

Electromagnetic transition properties and isospin excitation in the cross-conjugate nuclei ^{44}Ti and $^{52}\text{Fe}^*$

ZHANG Jin-Fu(张进富)¹⁾ LÜ Li-Jun(吕立君) BAI Hong-Bo(白洪波)

Department of Physics, Chifeng University, Chifeng 024001, China

Abstract The interacting boson model with isospin (IBM-3) is applied to study the band structure and electromagnetic transition properties of the low-lying states in the cross-conjugate nuclei ^{44}Ti and ^{52}Fe . The isospin excitation states with $T=0, 1$ and 2 are identified and compared with available data. The E2 and M1 matrix elements for the low-lying states have been investigated. According to this study, the 2_3^+ state is the lowest mixed symmetry state in the cross-conjugate nuclei ^{44}Ti and ^{52}Fe . The excitation energy of the second 0_2^+ and 2_2^+ states with $T=0$ in the nucleus ^{52}Fe are identified. The agreement between the model calculations and data is reasonably good.

Key words spectrum, isospin, matrix element, mixed symmetry states

PACS 21.60.Fw, 21.10.Re

1 Introduction

With the development of radioactive ion beam facilities and large detector arrays, the study of the structure of the $f_{7/2}$ - shell heavy nuclei ($A \geq 40$) with $N = Z$ have gained renewed interest. The main reason is that (i) the structure of these nuclei provides a sensitive test for the isospin symmetry of the nuclear force, (ii) these nuclei may give new insights into neutron – proton (np pair) correlations that are unknown up to the present, and (iii) these nuclei are important for the rp-process nucleosynthesis. Many experimental and theoretical works [1–38] have been carried out recently for the investigation and understanding of the structure of the atomic nuclei with $N=Z$. The initial works focused on heavy odd-odd $N=Z$ nuclei [1–7]. However, many experimental studies have been done recently for even-even $N=Z$ heavy nuclei [9, 11, 13, 24] ($A \geq 40$ up to $A=88$). The neutron-proton correlations in the $T=0$ channel are an interesting aspect of the $N=Z$ nuclei, where the $T=0$ pairing may lead to a new collective mode [39, 40]. Nuclei with $N=Z$ are expected to exhibit interesting deformation characteristics; for example, the reduction of the moment of inertia and the bankbanding phenomenon, etc.

The interacting boson model (IBM) of nuclei, introduced by Arima and Iachello [41], is phenomenologically successful in describing the spectra of medium heavy nuclei and heavy nuclei. This model treats pairs of valence nucleons (particles/holes) as bosons with angular momentum $l=0$ (s bosons) or $l=2$ (d bosons). In the original version of the interacting boson model, IBM-1, no distinction is made between neutron bosons and proton bosons. IBM-1 has been extended to IBM-2 by distinguishing neutron bosons from proton bosons, and proved to be a first approximation for IBM-2. IBM-2 is effective for nuclear states of valence protons and valence neutrons filling the different single particle orbits. However, in light nuclei, the valence protons and valence neutrons are in the same single particle orbit; therefore isospin effects have to be included. To this end, IBM-3 [42] and IBM-4 [43] were proposed. IBM-4 describes nuclei in the sd - shell where the nucleons couple in the LS scheme [43, 44]. In the pf shells, the nucleon couplings are treated in the jj scheme, and IBM-3 has to be used here. As has been shown [42–44], IBM-3 is introduced for light nuclei. But there are some medium heavy nuclei with protons and neutrons filling the same single particle orbit; for example, the

Received 31 January 2010

* Supported by National Natural Science Foundation (10265001, 10765001) and Inner Mongolian Nation Natural Science Foundation (200607010111)

1) E-mail: zhjinfu@sohu.com

©2010 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

proton rich nuclei with $N \sim Z \sim 40$, and the lighter isotopes of tellurium, xenon, barium. As far as the nuclei with $N \sim Z \sim 40$ are concerned [45], IBM-3 may be used. Furthermore, with more data accumulating, this method may be used to describe the case of many $N=Z$ even-even nuclei.

