Effect of spin-orbit interaction on entanglement of two-qutrit Heisenberg XYZ systems in an inhomogeneous magnetic field *

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Abstract We investigate the properties of spin-orbit interaction in the ground state and thermal entanglement of a two-qutrit Heisenberg XYZ system in the presence of an inhomogeneous magnetic field. Using negativity as entanglement measure, we give the dependence of entanglement on each parameter in detail. The result shows that one can get appropriate entanglement by adjusting the magnetic field, Dzyaloshinskii-Moriya interaction and anisotropy parameter simultaneously.

Key words entanglement, negativity, Heisenberg XYZ chain

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1 Introduction

Quantum entanglement [1, 2] has recently become an important resource in quantum information processing [3] (QIP). The study of QIP has opened up an exciting new area and made great progress in recent years. This motivates us to ask how much entanglement exists in a realistic system such as a solid at a finite temperature. The 1D Heisenberg model [4] is a realistic and extensively studied solid state system [5–16]. But most of those studies use concurrence as entanglement measure.

Recently Vidal et al. presented a measure of entanglement called negativity [17, 18] that might be suitable for spin-1 systems [12]. In this paper we also investigate the effects of the Dzyaloshinskii-Moriya (DM) interaction on the entanglement properties of Heisenberg chains. The spin-orbit interaction causes another type of anisotropy. This interaction has a number of important consequences and may cause a number of unconventional phenomena [19, 20].

In this paper, the entanglement of two-qutrit Heisenberg XYZ systems in an inhomogeneous magnetic field with DM interactions is investigated. In Section 2, we briefly give the Hamiltonian of the model and the definition of the negativity. In Section 3, we investigate the properties of the ground states and the thermal states. Finally, Section 4 contains the concluding remarks.

2 General formalism

The Hamiltonian for a two-qutrit Heisenberg XYZ chain with DM interactions in an inhomogeneous magnetic field is given by

$$H = \sum_{i=1}^{2}\Big[J(1+\gamma)S_i^xS_{i+1}^x + J(1-\gamma)S_i^yS_{i+1}^y + J_zS_i^zS_{i+1}^z + \overrightarrow{D}\cdot(\overrightarrow{S}_i + \overrightarrow{S}_{i+1}) + (B+b)S_i^z + (B-b)S_{i+1}^z\Big], \tag{1}$$

where $S^\alpha(\alpha=x,y,z)$ denotes the spin-1 operator, and its components take the form [21]:

$$S_i^x = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix},\quad S_i^y = \frac{1}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix},$$

$$S_i^z = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}. \tag{2}$$

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$J(1+\gamma)$, $J(1-\gamma)$ and J_z are the couplings which take care of both the ferro- and anti-ferromagnetic couplings between two qutrit degrees of freedoms. $\overrightarrow{D}\cdot(\overrightarrow{S}_i+\overrightarrow{S}_{i+1})$ is the DM coupling term arising from spin-orbit coupling and here we take $\overrightarrow{D}=D\overrightarrow{z}$. B and b describe the magnetic fields, applied along the z-direction, so that b controls the degree of inhomogeneity. The periodic boundary condition $S_1=S_{N+1}$ is also applied.

Now, we briefly introduce the definition of the negativity for a state ρ. According to the Peres-Horodecki criterion, a non-entangled state has necessarily a positive partial transpose (PPT) [22]. The Peres-Horodecki criterion gives a qualitative way to judge if the state is entangled or not. Negativity was first introduced by Vidal and Werner and is defined as [17]

$$N(\rho)=\frac{||\rho^{\mathrm{T_A}}||_1-1}{2}, \tag{3}$$

where $||\rho^{\mathrm{T_A}}||_1$ denotes the trace norm of the partial transpose $\rho^{\mathrm{T_A}}$,

$$||\rho^{\mathrm{T_A}}||_1=\mathrm{tr}\sqrt{(\rho^{\mathrm{T_A}})^{\dagger}\rho^{\mathrm{T_A}}}. \tag{4}$$

The state at thermal equilibrium [3] is represented by the Gibb's density operator $\rho(T)=Z^{-1}\exp(-H/k_{\mathrm{B}}T)$, where $Z=\mathrm{tr}[\exp(-H/k_{\mathrm{B}}T)]$ is the partition function. k_{B} is the Boltzmann's constant and is set to be 1 hereafter.

3 Results and discussion

As the density matrix has nine dimensions, it is very tedious to write the eigenvalues and eigenstates of $\rho^{\mathrm{T_A}}$. To see the effects of every parameter on the entanglement, we will discuss the dependence of N on the parameter in detail as follows.

