Strong coupling, charm- and bottom-quark masses from electron-positron annihilation

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Abstract The cross section for electron-positron annihilation into hadrons allows for a precise determination of the strong coupling constant and the charm- and bottom-quark masses. Recent theoretical and experimental results are presented with emphasis on the energy region accessible by B-meson factories and below.

Key words strong coupling, quark masses

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1 Introduction

The precise determination of the Standard Model (SM) parameters constitutes one of the important tasks of present day's particle physics. It is crucial for all tests of the SM, for the search for deviations and thus for physics "beyond" and, last but not least, it is a crucial input in all attempts to construct a Grand Unified Theory and thus to reduce the number of independent coupling constants.

With this motivation in mind the following discussion is focused on the implications of measurements at e^+e^- colliders in the low energy region, i.e. at B-meson factories or at even lower energies. It will be interesting to see, that these measurements are even sensitive to the strong coupling constant, although not (yet?) competitive with determinations through Z- or τ-decays, to name just a few. Quark mass determinations, however, in particular the combination of new theoretical results with improved measurements of charm- and bottom-quark production in the low energy region have recently reached a new level of precision that will be described below.

2 The strong coupling from electron-positron annihilation at low energy

During the past years the theoretical predictions for the e^+e^- annihilation cross section into hadrons have improved significantly (For a review see e.g. [1]). The massless approximation for this quantity has been pushed to $\mathcal{O}(\alpha_s^4)$ [2] and the corresponding terms in the small mass limit, of order $\alpha_s^4 m_Q^2/s$ are also available [3]. In order α_s^3 the massless approximation has been evaluated long time ago [4–6] as well as the dominant mass suppressed terms of order m_Q^2/s [7] and m_Q^4/s^2 [8]. In fact, a fairly accurate numerical parameterization of the full mass dependence in order α_s^3 has become available recently [9], which is based on the knowledge of the vacuum polarization function close to threshold, at high energies and around $q^2=0$. The corresponding $\mathcal{O}(\alpha_s^2)$ analysis had been performed more than a decade ago in Ref. [10]. The formulae up to $\mathcal{O}(\alpha_s^2)$ are encoded in the FORTRAN routine rhad which is publicly available [11] and which is sufficiently precise for the energy region under discussion.

The remarkable agreement between theory and experiment is demonstrated in Fig. 1 for the energy range accessible between 2 and 11 GeV. Already at fairly low energies, and in particular between $\Psi(2S)$ and $\Psi(3770)$, theory and experiment are in good agreement within the systematic uncertainty of about 3%. The results of a recent measurement [12] with a tiny statistical (0.5%–1%) and a small systematic (3.5%) error at $E_{\rm cm}=2.60, 3.07$ and 3.65 GeV are displayed in Table 1, together with the corresponding results for α_s.

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Table 1. Experimental results for the R-ratio at three different energies and α_s values with statistical and systematical error (from [12]).

E_{cm}/GeV	R	$\alpha_s^{(3)}(s)$	$\alpha_s^{(4)}(25\ \mathrm{GeV}^2)$	$\alpha_s^{(5)}(M_Z^2)$
2.60	2.18	$0.266^{+0.030+0.125}_{-0.030-0.126}$		
3.07	2.13	$0.192^{+0.029+0.103}_{-0.029-0.101}$	$0.209^{+0.044}_{-0.050}$	$0.117^{+0.012}_{-0.017}$
3.65	2.14	$0.207^{+0.015+0.104}_{-0.015-0.104}$		