In this work, we studied the electromagnetic transition properties, the isospin and mixed symmetry structure of the cross-conjugate nuclei ^{44}Ti and ^{52}Fe in the framework of IBM-3, in an attempt to identify mixed symmetry states and the isospin excitation states. The investigation of the relationship between particle-type and hole-type states of the cross-conjugate nuclei ^{44}Ti and ^{52}Fe contributes to an understanding of the behavior of nuclear force.

2 The IBM-3 Hamiltonian

To present our results, we give here a few important steps of the IBM-3. Details can be found in many previous publications [19, 25, 39, 41–43]. As in IBM-1, the IBM-3 model includes s and d bosons with no intrinsic spin, however with isospin. To take into account the isospin conservation in the framework of boson models, besides the proton-proton and neutron-neutron pairs (π , ν bosons), a proton-neutron pair (δ bosons) is also introduced. The building blocks of IBM-3 are s^+ , s^+ , s^+ , d^+ , d^+ and d^+ . The three s-bosons and three d-bosons form an isospin $T=1$ triplet ($T_z=1, 0, -1$ corresponding to pp, pn and nn pairs, respectively). The wavefunction has also to be classified by the $U_C(3) \supset SU(2)_T$ group chain, where $SU(2)_T$ is the usual isospin group. The corresponding creation and destruction operators of the bosons are

$$b_{l m_l, 1 m_T}^+, \quad b_{l m_l, 1 m_T}, \quad (1)$$

where $l=0, 2$ and $-l \leq m_l \leq l$, $-1 \leq m_T \leq 1$, respectively. The 324 bilinear combinations of $b_{l m_l, T m_T}^+$, $b_{l m_l, T m_T}$ generate the unitary group $U(18)$ of the IBM-3. In the coupled tensor form, the operators can be written as

$$\begin{aligned} & \left(b_{l_1}^+ \times \tilde{b}_{l_1} \right)_{M_L, M_T}^{(L, T)} \\ &= \sum_{m m' \mu \mu'} \langle l m l' m' | L M_L \rangle \langle 1 \mu 1 \mu' | T M_T \rangle b_{l m, 1 \mu}^+ \tilde{b}_{l' m', 1 \mu'}. \end{aligned} \quad (2)$$

Where the symbol $\langle | \rangle$ is the Clebsch-Gordan coefficient. The dynamical symmetry group for IBM-3 is $U(18)$, which starts with $U_{sd}(6) \times U_C(3)$ and must contain $SU_T(2)$ and $O(3)$ as subgroups because the

isospin and the angular momentum are good quantum numbers. The natural chains of the IBM-3 group $U(18)$ are [19]

$$\begin{aligned} & U(18) \supset (U_C(3) \supset SU_T(2)) \\ & \times (U_{sd}(6) \supset U_d(5) \supset O_d(5) \supset O_d(3)), \\ & U(18) \supset (U_C(3) \supset SU_T(2)) \\ & \times (U_{sd}(6) \supset O_{sd}(6) \supset O_d(5) \supset O_d(3)), \\ & U(18) \supset (U_C(3) \supset SU_T(2)) \\ & \times (U_{sd}(6) \supset SU_{sd}(3) \supset O_d(3)). \end{aligned} \quad (3)$$

