

# Muon $g-2$ discrepancy: new physics or a relatively light Higgs? \*

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**Abstract** After a brief review of the muon  $g-2$  status, we discuss hypothetical errors in the Standard Model prediction that might explain the present discrepancy with the experimental value. None of them seems likely. In particular, a hypothetical increase of the hadroproduction cross section in low-energy  $e^+e^-$  collisions could bridge the muon  $g-2$  discrepancy, but it is shown to be unlikely in view of current experimental error estimates. If, nonetheless, this turns out to be the explanation of the discrepancy, then the 95% CL upper bound on the Higgs boson mass is reduced to about 135 GeV which, in conjunction with the experimental 114.4 GeV 95% CL lower bound, leaves a narrow window for the mass of this fundamental particle.

**Key words** muon anomalous magnetic moment, Standard Model Higgs boson

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## 1 Introduction: status of $a_\mu$

The anomalous magnetic moment of the muon,  $a_\mu$ , is one of the most interesting observables in particle physics. Indeed, as each sector of the Standard Model (SM) contributes in a significant way to its theoretical prediction, the precise  $a_\mu$  measurement by the E821 experiment at Brookhaven [1, 2] allows us to test the entire SM and scrutinize viable “new physics” appendages to this theory [3, 4].

The SM prediction of the muon  $g-2$  is conveniently split into QED, electroweak (EW) and hadronic (leading- and higher-order) contributions:  $a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HLO}} + a_\mu^{\text{HHO}}$ . The QED prediction, computed up to four (and estimated at five) loops, currently stands at  $a_\mu^{\text{QED}} = 116584718.08(15) \times 10^{-11}$  [5], while the EW effects provide  $a_\mu^{\text{EW}} = 154(2) \times 10^{-11}$  [6]. The latest calculations of the hadronic leading-order contribution, via the hadronic  $e^+e^-$  annihilation data, are in agreement:  $a_\mu^{\text{HLO}} = 6894(40) \times 10^{-11}$  [7] (this preliminary result, presented at this workshop, updates the value  $6894(46) \times 10^{-11}$

of Ref. [8]) and  $6903(53) \times 10^{-11}$  [9]. These determinations include the 2008  $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$  cross section data from KLOE [10] (see also [11]). A somewhat larger value,  $6955(41) \times 10^{-11}$  [12], was recently obtained including also the 2009  $\pi^+\pi^-(\gamma)$  data of BABAR [13].

The higher-order hadronic term is further divided into two parts:  $a_\mu^{\text{HHO}} = a_\mu^{\text{HHO}}(\text{vp}) + a_\mu^{\text{HHO}}(\text{lbl})$ . The first one,  $-98(1) \times 10^{-11}$  [8], is the  $O(\alpha^3)$  contribution of diagrams containing hadronic vacuum polarization insertions [14]. The second term, also of  $O(\alpha^3)$ , is the hadronic light-by-light contribution; as it cannot be determined from data, its evaluation relies on specific models. The latest determinations of this term,  $116(39) \times 10^{-11}$  [9, 15] and  $105(26) \times 10^{-11}$  [16], are in very good agreement. If we add the latter to  $a_\mu^{\text{HLO}}$ , for example the value of Ref. [7], and the rest of the SM contributions, we obtain  $a_\mu^{\text{SM}} = 116591773(48) \times 10^{-11}$ . The difference with the experimental value  $a_\mu^{\text{EXP}} = 116592089(63) \times 10^{-11}$  [2] (note the tiny shift upwards, with respect to the value reported in [1], due to the updated value of the muon-proton magnetic moment

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ratio [17] is  $\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = +316(79) \times 10^{-11}$ , i.e.,  $4.0\sigma$  (all errors were added in quadrature). Slightly smaller discrepancies are found employing the  $a_\mu^{\text{HLO}}$  values reported in [12] (which also includes the recent  $\pi^+\pi^-(\gamma)$  data of BABAR) and [9]:  $3.2\sigma$  and  $3.6\sigma$ , respectively. We will use the  $a_\mu^{\text{HLO}}$  value of Ref. [7] (which also provides the hadronic contribution to the effective fine-structure constant later required for our analysis), but we expect that a consistent inclusion of the recent  $\pi^+\pi^-(\gamma)$  BABAR data would not change our basic conclusions. For reviews of  $a_\mu$  see Refs. [7, 9, 18].