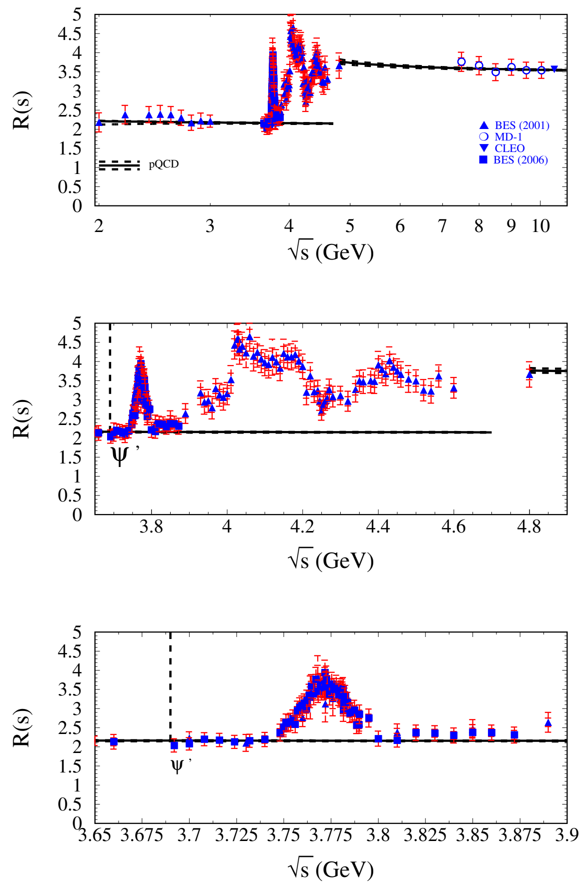

Fig. 1. $R(s)$ for different energy intervals around the charm threshold region. The solid line corresponds to the theoretical prediction, the uncertainties obtained from the variation of the input parameters and of μ are indicated by the dashed curves. The inner and outer error bars give the statistical and systematical uncertainty, respectively. (Data are taken from BES [13, 14], MD1 [15] and CLEO [16]).

Inclusion of the α_s^4 corrections would have moved the α_s values e.g. from $0.266 \pm 0.030 \pm 0.120$ by 0.020 upwards, a shift still significantly smaller than the systematic but well comparable to the statistical error.

Precise measurements have been performed also in the energy region between about 7 and 10.5 GeV, first by the MD-1-collaboration and, more recently, by CLEO [17] (Fig. 1). The data points with their systematic errors of about 2% agree nicely with the theory prediction. Indeed the measurement allows for a determination [18] of the strong coupling with $\alpha_s^{(5)}(M_Z) = 0.110^{+0.014}_{-0.017}$ in good agreement with other determinations [19].

Combining, finally, results from BES, MD-1 and CLEO one arrives at $\alpha_s^{(4)}(9\ \mathrm{GeV}) = 0.182^{+0.022}_{-0.025}$ which corresponds to $\alpha_s^{(5)}(M_Z) = 0.119^{+0.009}_{-0.011}$, again in good agreement with other determinations and with an error that starts to become competitive.

3 Charm- and bottom-quark masses

Quark mass determinations can be based on a variety of observations and theoretical calculations. The one presently most precise follows an idea advocated by the ITEP group more than thirty years ago [20], and has gained renewed interest after significant advances in higher order perturbative calculations discussed in this paper have been achieved. It exploits the fact that the vacuum polarization function $\Pi(q^2)$ and its derivatives, evaluated at $q^2 = 0$, can be considered short distance quantities with an inverse scale characterized by the distance between the reference point $q^2 = 0$ and the location of the threshold $q^2 = (3\ \mathrm{GeV})^2$ and $q^2 = (10\ \mathrm{GeV})^2$ for charm and bottom respectively. This idea has been taken up in [21] after the first three-loop evaluation of the moments became available [10, 22, 23] and has been further improved in [24] using four-loop results [25, 26] for the lowest moment. An analysis which is based on the most recent theoretical [9, 27, 28] and experimental progress has been performed in [29] and will be reviewed in the following.

Let us recall some basic notation and definitions. The vacuum polarization $\Pi_Q(q^2)$ induced by a heavy quark Q with charge Q_Q (ignoring in this short note the so-called singlet contributions), is an analytic function with poles and a branch cut at and above $q^2 = M^2_{J/\psi}$. Its Taylor coefficients $\bar{C}_n$, defined through

$$\Pi_Q(q^2) \equiv Q_Q^2 \frac{3}{16\pi^2} \sum_{n\geqslant 0} \bar{C}_n z^n \qquad (1)$$

can be evaluated in pQCD

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_Q}\right) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_Q} + \bar{C}_n^{(22)} l_{m_Q}^2\right) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m_Q} + \bar{C}_n^{(32)} l_{m_Q}^2 + \bar{C}_n^{(33)} l_{m_Q}^3\right) + \dots \quad (2)$$

Here $z \equiv q^2/4m_Q^2$, where $m_Q = m_Q(\mu)$ is the running $\overline{\rm MS}$ mass at scale μ. Using a once-subtracted dispersion relation

$$\Pi_Q(q^2) = \frac{1}{12\pi^2}\int_0^\infty {\rm d}s \frac{R(s)}{s(s-q^2)} \quad (3)$$

(with R_Q denoting the familiar R-ratio for the production of heavy quarks), the Taylor coefficients can be expressed through moments of R_Q. Equating perturbatively calculated and experimentally measured moments,

$$\mathcal{M}_n^{\rm exp} = \int \frac{{\rm d}s}{s^{n+1}} R_Q(s) \quad (4)$$

leads to an (n-dependent) determination of the quark mass

$$m_Q = \frac{1}{2}\left(\frac{9Q_Q^2}{4}\frac{C_n}{\mathcal{M}_n^{\rm exp}}\right)^{\frac{1}{2n}} . \quad (5)$$

The consistency of the results for different n and their stabilization with increasing orders in perturbation theory gives confidence in their reliability.