The subgroups $U_d(5)$, $O_{sd}(6)$ and $SU_{sd}(3)$, as in IBM-1, describe vibrational, γ -unstable and rotational nuclei respectively. Dynamical symmetries of the IBM-3 have been studied in Refs. [19, 25, 39, 42]. The isospin-invariant IBM-3 Hamiltonian can be written as [42]

$$H = \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + H_2, \quad (4)$$

where

$$\begin{aligned} H_2 = & \frac{1}{2} \sum_{L_2 T_2} C_{L_2 T_2} ((d^+ d^+)^{L_2 T_2} \cdot (\tilde{d} \tilde{d})^{L_2 T_2}) \\ & + \frac{1}{2} \sum_{T_2} B_{0 T_2} ((s^+ s^+)^{0 T_2} \cdot (\tilde{s} \tilde{s})^{0 T_2}) \\ & + \sum_{T_2} A_{2 T_2} ((s^+ d^+)^{2 T_2} \cdot (\tilde{d} \tilde{s})^{2 T_2}) \\ & + \frac{1}{\sqrt{2}} \sum_{T_2} D_{2 T_2} ((s^+ d^+)^{2 T_2} \cdot (\tilde{d} \tilde{d})^{2 T_2}) \\ & + \frac{1}{2} \sum_{T_2} G_{0 T_2} ((s^+ s^+)^{0 T_2} \cdot (\tilde{d} \tilde{d})^{0 T_2}), \end{aligned} \quad (5)$$

and

$$\begin{aligned} & (b_1^+ b_2^+)^{L_2 T_2} \cdot (\tilde{b}_3 \tilde{b}_4)^{L_2 T_2} \\ & = (-1)^{(L_2 + T_2)} \sqrt{(2L_2 + 1)(2T_2 + 1)} \\ & \times [(b_1^+ b_2^+)^{L_2 T_2} \cdot (\tilde{b}_3 \tilde{b}_4)^{L_2 T_2}]^{00}, \end{aligned} \quad (6)$$

is the dot product in both angular momentum and isospin. The tilted quantity is defined as

$$\tilde{b}_{l m, m_Z} = (-1)^{(l + m + 1 + m_Z)} b_{l - m - m_Z}, \quad (7)$$

where T_2 and L_2 represent the two-boson system isospin and angular momentum. The parameters A , B , C , D and G are the two-body matrix elements by $A_{T_2} = \langle sd20 | H_2 | sd20 \rangle$, with $T_2=0, 1, 2$, $B_{T_2} = \langle s^2 0 T_2 | H_2 | s^2 0 T_2 \rangle$, $G_{T_2} = \langle s^2 0 T_2 | H_2 | d^2 0 T_2 \rangle$, $D_{T_2} = \langle sd 2 T_2 | H_2 | d^2 2 T_2 \rangle$, and $C_{L_2 T_2} = \langle d^2 L_2 T_2 | H_2 | d^2 L_2 T_2 \rangle$, with $T_2=0, 2$, and $L_2=0, 2, 4$ and $C_{L_2 1} = \langle d^2 L_2 1 | H_2 | d^2 L_2 1 \rangle$ with $L_2=1, 3$. The parameters A_1 , C_{11} and C_{31} are Majorana parameters which are similar to the ones in the IBM-2. Microscopic studies of the IBM-3 parameters [46, 47] show that the IBM-

3 Hamiltonian depends not only on the boson number but also on the isospin value. The dependence on isospin is more dramatic than that on the boson number. To have a good understanding of the symmetry structures of nuclei, we have rewritten the Hamiltonian in terms of a linear combination of the corresponding Casimir operators [19]. In the Casimir operator form, the Hamiltonian is given by

$$\begin{aligned}
 H_{\text{Casimir}} = & \lambda C_{2U_{sd}(6)} + a_T T(T+1) + \varepsilon C_{1U_d(5)} \\
 & + \gamma C_{2O_{sd}(6)} + \eta C_{2SU_{sd}(3)} + \beta C_{2U_d(5)} \\
 & + \delta C_{2O_d(5)} + a_L C_{O_d(3)}. \quad (8)
 \end{aligned}$$

The C_{nG} denotes the n th order Casimir operator of the algebra G . The λ parameter can be used to determine the position of the mixed symmetry states. The parameters in the Hamiltonian can be determined by fitting the experimental spectra. The low-lying levels of the cross-conjugate nuclei ^{44}Ti and ^{52}Fe can be described by the following Hamiltonians,

$$\begin{aligned}
 H_{\text{Casimir}} = & -0.535 C_{2U_{sd}(6)} + 1.64 T(T+1) \\
 & + 0.4 C_{1U_d(5)} + 0.093 C_{2U_d(5)} \\
 & + 0.02 C_{2O_d(5)} + 0.01 C_{O_d(3)}. \quad (9)
 \end{aligned}$$

From (9), we found that the coefficient of $C_{1U_d(5)}$ is very large. So the spectra are dominated by the vibrational $C_{1U_d(5)}$ term.

