

Molecular and compact four-quark states^{*}

A. Valcarce^{1;1)} J. Vijande^{2;2)}

¹ Departamento de Física Fundamental, Universidad de Salamanca, Salamanca, Spain

² Departamento de Física Atómica, Molecular y Nuclear,
Universidad de Valencia (UV) and IFIC (UV-CSIC), Valencia, Spain

Abstract We study charmonium ($c\bar{c}n\bar{n}$), bottomonium ($b\bar{b}n\bar{n}$) and exotic ($cc\bar{n}\bar{n}$ and $bb\bar{n}\bar{n}$) four-quark states by means of a standard non-relativistic quark potential model. We look for possible bound states. Among them we are able to distinguish between meson-meson molecules and compact four-quark states.

Key words tetraquarks, quark-models, charmonium, exotics

PACS 14.40.Nd, 14.40.Lb, 14.40.-n

1 Introduction

The discoveries on several fronts of unusual charmonium states like X(3872) and Y(4260) and open-charm mesons with unexpected masses like D_{sJ}^* (2317) and D_0^* (2308), have re-invigorated the study of the hadron spectra. Their anomalous nature has triggered several interpretations, among them, the existence of compact four-quark states or meson-meson molecules. This challenging situation resembles the long-standing problem of the light-scalar mesons, where it has been suggested that some resonances may not be ordinary $q\bar{q}$ states, though there is little agreement on what they actually are [1]. In this case, four-quark states have been justified to coexist with $q\bar{q}$ states because they can couple to $J^{PC} = 0^{++}$ without orbital excitation [2].

Four-quark systems present a richer color structure than standard baryons or mesons. Although the color wave function for ordinary mesons and baryons leads to a single vector, working with four-quark states there are different vectors driving to a singlet color state out of colorless or colored quark-antiquark two-body components. Thus, in dealing with four-

quark states an important question is whether we are in front of a colorless meson-meson molecule or a compact state (i.e., a system with two-body colored components). Whereas the first structure would be natural in the naive quark model, the second one would open a new area in hadron spectroscopy.

We have derived the necessary formalism to evaluate the physical channels probability (singlet-singlet color states) in an arbitrary four-quark wave function. For this purpose we have expanded any hidden-color vector of the four-quark state color basis, i.e., vectors with non-singlet internal color couplings, in terms of singlet-singlet color vectors. Such a procedure gives rise to a wave function expanded in terms of color singlet-singlet nonorthogonal vectors, where the determination of the probability of physical channels becomes cumbersome. However, it allows to differentiate among unbound, compact and molecular four-quark states.

2 Formalism

There are three different ways of coupling two quarks and two antiquarks into a colorless state:

$$[(q_1 q_2)(\bar{q}_3 \bar{q}_4)] \equiv \{|\bar{3}_{12} 3_{34}\rangle, |6_{12} \bar{6}_{34}\rangle\} \equiv \{|\bar{3}3\rangle_c^{12}, |6\bar{6}\rangle_c^{12}\} \quad (1a)$$

$$[(q_1 \bar{q}_3)(q_2 \bar{q}_4)] \equiv \{|\bar{1}_{13} 1_{24}\rangle, |8_{13} 8_{24}\rangle\} \equiv \{|\bar{1}1\rangle_c, |88\rangle_c\} \quad (1b)$$

$$[(q_1 \bar{q}_4)(q_2 \bar{q}_3)] \equiv \{|\bar{1}_{14} 1_{23}\rangle, |8_{14} 8_{23}\rangle\} \equiv \{|\bar{1}'1'\rangle_c, |8'8'\rangle_c\}, \quad (1c)$$

Received 26 January 2010

^{*} This work has been partially funded by Ministerio de Ciencia y Tecnología under Contract No. FPA2007-65748, by EU FEDER, and by Junta de Castilla y León under Contracts No. SA016A17 and GR12

1) E-mail: valcarce@usal.es

2) E-mail: javier.vijande@uv.es

©2010 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

each forms an orthonormal basis. Each coupling scheme allows us to define a color basis where the four-quark problem can be solved. The first basis, Eq. (1a), being the most suitable one to deal with the Pauli principle is made entirely of vectors containing hidden-color components. The other two, Eqs. (1b) and (1c), are hybrid bases containing singlet-singlet (physical) and octet-octet (hidden-color) vectors.