Significant progress has been made in the perturbative evaluation of the moments since the first analysis of the ITEP group. The $\mathcal{O}(\alpha_s^2)$ contribution (three loops) has been evaluated more than 13 years ago [10, 22, 23], as far as the terms up to $n=8$ are concerned, recently even up to $n=30$ [30, 31]. About ten years later the lowest two moments ($n = 0,1$) of the vector correlator were evaluated in $\mathcal{O}(\alpha_s^3)$, i. e. in four-loop approximation [25, 26]. The corresponding two lowest moments for the pseudoscalar correlator were obtained in [32] in order to derive the charmed quark mass from lattice simulations [33]. In [27, 28] the second and third moments were evaluated for vector, axial and pseudoscalar correlators. Combining, finally, these results with information about the threshold and high-energy behavior in the form of a Padé approximation, the full q^2/dependence of all four correlators was reconstructed and the next moments, from four up to ten, were obtained with adequate accuracy.

Most of the experimental input had already been compiled and exploited in [24], where it is described in more detail. However, until recently the only measurement of the cross section above but still close B-meson threshold was performed by the CLEO collaboration more than twenty years ago [34]. Its large systematic uncertainty was responsible for a sizable fraction of the final error on m_b. This measurement has been recently superseded by a measurement of BABAR [35] with a systematic error between 2 and 3% (Fig. 2). In [29] the radiative corrections were unfolded and used to obtain a significantly improved determination of the moments.

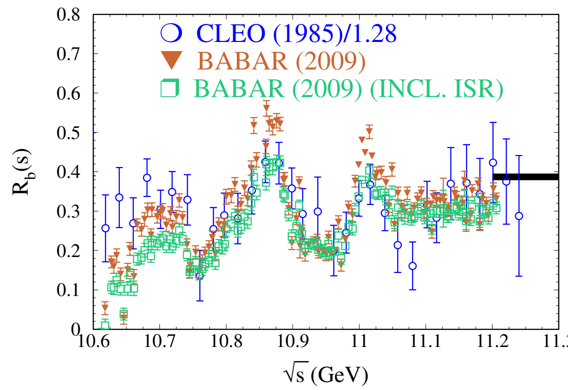

Fig. 2. Comparison of rescaled CLEO data for R_b with BABAR data before and after deconvolution. The black bar on the right corresponds to the theory prediction

The final results for m_c(3 GeV) and m_b(10 GeV) are listed in Tables 2 and 3. Despite the significant differences in the composition of the errors, the results for different values of n are perfectly consistent. For charm the result from $n = 1$ has the smallest dependence on the strong coupling and the smallest total error, which we take as our final value

$$m_c(3~{\rm GeV}) = 986(13)~{\rm MeV}, \quad (6)$$

and consider its consistency with $n = 2$, 3 and 4 as additional confirmation.

Table 2. Results for m_c(3 GeV) in MeV obtained from Eq. (5). The errors are from experiment, α_s, variation of μ and the gluon condensate.

n	m_c(3 GeV)	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

The stability and consistency of our result is also demonstrated in Fig. 3,where the results are compared for different moments, $n = 1, 2, 3, 4$ and different orders in perturbation theory.

Table 3. Results for $m_b(10 \text{ GeV})$ and $m_b(m_b)$ in MeV obtained from Eq. (5). The errors are from experiment, α_s and the variation of μ.