Another important aspect of nuclear structure is its transition properties. The general one-boson E2 operator in IBM-3 consists of isovector and isoscalar parts. So the quadrupole operator was expressed as [48]

$$T(\text{E2}) = T^0(\text{E2}) + T^1(\text{E2}), \quad (10)$$

where

$$\begin{aligned}
 T^0(\text{E2}) = & \alpha_0 \sqrt{3} [s^+ \tilde{d}]^{20} + (d^+ \tilde{s})^{20} \\
 & + \beta_0 \sqrt{3} [(d^+ \tilde{d})^{20}], \quad (11)
 \end{aligned}$$

$$\begin{aligned}
 T^1(\text{E2}) = & \alpha_1 \sqrt{2} [s^+ \tilde{d}]^{21} + (d^+ \tilde{s})^{21} \\
 & + \beta_1 \sqrt{2} [(d^+ \tilde{d})^{21}]. \quad (12)
 \end{aligned}$$

The transition M1 is also a one-boson operator with an isoscalar part and an isovector part,

$$T(\text{M1}) = T^0(\text{M1}) + T^1(\text{M1}), \quad (13)$$

where

$$T^0(\text{M1}) = g_0 \sqrt{3} (d^+ \tilde{d})^{10} = g_0 L / \sqrt{10}, \quad (14)$$

$$T^1(\text{M1}) = g_1 \sqrt{2} (d^+ \tilde{d})^{11}, \quad (15)$$

and g_0 and g_1 are the isoscalar and isovector g -factors, respectively, and L is the angular momentum operator. In order to analyze the contributions from the

isoscalar and isovector parts in the M1 and E2 transitions, we note the terms in the zero isospin z component of the transition operators as [27]

$$\begin{aligned}
 T_{sd}^0(\text{E2}) = & [(s^+ \tilde{d})^2 + (d^+ \tilde{s})^2]_{\pi} + [(s^+ \tilde{d})^2 + (d^+ \tilde{s})^2]_{\delta} \\
 & + [(s^+ \tilde{d})^2 + (d^+ \tilde{s})^2]_{\nu}, \quad (16)
 \end{aligned}$$

$$T_{dd}^0(\text{E2}) = [(d^+ \tilde{d})^2]_{\pi} + [(d^+ \tilde{d})^2]_{\delta} + [(d^+ \tilde{d})^2]_{\nu}, \quad (17)$$

$$\begin{aligned}
 T_{sd}^1(\text{E2}) = & [(s^+ \tilde{d})^2 + (d^+ \tilde{s})^2]_{\pi} \\
 & - [(s^+ \tilde{d})^2 + (d^+ \tilde{s})^2]_{\nu}, \quad (18)
 \end{aligned}$$

$$T_{dd}^1(\text{E2}) = [(d^+ \tilde{d})^2]_{\pi} - [(d^+ \tilde{d})^2]_{\nu}, \quad (19)$$

$$T_{dd}^0(\text{M1}) = [(d^+ \tilde{d})^1]_{\pi} + [(d^+ \tilde{d})^1]_{\delta} + [(d^+ \tilde{d})^1]_{\nu}, \quad (20)$$

$$T_{dd}^1(\text{M1}) = [(d^+ \tilde{d})^1]_{\pi} - [(d^+ \tilde{d})^1]_{\nu}. \quad (21)$$

So we rewrite the total transition operators as

$$\begin{aligned}
 T(\text{E2}) = & \alpha_0 T_{sd}^0(\text{E2}) + \beta_0 T_{dd}^0(\text{E2}) + \alpha_1 T_{sd}^1(\text{E2}) \\
 & + \beta_1 T_{dd}^1(\text{E2}), \quad (22)
 \end{aligned}$$

$$T(\text{M1}) = \sqrt{\frac{3}{4\pi}} \{g_0 T_{dd}^0(\text{M1}) + g_1 T_{dd}^1(\text{M1})\}. \quad (23)$$

We will use these results in our investigation.