An arbitrary four-quark wave function can be expressed in terms of physical components, singlet-singlet color states, by means of an infinite expansion than can be resummed as [3],

$$|\Psi\rangle = \frac{1}{2} \left(P\hat{Q} + \hat{Q}P \right) \frac{1}{1 - \cos^2 \alpha} |\Psi\rangle + \frac{1}{2} \left(\hat{P}Q + Q\hat{P} \right) \frac{1}{1 - \cos^2 \alpha} |\Psi\rangle. \quad (2)$$

Thus, one obtains two hermitian operators that are well-defined projectors on the two physical singlet-

singlet color states

$$\mathcal{P}_{|11\rangle_c} = \left(P\hat{Q} + \hat{Q}P \right) \frac{1}{2(1 - \cos^2 \alpha)},$$

$$\mathcal{P}_{|1'1'\rangle_c} = \left(\hat{P}Q + Q\hat{P} \right) \frac{1}{2(1 - \cos^2 \alpha)}. \quad (3)$$

Thus, given an arbitrary state $|\Psi\rangle$ its projection on a particular subspace E is given by $|\Psi\rangle|_E = P_E|\Psi\rangle$. Then, the probability of finding such a state on this subspace is

$$E \langle \Psi | \Psi \rangle |_E = \langle \Psi | P_E^\dagger P_E | \Psi \rangle = \langle \Psi | P_E^2 | \Psi \rangle = \langle \Psi | P_E | \Psi \rangle. \quad (4)$$

Therefore, once the projection operators have been constructed [Eq. (3)], the probabilities for finding singlet-singlet components are given by,

$$P^{|\Psi\rangle}([11]) = \langle \Psi | \mathcal{P}_{|11\rangle_c} | \Psi \rangle,$$

$$P^{|\Psi\rangle}([1'1']) = \langle \Psi | \mathcal{P}_{|1'1'\rangle_c} | \Psi \rangle. \quad (5)$$

Using Eq. (3) it can be easily checked that $P^{|\Psi\rangle}([11]) + P^{|\Psi\rangle}([1'1']) = 1$, where

$$P^{|\Psi\rangle}([11]) = \frac{1}{2(1 - \cos^2 \alpha)} \left[\langle \Psi | P\hat{Q} | \Psi \rangle + \langle \Psi | \hat{Q}P | \Psi \rangle \right],$$

$$P^{|\Psi\rangle}([1'1']) = \frac{1}{2(1 - \cos^2 \alpha)} \left[\langle \Psi | \hat{P}Q | \Psi \rangle + \langle \Psi | Q\hat{P} | \Psi \rangle \right]. \quad (6)$$

3 Results

The stability of a four-quark state can be analyzed in terms of Δ_E , the energy difference between its mass and that of the lowest two-meson threshold,

$$\Delta_E = E_{4q} - E(M_1, M_2), \quad (7)$$

where E_{4q} stands for the four-quark energy and $E(M_1, M_2)$ for the energy of the two-meson threshold. Thus, $\Delta_E < 0$ indicates all fall-apart decays are forbidden, and therefore one has a proper bound state. $\Delta_E \geq 0$ will indicate that the four-quark solution corresponds to an unbound threshold (two free mesons). Thus, an energy above the threshold would simply mean that the system is unbound within our variational approximation, suggesting that the minimum of the Hamiltonian is at the two-meson threshold.

Another helpful tool analyzing the structure of a four-quark state is the value of the root mean square radii: $\langle x^2 \rangle^{1/2}$, $\langle y^2 \rangle^{1/2}$ and $\langle z^2 \rangle^{1/2}$. Bound four-quark states can be distinguished from two free mesons by

means of their root mean square radius

$$\text{RMS}_{4q(2q)} = \left(\frac{\sum_{i=1}^{4(2)} m_i \langle (r_i - R)^2 \rangle}{\sum_{i=1}^{4(2)} m_i} \right)^{1/2}, \quad (8)$$

and in particular, their corresponding ratio,

$$\Delta_R = \frac{\text{RMS}_{4q}}{\text{RMS}_{M_1} + \text{RMS}_{M_2}}, \quad (9)$$

where $\text{RMS}_{M_1} + \text{RMS}_{M_2}$ stands for the sum of the radii of the mesons corresponding to the lowest threshold.

We show in Table 1 the results obtained for several different four-quark states in the bottom and charm sectors. One can see how independently of their binding energy, all of them present a sizable octet-octet component when the wave function is expressed in the (1b) coupling. Let us first of all concentrate on the two unbound states, $\Delta_E > 0$, one with $S_T = 0$ and one with $S_T = 1$, given in Table 1. The octet-octet component of basis (1b) can be expanded in terms of the vectors of basis (1c) as explained in the previous section. Thus, once expressions (6) are considered one finds that the probabilities are concentrated into a single physical channel, MM or MM*.