The term  $a_\mu^{\text{HLO}}$  can alternatively be computed incorporating hadronic  $\tau$ -decay data, related to those of hadroproduction in  $e^+e^-$  collisions via isospin symmetry [19]. The long-standing difference between the  $e^+e^-$ - and  $\tau$ -based determinations of  $a_\mu^{\text{HLO}}$  [20] has been recently somewhat lessened by a re-analysis [21] where the isospin-breaking corrections [22] were revisited taking advantage of more accurate data and new theoretical investigations (recent  $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$  data from the Belle experiment [23] were also included). In spite of this, the  $\tau$ -based value remains higher than the  $e^+e^-$ -based one, leading to a smaller ( $1.9\sigma$ ) difference  $\Delta a_\mu$ . On the other hand, recent analyses of the pion form factor claim that the  $\tau$  and  $e^+e^-$  data are consistent after isospin violation effects and vector meson mixings are considered, further confirming the  $e^+e^-$ -based discrepancy [24].

The  $3\text{--}4\sigma$  discrepancy between the theoretical prediction and the experimental value of the muon  $g-2$  can be explained in several ways. It could be due, at least in part, to an error in the determination of the hadronic light-by-light contribution. However, if this were the only cause of the discrepancy,  $a_\mu^{\text{HLO}}$  (lbl) would have to move up by many standard deviations (roughly ten) to fix it. Although the errors assigned to  $a_\mu^{\text{HLO}}$  (lbl) are only educated guesses, this solution seems unlikely, at least as the dominant one.

Another possibility is to explain the discrepancy  $\Delta a_\mu$  via the QED, EW and hadronic higher-order vacuum polarization contributions; this looks very improbable, as one can immediately conclude inspecting their values and uncertainties reported above. If we assume that the  $g-2$  experiment E821 is correct, we are left with two options: possible contributions of physics beyond the SM, or an erroneous determination of the leading-order hadronic contribution  $a_\mu^{\text{HLO}}$  (or both). The first of these two explanations has been extensively discussed in the literature; updating Ref. [25] we will study whether the second one

is realistic or not, and analyze its implications for the EW bounds on the mass of the Higgs boson.

## 2 Connection with the Higgs mass

The hadronic leading-order contribution  $a_\mu^{\text{HLO}}$  can be computed via the dispersion integral [26]

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma(s), \quad (1)$$

where  $\sigma(s)$  is the total cross section for  $e^+e^-$  annihilation into any hadronic state, with vacuum polarization and initial state QED corrections subtracted off (for a detailed discussion of these radiative corrections and the precision of the Monte Carlo generators used to analyze the hadronic cross section measurements see [27]), and  $s$  is the squared momentum transfer. The well-known kernel function  $K(s)$  (see [28]) is positive definite, decreases monotonically for increasing  $s$  and, for large  $s$ , behaves as  $m_\mu^2/(3s)$  to a good approximation. About 90% of the total contribution to  $a_\mu^{\text{HLO}}$  is accumulated at center-of-mass energies  $\sqrt{s}$  below 1.8 GeV and roughly three-fourths of  $a_\mu^{\text{HLO}}$  is covered by the two-pion final state which is dominated by the  $\rho(770)$  resonance [12]. Exclusive low-energy  $e^+e^-$  cross sections were measured at colliders in Frascati, Novosibirsk, Orsay, and Stanford, while at higher energies the total cross section was determined inclusively.