3 Results and discussion of the cross-conjugate nuclei ^{44}Ti and ^{52}Fe

We perform a standard IBM-3 calculation for the cross-conjugate nuclei ^{44}Ti and ^{52}Fe . For the spectroscopy of the low-lying states in the pf-shell nucleus ^{44}Ti , ^{40}Ca is taken as the closed shell core. Accordingly, both proton and neutron bosons are of the particle type. For the nucleus ^{52}Fe , we have chosen ^{28}Ni as the closed shell core, and both proton and neutron bosons are of the hole type. The parameters in the Hamiltonian, which are determined by a best fit to the experimental levels, are listed in Table 1. The comparison between the calculated and the measured levels [49, 50] for ^{44}Ti and ^{52}Fe are shown in Fig. 1. The energy levels have been ordered into groups according to the isospin and $U(6)$ symmetry labels.

In general, a good reproduction of the low-lying structural characters observed in the experimental data can be found, especially for those states with $T=0$ in the IBM-3 calculation, as shown in Fig. 1. We predict that the excitation energy of the second 0_2^+ and 2_2^+ states with $T=0$ in ^{52}Fe are 1.916 MeV and 2.176 MeV, respectively. The calculation suggests that the excitation energy of the 3^+ states with $T=1$ in the nuclei ^{44}Ti and ^{52}Fe approach 7 MeV. In ^{44}Ti , the calculation suggests that the 0^+ state at 9.298 MeV is one with $T=2$. In the low-lying states,

the 1^+ state is of particular interest. In this paper, the IBM-3 calculation gives the 1^+ level at 7.104 MeV for the nuclei ^{44}Ti and ^{52}Fe . The theoretical energy level is in good agreement with the data for the nucleus ^{44}Ti . In ^{52}Fe , no experimental evidence for this

level is available. We suggest the 1^+ state level around this energy.

We have analyzed the wavefunction of the low-lying states, as shown in Table 2. It is found that the main components of the wavefunction for these states in the ground-state band are all basically s^N , $s^{N-1}d$, $s^{N-2}d^2$, $s^{N-3}d^3$ and so on configurations. For instance,

$$\begin{aligned} |0_1^+\rangle &= 0.8165 |s_\nu^1 s_\pi^1\rangle - 0.5774 |s_\delta^2\rangle + \dots, \\ |2_1^+\rangle &= -0.5774 |s_\nu^1 d_\pi^1\rangle - 0.5774 |s_\pi^1 d_\nu^1\rangle \\ &\quad + 0.5774 |s_\delta^1 d_\delta^1\rangle + \dots, \\ |4_1^+\rangle &= 0.8165 |d_\nu^1 d_\pi^1\rangle - 0.5774 |d_\delta^2\rangle + \dots. \end{aligned}$$

For the $N=Z$ nuclei, ^{44}Ti and ^{52}Fe , the lowest isospin value is $T=0$. From Table 2, we found that the experimental $T=0$, $T=1$ and $T=2$ energy levels are reproduced by the IBM-3 calculation. The IBM-3 calculation predicts other member levels for the $T=1$ and $T=2$ bands also. The state 0_1 is a s-boson state only and contains a significant s_δ^2 contribution. We found that the two-d-boson states 0_2^+ , 2_2^+ and 4_1^+ come