In other words, the octet-octet component of the basis (1b) or (1c) is a consequence of having identical quarks and antiquarks. Thus, four-quark unbound states are represented by two isolated mesons. This conclusion is strengthened when studying the

root mean square radii, leading to a picture where the two quarks and the two antiquarks are far away, $\langle x^2 \rangle^{1/2} \gg 1$ fm and $\langle y^2 \rangle^{1/2} \gg 1$ fm, whereas the quark-antiquark pairs are located at a typical distance for a meson, $\langle z^2 \rangle^{1/2} \leq 1$ fm.

Table 1. Four-quark state properties for selected quantum numbers. All states have positive parity and total orbital angular momentum $L = 0$. Energies are given in MeV and distances in fm. The notation $M_1 M_2 |_\ell$ stands for mesons M_1 and M_2 with a relative orbital angular momentum ℓ . $P[|\bar{3}\bar{3}\rangle_c^{12}(|\bar{6}\bar{6}\rangle_c^{12})]$ stands for the probability of the $\bar{3}\bar{3}(\bar{6}\bar{6})$ components given in Eq. (1a) and $P[|11\rangle_c(|88\rangle_c)]$ for the $11(88)$ components given in Eq. (1b). P_{MM} , P_{MM^*} and $P_{M^*M^*}$ stand for the probability of two pseudoscalar, pseudoscalar-vector or two vector mesons.

(S_T, I)	(0,1)	(1,1)	(1,0)	(1,0)	(0,0)
Flavor	$cc\bar{n}\bar{n}$	$cc\bar{n}\bar{n}$	$cc\bar{n}\bar{n}$	$bb\bar{n}\bar{n}$	$bb\bar{n}\bar{n}$
Energy	3877	3952	3861	10395	10948
Threshold	$DD _S$	$DD^* _S$	$DD^* _S$	$BB^* _S$	$B_1B _P$
Δ_E	+5	+15	-76	-217	-153
$P[\bar{3}\bar{3}\rangle_c^{12}]$	0.333	0.333	0.881	0.974	0.981
$P[\bar{6}\bar{6}\rangle_c^{12}]$	0.667	0.667	0.119	0.026	0.019
$P[11\rangle_c]$	0.556	0.556	0.374	0.342	0.340
$P[88\rangle_c]$	0.444	0.444	0.626	0.658	0.660
P_{MM}	1.000	—	—	—	0.254
P_{MM^*}	—	1.000	0.505	0.531	—
$P_{M^*M^*}$	0.000	0.000	0.495	0.469	0.746
$\langle x^2 \rangle^{1/2}$	60.988	13.804	0.787	0.684	0.740
$\langle y^2 \rangle^{1/2}$	60.988	13.687	0.590	0.336	0.542
$\langle z^2 \rangle^{1/2}$	0.433	0.617	0.515	0.503	0.763
RMS_{4q}	30.492	6.856	0.363	0.217	0.330
Δ_R	69.300	11.640	0.799	0.700	0.885

Let us now turn to the bound states shown in Table 1, $\Delta_E < 0$, one in the charm sector and two in the bottom one. In contrast to the results obtained for unbound states, when the octet-octet component of basis (1b) is expanded in terms of the vectors of basis (1c), equations (6) indicate a picture where the probabilities in all allowed physical channels are relevant. It is clear that the bound state must be generated by an interaction that it is not present in the asymptotic channel, sequestering probability from a single singlet-singlet color vector from the interaction between color octets. Such systems are clear examples of compact four-quark states, in other words, they cannot be expressed in terms of a single physical channel. Moreover, as can be seen in Table 1, their typical sizes point to compact objects 20% smaller than a standard two-meson system.

We have studied the dependence of the probability of a physical channel on the binding energy. For this purpose we have considered the simplest system from the numerical point of view, the $(S_T, I) = (0, 1)$ $cc\bar{n}\bar{n}$ state. Unfortunately, this state is unbound

for any reasonable set of parameters. Therefore, we bind it by multiplying the interaction between the light quarks by a fudge factor. Such a modification does not affect the two-meson threshold while it decreases the mass of the four-quark state. The results are illustrated in Fig. 1, showing how in the $\Delta_E \rightarrow 0$ limit, the four-quark wave function is almost a pure single physical channel. Close to this limit one would find what could be defined as molecular states. Moreover, the size of the four-quark state increases when $\Delta_E \rightarrow 0$. When the probability concentrates into a single physical channel ($P_{MM} \rightarrow 1$) the size of the system gets larger than the sum of two isolated mesons [3]. We have identified the subsystems responsible for increasing the size of the four-quark state. Quark-quark ($\langle x^2 \rangle^{1/2}$) and antiquark-antiquark ($\langle y^2 \rangle^{1/2}$) distances grow rapidly while the quark-antiquark distance ($\langle z^2 \rangle^{1/2}$) remains almost constant. This reinforces our previous result, pointing to the appearance of two-meson-like structures whenever the binding energy goes to zero.