n	$m_b(10 \text{ GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

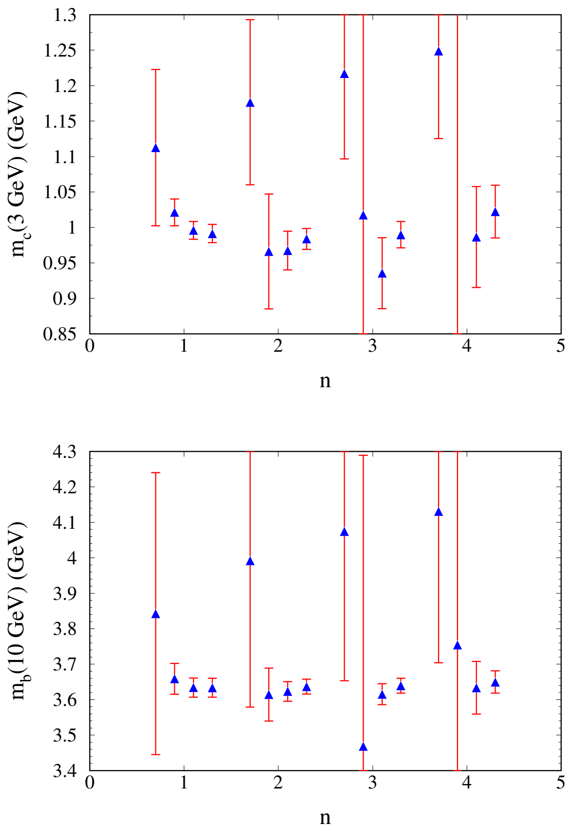

Fig. 3. m_c (3 GeV) (upper Fig.) and m_b (10 GeV) (lower Fig.) for $n = 1, 2, 3$ and 4. For each value of n the results from left to right correspond the inclusion of terms of order α_s^0, α_s^1, α_s^2 and α_s^3 in the coefficients $\bar{C}_n$ (cf. Eq. (2)). Note, that for $n = 3$ and $n = 4$ the central values and uncertainties can not be determined with the help of Eq. (5) in those cases where only the two-loop corrections of order α_s^1 are included into the coefficients $\bar{C}_n$ as the equation cannot be solved for $m_c(3 \text{ GeV})$.

Transforming this to the scale-invariant mass $m_c(m_c)$ [36], including the four-loop coefficients of the renormalization group functions one finds [29] $m_c(m_c) = 1279(13)$ MeV. Let us recall at this point that a recent study [33], combining a lattice simulation for the data for the pseudoscalar correlator with the perturbative three- and four-loop result [23, 28, 32] has led to $m_c(3 \text{ GeV}) = 986(10)$ MeV in remarkable agreement with [24, 29].

The treatment of the bottom quark case proceeds along similar lines. However, in order to suppress the theoretically evaluated input above 11.2 GeV (which corresponds to roughly 60% for the lowest, 40% for the second and 26% for the third moment), the result from the second moment has been adopted as our final result,

$$m_b(10 \text{ GeV}) = 3610(16) \text{ MeV}, \tag{7}$$

corresponding to $m_b(m_b) = 4163(16)$ MeV.

As shown in Figs. 4, 5 the results presented in [29] constitute the most precise values for the charm- and bottom-quark masses available to date. Nevertheless it is tempting to point to the dominant errors and thus identify potential improvements. In the case of the charmed quark the error is dominated by the parametric uncertainty in the strong coupling

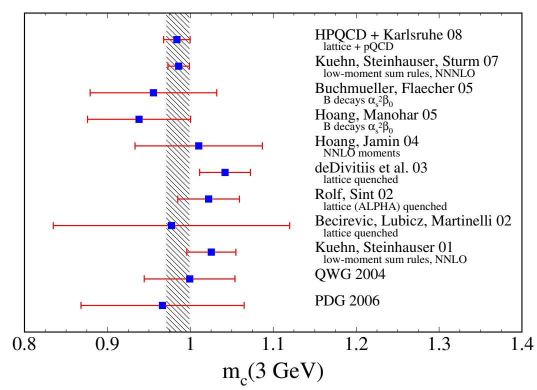

Fig. 4. Comparison of recent determinations of $m_c(m_c)$.

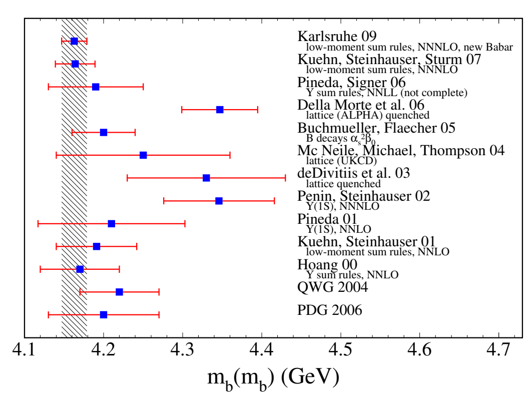

Fig. 5. Comparison of recent determinations of $m_b(m_b)$

$\alpha_s(M_Z) = 0.1185 \pm 0.002$. Experimental and theoretical errors are comparable, the former being dominated by the electronic width of the narrow resonances. In principle this error could be further reduced by the high luminosity measurements at BESS III. A further reduction of the (already tiny) theory error, e. g. through a five-loop calculation looks difficult. Further confidence in our result can be obtained from the comparison with the aforementioned lattice evaluation.

To summarize: Charm and bottom quark mass determinations have made significant progress during the past years. A further reduction of the theoretical and experimental error seems difficult at present. However, independent experimental results on the R ratio would help to further consolidate the present situation. The confirmation by a recent lattice analysis with similarly small uncertainty gives additional confidence in the result for m_c.

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