Let's now assume that the discrepancy  $\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = +316(79) \times 10^{-11}$ , is due to – and only to – hypothetical errors in  $\sigma(s)$ , and let us increase this cross section in order to raise  $a_\mu^{\text{HLO}}$ , thus reducing  $\Delta a_\mu$ . This simple assumption leads to interesting consequences. An upward shift of the hadronic cross section also induces an increase of the value of the hadronic contribution to the effective fine-structure constant at  $M_Z$  [29],

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z) = \frac{M_Z^2}{4\alpha\pi^2} P \int_{4m_\pi^2}^{\infty} ds \frac{\sigma(s)}{M_Z^2 - s} \quad (2)$$

( $P$  stands for Cauchy's principal value). This integral is similar to the one we encountered in Eq. (1) for  $a_\mu^{\text{HLO}}$ . There, however, the weight function in the integrand gives a stronger weight to low-energy data. Let us define

$$a_i = \int_{4m_\pi^2}^{s_u} ds f_i(s) \sigma(s) \quad (3)$$

( $i = 1, 2$ ), where the upper limit of integration is  $s_u < M_Z^2$ , and the kernels are  $f_1(s) = K(s)/(4\pi^3)$  and  $f_2(s) = [M_Z^2/(M_Z^2 - s)]/(4\alpha\pi^2)$ . The integrals  $a_i$  with  $i = 1, 2$  provide the contributions to  $a_\mu^{\text{HLO}}$  and  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ , respectively, from  $4m_\pi^2$  up to  $s_u$  (see Eqs. (1,2)). An increase of the cross section  $\sigma(s)$  of the form

$$\Delta\sigma(s) = \epsilon\sigma(s) \quad (4)$$

in the energy range  $\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$ , where  $\epsilon$  and  $\delta$  are positive constants and  $2m_\pi + \delta/2 < \sqrt{s_0} < \sqrt{s_u} - \delta/2$ , increases  $a_1$  by  $\Delta a_1(\sqrt{s_0}, \delta, \epsilon) = \epsilon \int_{\sqrt{s_0} - \delta/2}^{\sqrt{s_0} + \delta/2} 2t \sigma(t^2) f_1(t^2) dt$ . If we assume that the muon  $g-2$  discrepancy is entirely due to this increase in  $\sigma(s)$ , so that  $\Delta a_1(\sqrt{s_0}, \delta, \epsilon) = \Delta a_\mu$ , the parameter  $\epsilon$  becomes

$$\epsilon = \frac{\Delta a_\mu}{\int_{\sqrt{s_0} - \delta/2}^{\sqrt{s_0} + \delta/2} 2t f_1(t^2) \sigma(t^2) dt}, \quad (5)$$

and the corresponding increase in  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$  is

$$\Delta a_2(\sqrt{s_0}, \delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0} - \delta/2}^{\sqrt{s_0} + \delta/2} f_2(t^2) \sigma(t^2) t dt}{\int_{\sqrt{s_0} - \delta/2}^{\sqrt{s_0} + \delta/2} f_1(t^2) \sigma(t^2) t dt}. \quad (6)$$

The shifts  $\Delta a_2(\sqrt{s_0}, \delta)$  were studied in Ref. [25] for several bin widths  $\delta$  and central values  $\sqrt{s_0}$ .

The present global fit of the LEP Electroweak Working Group (EWWG) leads to the Higgs boson mass  $M_H = 87_{-26}^{+35}$  GeV and the 95% confidence level (CL) upper bound  $M_H^{\text{UB}} \simeq 157$  GeV [30]. This result is based on the recent preliminary top quark mass  $M_t = 173.1(1.3)$  GeV [31] and the value  $\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02758(35)$  [32]. The LEP direct-search 95% CL lower bound is  $M_H^{\text{LB}} = 114.4$  GeV [33]. Although the global EW fit employs a large set of observables,  $M_H^{\text{UB}}$  is strongly driven by the comparison of the theoretical predictions of the W boson mass and the effective EW mixing angle  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  with their precisely measured values. Convenient formulae providing the  $M_W$  and  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  SM predictions in terms of  $M_H$ ,  $M_t$ ,  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ , and  $\alpha_s(M_Z)$ , the strong coupling constant at the scale  $M_Z$ , are given in [34]. Combining these two predictions via a numerical  $\chi^2$ -analysis and using the present world-average values  $M_W =$

$80.399(23)$  GeV [35],  $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23153(16)$  [36],  $M_t = 173.1(1.3)$  GeV [31],  $\alpha_s(M_Z) = 0.118(2)$  [37], and the determination  $\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02758(35)$  [32], we get  $M_H = 92_{-28}^{+37}$  GeV and  $M_H^{\text{UB}} = 158$  GeV. We see that indeed the  $M_H$  values obtained from the  $M_W$  and  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  predictions are quite close to the results of the global analysis.