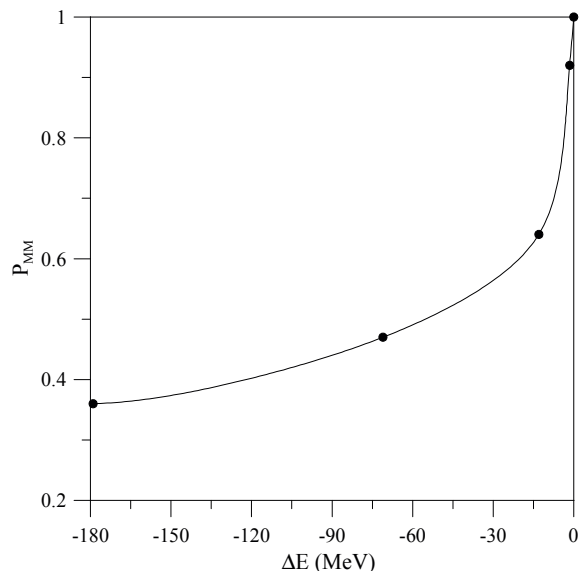


Fig. 1. P_{MM} as a function of ΔE .

Although the present analysis has been performed by means of a particular quark interacting potential [4], the conclusions derived are independent of the quark-quark interaction used. They mainly rely on using the same hamiltonian to describe tensors of different order, two and four-quark components in the present case. When dealing with a complete basis, any four-quark deeply bound state has to be compact. Only slightly bound systems could be considered as molecular. Unbound states correspond to a two-meson system. A similar situation would be found in the two baryon system, the deuteron could be considered as a molecular-like state with a small percentage of its wave function on the $\Delta\Delta$ channel, whereas the H -dibaryon would be a compact six-quark state. When working with central forces, the only way of getting a bound system is to have a strong interaction between the constituents that are far apart in the asymptotic limit (quarks or antiquarks in the present case). In this case the short-range interaction will capture part of the probability of a two-meson threshold to form a bound state. This can be reinterpreted as an infinite sum over physical states. This is why

the analysis performed here is so important before any conclusion can be made concerning the existence of compact four-quark states beyond simple molecular structures.

If the prescription of using the same hamiltonian to describe all tensors in the Fock space is relaxed, new scenarios may appear. Among them, the inclusion of many-body forces is particularly relevant. In Ref. [5] the stability of $QQ\bar{q}\bar{n}$ and $Q\bar{Q}n\bar{n}$ systems was analyzed in a simple string model considering only a multiquark confining interaction given by the minimum of a flip-flop or a butterfly potential in an attempt to discern whether confining interactions not factorizable as two-body potentials would influence the stability of four-quark states. The ground state of systems made of two quarks and two antiquarks of equal masses was found to be below the dissociation threshold. While for the cryptoexotic $Q\bar{Q}n\bar{n}$ the binding decreases when increasing the mass ratio m_Q/m_n , for the flavor exotic $QQ\bar{q}\bar{n}$ the effect of mass symmetry breaking is opposite. Others scenarios may emerge if different many-body forces, like many-body color interactions [6] or 't Hooft instanton-based three-body interactions [7], are considered.

4 Conclusions

We have discussed the formalism to express the wave function of a general four-quark state in terms of physical channels, i.e., those constructed by using color singlet-singlet states. We have studied charmonium ($c\bar{c}n\bar{n}$), bottomonium ($b\bar{b}n\bar{n}$) and exotic ($c\bar{c}n\bar{n}$ and $b\bar{b}n\bar{n}$) four-quark states by means of a standard non-relativistic quark potential model. We look for possible bound states. Among them we are able to distinguish between meson-meson molecules and compact four-quark states. The importance of performing a complete analysis of the system, energy and wave function, in the vicinity of a two-meson threshold has been emphasized.

References

- 1 Amsler C, Tornqvist N A. Phys. Rep., 2004, **389**: 61
- 2 Jaffe R L. Phys. Rep., 2005, **409**: 1
- 3 Vijande J, Valcarce A. Phys. Rev. C, 2005, **72**: 025206
- 4 Vijande J, Fernández F, Valcarce A. J. Phys. G, 2005, **31**: 481
- 5 Vijande J, Valcarce A, Richard J M. Phys. Rev. D, 2007, **76**: 114013; Ay C, Richard J M, Rubinstein J H. Phys. Lett. B, 2009, **674**: 227
- 6 Dmitrašinović V. Phys. Lett. B, 2001, **499**: 135; Phys. Rev. D, 2003, **67**: 114007
- 7 't Hooft G. Phys. Rev. D, 1976, **14**: 3432