3.1 Ground state entanglement

In this section, we discuss the entanglement properties of the ground state at temperature T=0. The quantum phase transition (QPT) may be determined from the density matrix at T=0. The negativity (N) as a function of b for four values of D (D=0, 1, 2, 4) is plotted in Fig. 1. It is shown that the negativity increases with the increase of b until it reaches a peak, and then it drops with the increasing value of b. We also find that for a higher value of D, the system has a strong entanglement. It is interesting that such an increase and decrease disappear for D=4, and this means DM interaction not only increases the entanglement but also makes such fluctuation properties vanish.

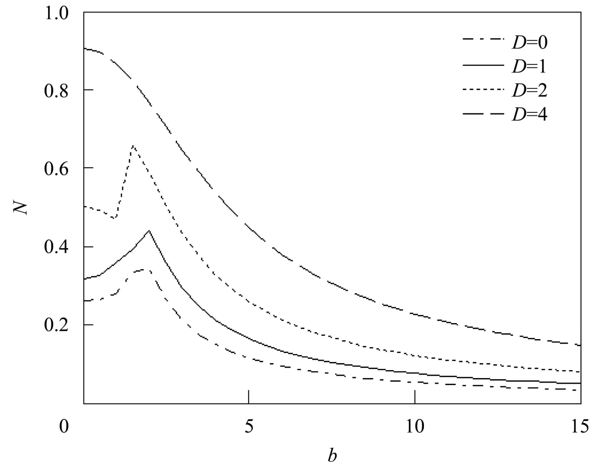

Fig. 1. The negativity versus b for different D (J=1, γ=0.5, J_z=−1, B=0.8).

The negativity (N) as a function of D is plotted in Fig. 2. In Fig. 2(a) we observe that the inhomogeneous magnetic field reduces entanglement. For some lower values of b (such as b <1.2) the entanglement experiences a decay and a revival with the increase of D. We see that the properties of the curve are similar when the value of b is lower than some distinct value. But for b=2 the entanglement increases with the increase of D. In Fig. 2(b) and (c) we find the entanglement increases with the increase of D. But when γ=0.9, the entanglement quickly reduces to zero and the critical $D_{\mathrm{c}}\approx 1$. When D crosses its critical value D_{c} the entanglement undergoes a revival and becomes a stable value. Obviously from Fig. 3(c), we find the entanglement increases with the increase of the value of the anisotropic parameter J_z. We also see that negativity becomes a more stable value after D reaches 4.

3.2 Thermal entanglement

Then we discuss the properties of a natural entanglement associated with thermal equilibrium state of the system at finite temperature. In Fig. 3 we give the plot of negativity as a function of T for different b. In Fig. 3(a) we set J_z=2, and in Fig. 3(b) we set J_z=−2. Obliviously for different J_z, the results are different. We can clearly see that the inhomogeneity of the magnetic field causes the decrease of negativity. For Fig. 3(a) we can notice that the critical temperature increases with the increase of b. It is more complicated in picture (b) because for some higher inomogeneity b (such as b=2) may get better entanglement than lower value of b at some region. It is clear in Fig. 3(b) that the critical temperature T_{c} is independent of b and that is different from Fig. 3(a).

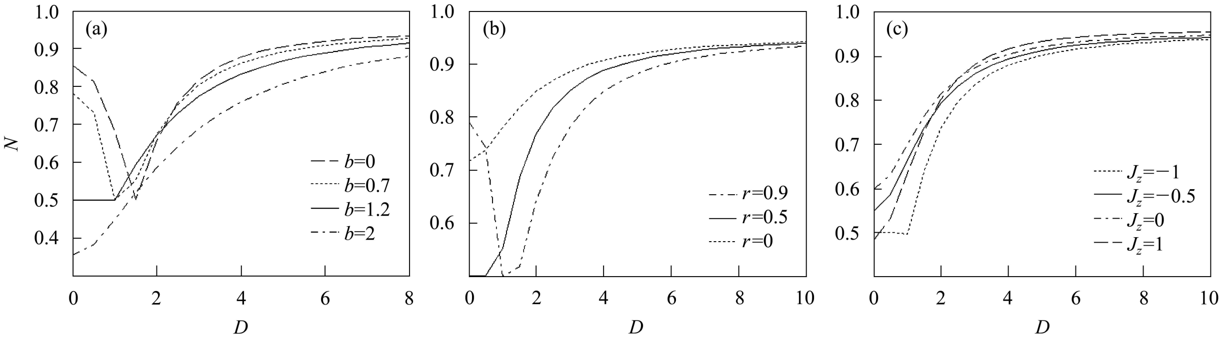

Fig. 2. (a) The negativity versus D for different b (J=1, γ=0.8, J_z=−1). (b) The negativity versus D for different γ (J=1, J_z=−1, b=0.75). (c) The negativity versus D for different J_z (J=1, J_z=1, b=0.75).