The  $M_H$  dependence of  $a_\mu^{\text{SM}}$  is too weak to provide  $M_H$  bounds from the comparison with the measured value. On the other hand,  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$  is one of the key inputs of the EW fits. For example, employing the latest (preliminary) value  $\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02760(15)$  presented at this workshop [7] instead of  $0.02758(35)$  [32], the  $M_H$  prediction derived from  $M_W$  and  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  shifts to  $M_H = 96_{-25}^{+32}$  GeV and  $M_H^{\text{UB}} = 153$  GeV. To update the analysis of Ref. [25] we considered the new values of  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$  obtained shifting  $0.02760(15)$  [7] by  $\Delta a_2(\sqrt{s_0}, \delta)$  (including their uncertainties, as discussed in [25]), and computed the corresponding new values of  $M_H^{\text{UB}}$  via the combined  $\chi^2$ -analysis based on the  $M_W$  and  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  inputs (for both  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$  and  $a_\mu^{\text{HLO}}$  we used the values reported in Ref. [7]). Our results show that an increase  $\epsilon\sigma(s)$  of the hadronic cross section (in  $\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$ ), adjusted to bridge the muon  $g-2$  discrepancy  $\Delta a_\mu$ , decreases  $M_H^{\text{UB}}$ , further restricting the already narrow allowed region for  $M_H$ . We conclude that these hypothetical shifts conflict with the lower limit  $M_H^{\text{LB}}$  when  $\sqrt{s_0} \gtrsim 1.2$  GeV, for values of  $\delta$  up to several hundreds of MeV. In [25] we pointed out that there are more complex scenarios where it is possible to bridge the  $\Delta a_\mu$  discrepancy without significantly affecting  $M_H^{\text{UB}}$ , but they are considerably more unlikely than those discussed above.

If  $\tau$  data are used instead of  $e^+e^-$  ones in the calculation of the dispersive integral in Eq. (1),  $a_\mu^{\text{HLO}}$  increases to  $7053(45) \times 10^{-11}$  [21] and the discrepancy drops to  $\Delta a_\mu = +157(82) \times 10^{-11}$ , i.e.  $1.9\sigma$ . While using  $\tau$  data reduces the  $\Delta a_\mu$  discrepancy, it increases  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$  by approximately  $2 \times 10^{-4}$  <sup>1)</sup>, leading to a sharply lower  $M_H$  prediction [38]. Indeed, increasing the previously employed value  $\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02760(15)$  [7] by  $2 \times 10^{-4}$  and using the same above-discussed previous inputs of the  $\chi^2$ -analysis, we find an  $M_H^{\text{UB}}$  value of only 138 GeV. If the remaining  $1.9\sigma$  discrepancy  $\Delta a_\mu$  is bridged by a further increase  $\Delta\sigma(s) = \epsilon\sigma(s)$  of the hadronic cross section,  $M_H^{\text{UB}}$  decreases to even lower values, leading to a scenario in near conflict with  $M_H^{\text{LB}}$ .

Recent analyses of the pion form factor below 1 GeV claim that  $\tau$  data are consistent with the  $e^+e^-$

1) This number represents our rough update of the value reported in Ref. [20].

ones after isospin violation effects and vector meson mixings are considered [24]. In this case one could use the  $e^+e^-$  data below  $\sim 1$  GeV, confirmed by the  $\tau$  ones, and assume that  $\Delta a_\mu$  is accommodated by hypothetical errors in the  $e^+e^-$  measurements occurring above  $\sim 1$  GeV, where disagreement persists between these two data sets. Our analysis shows that this assumption would lead to  $M_H^{\text{UB}}$  values inconsistent with  $M_H^{\text{LB}}$ .

