.89	1987.51	2339.12
	Experiment	768.0	1450.0	2110.0	
K*	Theory	893.02	1662.45	2289.0	2678.97
	Experiment	892.0	1680.0		
ϕ	Theory	1021.0	1687.56	2128.19	2456.14
	Experiment	1020.0	1680.0		

the wavefunction is $\frac{1}{\sqrt{2}}(\mathrm{u}\bar{\mathrm{u}}-\mathrm{d}\bar{\mathrm{d}})$ and the charge content is

$$\left|\frac{1}{\sqrt{2}}\left(\frac{2}{3}+\frac{1}{3}\right)\right|^2=\frac{1}{2}.$$

Similarly, the charge content for ω and ϕ are $\frac{1}{18}$ and $\frac{1}{9}$respectively. Using Eq. (8)we have computed leptonic decay widths of vector mesons. Table 4 and Table 5 give the calculated square of the wavefunction at the origin ($|\psi(0)|^2$) and the leptonic decay widths (Γ) of VM in comparison with experiment [16] which are in good agreement.

Table 4. Calculated values of the square of the wavefunction at the origin in comparison with experiment.

meson	experimental$\lvert\psi(0)\rvert^2$	calcualted $\lvert\psi(0)\rvert^2$
ρ	3.16×10^6 MeV^3	2.97×10^6 MeV^3
ω	2.47×10^6 MeV^3	2.97×10^6 MeV^3
ϕ	4.44×10^6 MeV^3	4.39×10^6 MeV^3

Table 5. Leptonic decay widths Γ of VM in KeV.

meson	experimental Γ	calculated Γ
ρ	7.04 ± 0.06	6.7
ω	0.60 ± 0.02	0.74
ϕ	1.27 ± 0.03	1.26

4 Conclusions

In this work, we have obtained the VM masses and their radially excited states and their leptonic decay widths in the frame work of NRQM. It is shown that the computation of mesonic masses/mass splittings using OGEP is adequate for obtaining the masses of VM. The contribution from the off-diagonal elements is found to be significant. The calculation clearly indicates that masses of the VM converge to the experimental values when the diagonalisation is performed in a larger basis. The same set of parameters used to obtain masses of VM reproduce the leptonic decay widths. This work could be extended to heavy meson sector. Work in this direction is in progress.

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References

1 Gromes D. Nucl. Phys. B, 1977, **130**: 18–28
2 Greenberg O W. Ann. Rev. Nucl. Part. Sci., 1978, **28**: 327–378
3 Bhaduri R K, Cohler L E, Nogami Y. Nuovo Cimento. A, 1981, **65**: 376–390
4 Semay C, Silvestre-Brac B. Nucl. Phys. A, 1977, **618**: 455–482; Nucl. Phys. A, 1999, **647**: 72–96
5 Burakovsky L, Goldman T. Nucl. Phys. A, 1977, **625**: 220–230
6 De Rujula A, Georgi H, Glashow S L. Phys. Rev. D, 1975, **12**: 147–162
7 Bhavyashri, Vijaya Kumar K B, Hanumaiah B, Pepin S. J. Phys. G, 2005, **31**: 981-986
8 Bhavyashri, Sarangi S, Godfrey Saldanha, Vijaya Kumar K B. Pramana, 2008, **70**: 75–85
9 Vijaya Kumar K B, Bhavyshri, Ma Y L, Monteiro A P. Int. J. Mod. Phys. A, 2009, **24**: 4209–4220
10 Khadkikar S B, Gupta S K. Phys. Lett. B, 1983, **124**: 523–526
11 Capstick S, Isgur N. Phys. Rev. D, 1986, **34**: 2809–2835
12 Narodetskii I M et al. J. Phys. G, 1992, **18**: 1901–1909
13 Van Royen R, Weisskopf V F. Nuovo Cimento. A, 1967, **50**: 617–645
14 Quigg C, Rosner J L. Phys. lett. B, 1977, **71**: 153–157
15 Quigg C, Rosner J L. Phys. Rep., 1979, **56**: 167–235
16 Amsler C et al. PDG, Phys. Lett. B, 1988, **667**: 1–298
17 Ferreira P L, Zagury N. Nuovo Cimento. A, 1977, **20**: 511–522
18 Godfrey S, Isgur N. Phys. ReV. D, 1985, **32**: 189–231