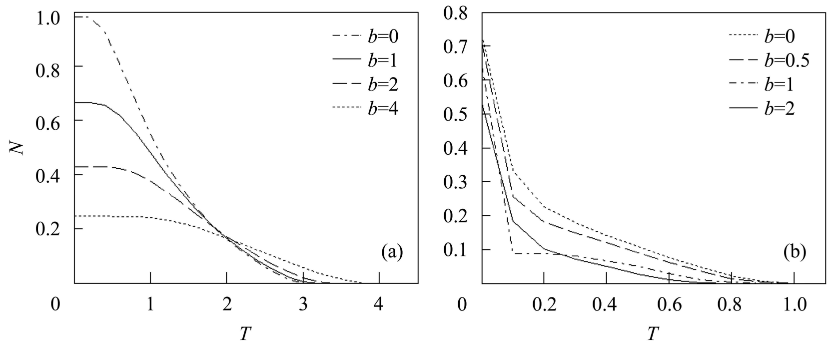

Fig. 3. The negativity versus T for different b (J=1, γ=0.8, D=2). (a) J_z=2; (b) J_z=−2.

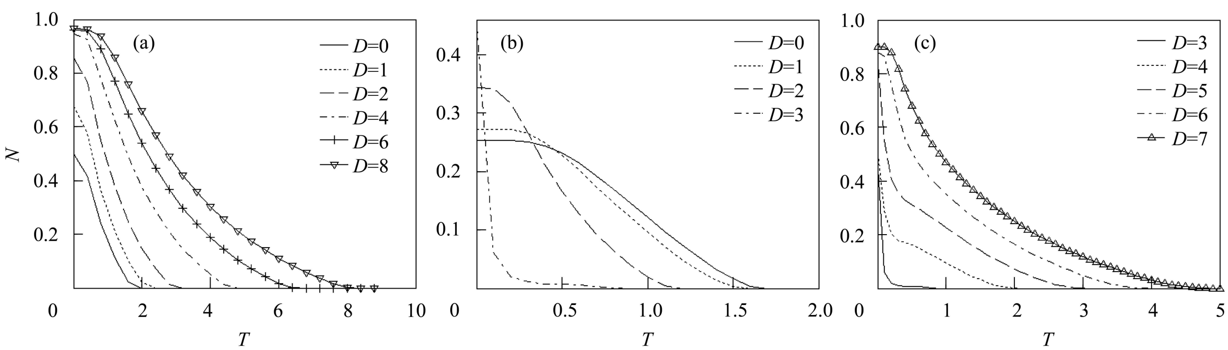

Fig. 4. The negativity versus T for different D (J=1, γ=0.8, B=1, b=0.5). (a) J_z=2; (b), (c) J_z=−2.

So we can get entanglement by adjusting b, D and J_z simultaneously.

In Fig. 4 we give the plot of negativity as a function of T for different D. In Fig. 4(a) we set J_z=2, and in Fig. 4(b), (c) we set J_z=−2. From Fig. 4(a) we find the entanglement increases with the increase of DM interaction. And also the value of critical temperature T_c increases. The negativity for T=0 increases with the increase of D until it reaches the maximum. In Fig. 4(b) and (c) we give the plot of negativity vs T for different D when J_z=−2. And we give two pictures in order to clearly show the entanglement properties. Obviously, the entanglement increases with the increase of DM interaction at a lower temperature and decreases with the increase of DM interaction at a higher temperature. It can also be seen that critical temperature decreases with the increase of the value of D. However, Fig. 4(c) gives different information. With the increase of the value of D, the entanglement and critical temperature increase, as they are absolutely different from Fig. 4(b). So we can manipulate the entanglement by suitably changing these parameters.

4 Conclusions

The entanglement of a two-qutrit Heisenberg XYZ system in the presence of an inhomogeneous magnetic field and spin-orbit interaction is investigated. We obtain some numerical results of this model by investigating the negativity. Since thermal entanglement is a natural type of entanglement for a system embedded in a thermal environment, we expect that the present results could be useful for solid state applications.

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