In the above analysis, the hadronic cross section  $\sigma(s)$  was shifted up by amounts  $\Delta\sigma(s) = \epsilon\sigma(s)$  adjusted to bridge  $\Delta a_\mu$ . Apart from the implications for  $M_H$ , these shifts may actually be inadmissibly large when compared with the quoted experimental uncertainties. Consider the parameter  $\epsilon = \Delta\sigma(s)/\sigma(s)$ . Clearly, its value depends on the choice of the energy range  $[\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2]$  where  $\sigma(s)$  is increased and, for fixed  $\sqrt{s}_0$ , it decreases when  $\delta$  increases. Its minimum value,  $\sim 5\%$ , occurs if  $\sigma(s)$  is multiplied by  $(1 + \epsilon)$  in the whole integration region, from  $2m_\tau$  to infinity. Such a shift would lead to  $M_H^{\text{UB}} \sim 75$  GeV, well below  $M_H^{\text{LB}}$ . Higher values of  $\epsilon$  are obtained for narrower energy bins, particularly if they do not include the  $\rho$ - $\omega$  resonance region. For example, a huge  $\epsilon \sim 55\%$  increase is needed to accommodate  $\Delta a_\mu$  with a shift of  $\sigma(s)$  in the region from  $2m_\tau$  up to 500 MeV (reducing  $M_H^{\text{UB}}$  to 146 GeV), while an increase in a bin of the same size but centered at the  $\rho$  peak requires  $\epsilon \sim 9\%$  (lowering  $M_H^{\text{UB}}$  to 135 GeV). As the quoted experimental uncertainty of  $\sigma(s)$  below 1 GeV is of the order of a few per cent (or less, in some specific energy regions), the possibility to explain  $\Delta a_\mu$  with these shifts  $\Delta\sigma(s)$  appears to be unlikely. Lower values of  $\epsilon$  are obtained if the shifts occur in energy ranges centered around the  $\rho$ - $\omega$  resonances, but also this possibility looks unlikely, since it requires variations of  $\sigma(s)$  of at least  $\sim 6\%$ . If, however, such shifts  $\Delta\sigma(s)$  indeed turn out to be the solution of the  $\Delta a_\mu$  discrepancy, then  $M_H^{\text{UB}}$  is reduced to about 135 GeV.

It is interesting to note that in the scenario where  $\Delta a_\mu$  is due to hypothetical errors in  $\sigma(s)$ , rather than “new physics”, the reduced  $M_H^{\text{UB}} \lesssim 135$  GeV induces some tension with the approximate 95% CL lower bound  $M_H \gtrsim 120$  GeV required to ensure vacuum stability under the assumption that the SM is valid up to the Planck scale [39] (note, however, that this lower bound somewhat decreases when the vacuum is allowed to be metastable, provided its lifetime is longer than the age of the universe [40]). Thus, one could argue that this tension is, on its own, suggestive of physics beyond the SM.

We remind the reader that the present values of

$\sin^2 \theta_{\text{eff}}^{\text{lept}}$  derived from the leptonic and hadronic observables are respectively  $(\sin^2 \theta_{\text{eff}}^{\text{lept}})_l = 0.23113(21)$  and  $(\sin^2 \theta_{\text{eff}}^{\text{lept}})_h = 0.23222(27)$  [36]. In Ref. [25] we pointed out that the use of either of these values as an input parameter leads to inconsistencies in the SM framework that already require the presence of “new physics”. For this reason, we followed the standard practice of employing as input the world-average value for  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$  determined in the SM global analysis. Since  $M_H^{\text{UB}}$  also depends sensitively on  $M_t$ , in [25] we provided simple formulae to obtain the new values derived from different  $M_t$  inputs.

A 3–4 $\sigma$  discrepancy between the theoretical prediction and the experimental value of the muon  $g-2$  would have interesting implications if truly due to “new physics” (i.e. beyond the SM expectations). Supersymmetry provides a natural interpretation of this discrepancy (see Ref. [4] for a review). For illustration purposes, we assume a single mass  $m_{\text{susy}}$  for sleptons, sneutrinos and gauginos that enter the  $a_\mu^{\text{susy}}$  calculation. Then one finds [41] (including leading two-loop effects)

$$a_\mu^{\text{susy}} \simeq \text{sgn}(\mu) \times 130 \times 10^{-11} \left( \frac{100 \text{ GeV}}{m_{\text{susy}}} \right)^2 \tan\beta, \quad (7)$$

where  $\text{sgn}(\mu) = \pm$  is the sign of the  $\mu$  term in supersymmetry models and  $\tan\beta > 3-4$  is the ratio of the two scalar vacuum expectation values,  $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ . The  $\tan\beta$  factor is an important source of enhancement. As experimental constraints on the Higgs mass have increased, so has the lower bound on  $\tan\beta$ . With larger  $\tan\beta$  now required, it appears inevitable that supersymmetric loops have a fairly major effect on the theoretical prediction of the muon  $g-2$  if  $m_{\text{susy}}$  is not too large. In fact, equating (7) and the discrepancy  $\Delta a_\mu$ , for example the value  $\Delta a_\mu = +316(79) \times 10^{-11}$  obtained using the  $a_\mu^{\text{HLO}}$  determination of Ref. [7], one finds  $\text{sgn}(\mu) = +$  and

$$m_{\text{susy}} \simeq 64_{-7}^{+10} \sqrt{\tan\beta} \text{ GeV}. \quad (8)$$

For  $\tan\beta \sim 4-50$ , these values are in keeping with mainstream supersymmetric expectations. Several alternative “new physics” explanations have also been suggested [3].

### 3 Conclusions

We examined a number of hypothetical errors in the SM prediction of the muon  $g-2$  that could be responsible for the present 3–4 $\sigma$  discrepancy  $\Delta a_\mu$  with the experimental value. None of them looks likely. In

particular, updating Ref. [25] we showed how an increase  $\Delta\sigma(s)=\epsilon\sigma(s)$  of the hadroproduction cross section in low-energy  $e^+e^-$  collisions could bridge  $\Delta a_\mu$ . However, such increases lead to reduced  $M_H$  upper bounds – even lower than 114.4 GeV (the LEP lower bound) if they occur in energy regions centered above  $\sim 1.2$  GeV). Moreover, their amounts are generally very large when compared with the quoted experimental uncertainties, even if the latter were significantly underestimated. The possibility to bridge the muon  $g-2$  discrepancy with shifts of the hadronic cross section therefore appears to be unlikely. If, nonetheless, this turns out to be the solution, then the 95% CL upper bound  $M_H^{\text{UB}}$  drops to about 135 GeV.

If  $\tau$ -decay data are used instead of  $e^+e^-$  ones in the calculation of  $a_\mu^{\text{SM}}$ , the muon  $g-2$  discrepancy decreases to  $\sim 2\sigma$ . While this reduces  $\Delta a_\mu$ , it raises the value of  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$  leading to  $M_H^{\text{UB}}=138$  GeV, thus increasing the tension with the LEP lower bound and suggesting a near conflict with it should one try to overcome the full discrepancy. One could also consider a scenario, suggested by recent studies, where the  $\tau$  data confirm the  $e^+e^-$  ones below  $\sim 1$  GeV,

while a discrepancy between them persists at higher energies. If, in this case,  $\Delta a_\mu$  is fixed by hypothetical errors in the  $e^+e^-$  measurements above  $\sim 1$  GeV, where the data sets disagree, one also finds values of  $M_H^{\text{UB}}$  inconsistent with the LEP lower bound.

If the  $\Delta a_\mu$  discrepancy is real, it points to “new physics”, like low-energy supersymmetry where  $\Delta a_\mu$  is reconciled by the additional contributions of supersymmetric partners and one expects  $M_H \lesssim 135$  GeV for the mass of the lightest scalar [42]. If, instead, the deviation is caused by an incorrect leading-order hadronic contribution, it leads to reduced  $M_H^{\text{UB}}$  values. This reduction, together with the LEP lower bound, leaves a narrow window for the mass of this fundamental particle. Interestingly, it also raises the tension with the  $M_H$  lower bound derived in the SM from the vacuum stability requirement.

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## References

- 1 Bennett G W et al. Phys. Rev. D, 2006, **73**: 072003; Phys. Rev. Lett., 2004, **92**: 161802; 2002, **89**: 101804; 2002, **89**: 129903(E)
- 2 Roberts B L. These proceedings, arXiv:1001.2898 [hep-ex]
- 3 Czarnecki A, Marciano W J. Phys. Rev. D, 2001, **64**: 013014
- 4 Stöckinger D. J. Phys. G, 2007, **34**: R45
- 5 Kinoshita T, Nio M. Phys. Rev. D, 2006, **73**: 013003; D, 2004, **70**: 113001; D, 2006, **73**: 053007; Aoyama T et al. Phys. Rev. Lett., 2007, **99**: 110406; Phys. Rev. D, 2008, **77**: 053012; D, 2008, **78**: 053005; D, 2008, **78**: 113006; arXiv:1001.3704 [hep-ph]; Laporta S, Remiddi E. Phys. Lett. B, 1993, **301**: 440; B, 1996, **379**: 283; Passera M. Phys. Rev. D, 2007, **75**: 013002; Kataev A L. Phys. Rev. D, 2006, **74**: 073011
- 6 Czarnecki A, Marciano W J, Vainshtein A. Phys. Rev. D, 2003, **67**: 073006; D, 2006, **73**: 119901(E); Czarnecki A, Krause B, Marciano W J. Phys. Rev. D, 1995, **52**: 2619; Phys. Rev. Lett., 1996, **76**: 3267
- 7 Teubner T. These proceedings
- 8 Hagiwara K et al. Phys. Lett. B, 2007, **649**: 173
- 9 Jegerlehner F, Nyffeler A. Phys. Rept., 2009, **477**: 1
- 10 Ambrosino F et al (KLOE collaboration). Phys. Lett. B, 2009, **670**: 285
- 11 Müller S E. These proceedings, arXiv:0912.2205 [hep-ex]
- 12 Davier M et al. arXiv:0908.4300 [hep-ph]; Davier M. These proceedings, arXiv:1001.2243 [hep-ph]
- 13 Aubert B et al (BABAR collaboration). Phys. Rev. Lett., 2009, **103**: 231801; Wang W. These proceedings
- 14 Krause B. Phys. Lett. B, 1997, **390**: 392
- 15 Nyffeler A. Phys. Rev. D, 2009, **79**: 073012; these proceedings, arXiv:1001.3970 [hep-ph]
- 16 Prades J, de Rafael E, Vainshtein A. arXiv:0901.0306 [hep-ph]
- 17 Mohr P J, Taylor B N, Newell D B. [CODATA 2006], Rev. Mod. Phys., 2008, **80**: 633
- 18 Jegerlehner F. The anomalous magnetic moment of the muon, Springer 2007; Miller J P, de Rafael E, Roberts B L. Rept. Prog. Phys., 2007, **70**: 795; Passera M. Nucl. Phys. Proc. Suppl., 2007, **169**: 213; J. Phys. G, 2005, **31**: R75; Melnikov K, A. Vainshtein A. Theory of the muon anomalous magnetic moment, Springer 2006; Davier M, Marciano W J. Ann. Rev. Nucl. Part. Sci., 2004, **54**: 115
- 19 Alemany R, Davier M, Höcker A. Eur. Phys. J. C, 1998, **2**: 123
- 20 Davier M et al. Eur. Phys. J. C, 2003, **27**: 497; C, 2003, **31**: 503
- 21 Davier M et al. arXiv:0906.5443 [hep-ph]
- 22 Marciano W J, Sirlin A. Phys. Rev. Lett., 1988, **61**: 1815; Sirlin A Nucl. Phys. B, 1982, **196**: 83; Cirigliano V, Ecker G, Neufeld H. Phys. Lett. B, **513**: 361; JHEP, 2002, **0208**: 002; Flores-Baez F et al. Phys. Rev. D, 2006, **74**: 071301; Flores-Baez F V et al. Phys. Rev. D, 2007, **76**: 096010; Lopez Castro G. These proceedings, arXiv:1001.3703 [hep-ph]
- 23 Fujikawa M et al (Belle collaboration). Phys. Rev. D, 2008, **78**: 072006
- 24 Benayoun M et al. Eur. Phys. J. C, 2008, **55**: 199; C, 2010, **65**: 211; arXiv:0907.5603 [hep-ph]; these proceedings, arXiv:0912.1248 [hep-ph]
- 25 Passera M, Marciano W J, Sirlin A. Phys. Rev. D, 2008, **78**: 013009; AIP Conf. Proc. 2009, **1078**: 378
- 26 Bouchiat C, Michel L. J. Phys. Radium, 1961, **22**: 121; Gourdin M, de Rafael E. Nucl. Phys. B, 1969, **10**: 667
- 27 Actis S et al. arXiv:0912.0749 [hep-ph]; Czyz H. These proceedings; Fedotovich G. These proceedings; Montagna G.

- These proceedings
- 28 Eidelman S, Jegerlehner F. *Z. Phys. C*, 1995, **67**: 585
- 29 Cabibbo N, Gatto R. *Phys. Rev.*, 1961, **124**: 1577
- 30 LEP EW Working Group, <http://lepewwg.web.cern.ch>
- 31 Tevatron EW Working Group, arXiv:0903.2503 [hep-ex]
- 32 Burkhardt H, Pietrzyk B. *Phys. Rev. D*, 2005, **72**: 057501
- 33 Barate R et al. *Phys. Lett. B*, 2003, **565**: 61
- 34 Degrassi G et al. *Phys. Lett. B*, 1998, **418**: 209; Degrassi G, Gambino P. *Nucl. Phys. B*, 2000, **567**: 3; Ferrogli A et al. *Phys. Rev. D*, 2002, **65**: 113002; Awramik M et al. *Phys. Rev. D*, 2004, **69**: 053006; *Phys. Rev. Lett.*, 2004, **93**: 201805; Awramik M, Czakon M, Freitas A. *JHEP*, 2006, **0611**: 048
- 35 ALEPH, CDF, D0, DELPHI, L3, OPAL and SLD collaborations, LEP and Tevatron Electroweak Working Groups, SLD Electroweak and Heavy Flavour Groups, arXiv:0911.2604 [hep-ex]
- 36 ALEPH, DELPHI, L3, OPAL, SLD collaborations, the LEP Electroweak Working Group, the SLD Electroweak and Heavy Flavour Groups, *Phys. Rept.*, 2006, **427**: 257
- 37 Amsler C et al (Particle Data Group). *Phys. Lett. B*, 2008, **667**: 1
- 38 Marciano W J. eConf, 2004, **C040802**: L009 [arXiv:hep-ph/0411179]
- 39 Altarelli G, Isidori G. *Phys. Lett. B*, 1994, **337**: 141; Casas J A, Espinosa J R, Quiros M. *Phys. Lett. B*, 1995, **342**: 171; *B*, 1996, **382**: 374
- 40 Isidori G, Ridolfi G, Strumia A. *Nucl. Phys. B*, 2001, **609**: 387
- 41 Kosower D A, Krauss L M, Sakai N. *Phys. Lett. B*, 1983, **133**: 305; Yuan T C et al. *Z. Phys. C*, 1984, **26**: 407; Moroi T. *Phys. Rev. D*, 1996, **53**: 6565; *D*, 1997, **56**: 4424; Ibrahim T, Nath P. *Phys. Rev. D*, 2000, **62**: 015004; Heine-meyer S, Stöckinger D, Weiglein G. *Nucl. Phys. B*, 2004, **690**: 62; *B*, 2004, **699**: 103
- 42 Degrassi G et al. *Eur. Phys. J. C*, 2003, **28**: 133