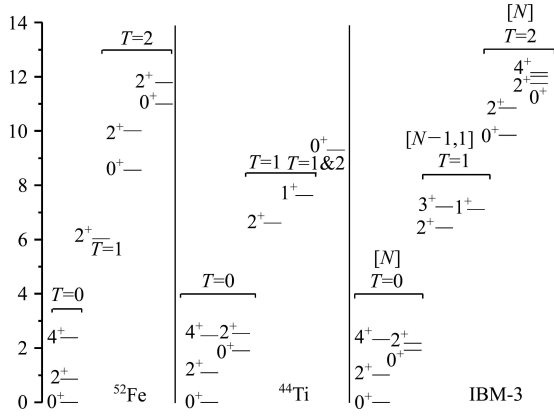


Fig. 1. Comparison of lowest excitation energy bands ($T = T_Z$, T_Z+1 and T_Z+2) of the IBM-3 calculation with experimental excitation energies of ^{52}Fe and ^{44}Ti .

Table 1. The parameters of the IBM-3 Hamiltonian used for the description of the cross-conjugate nuclei ^{44}Ti and ^{52}Fe .

$\varepsilon_{s\nu} = \varepsilon_{s\pi}$	$\varepsilon_{d\nu} = \varepsilon_{d\pi}$	$A_i (i=0,1,2)$	$C_{i0} (i=0,2,4)$	$C_{i2} (i=0,2,4)$	$C_{i1} (i=1,3)$	$B_i (i=0,2)$	$D_i (i=0,2)$	$G_i (i=0,2)$
0.07	1.075	-7.630	-7.724	2.116	-2.536	-7.630	0.000	0.000
		-2.210	-7.464	2.376	-2.436	2.210	0.000	0.000
		2.210	-7.324	2.516				

Table 2. Main components and amplitudes of the wavefunction in IBM-3 calculated for the cross conjugate nuclei ^{44}Ti and ^{52}Fe . The last three columns are the experimental and calculated values of the energy levels for the cross-conjugate nuclei ^{44}Ti and ^{52}Fe .

J^+	T	$U(6)$	$s_\nu s_\pi$	$s_\nu d_\pi$	$s_\pi d_\nu$	$d_\nu d_\pi$	d_δ^2	s_δ^2	$s_\delta d_\delta$	Expt.		Calc.
										^{44}Ti	^{52}Fe	
0	0	[2]	0.8165					-0.5774		0.000	0.000	0.000
2	0	[2]		-0.5774	-0.5774				0.5774	1.083	0.850	1.005
4	0	[2]				0.8156	-0.5774			2.454	2.386	2.316
0	0	[2]				-0.8165	0.5774			1.904		1.916
2	0	[2]				-0.8165	0.5774			2.531		2.176
2	1	[11]		0.7071	-0.7071					6.598	6.034	6.425
1	1	[11]				1.000				7.216		7.104
3	1	[11]				1.000						7.204
0	2	[2]	-0.5774					-0.8165		9.298	8.561	9.84
2	2	[2]		0.4082	0.4082				0.8165		10.006	10.845
0	2	[2]				0.5774	0.8165				10.99	11.756
2	2	[2]				0.5774	0.8165				11.78	12.016
4	2	[2]				0.5774	0.8165					12.156

from the same band because they have the same wavefunction structure. An analogous behavior is shown in the states with $T=2$, 0_2^+ , 2_2^+ and 4_1^+ . To identify the mixed symmetry states, we can make use of their general signatures: weak E2 and strong M1 transitions to symmetric states. Recently, it has been suggested that mixed symmetry states may form isomeric states under certain conditions [51]. The mixed symmetry state 2^+ in light nuclei with $N = Z$ has been identified in ^{24}Mg [31], ^{36}Ar [32], ^{44}Ti [20] and ^{48}Cr [22]. As shown in Table 2, among the low-lying states 2^+ , the IBM-3 calculation shows that the state 2_3^+ is the lowest mixed-symmetry state of composition sd. From these wavefunction expressions one can see that the composition of the states with $T=0$ and $T=1$ are two boson states and each state contains a δ boson component. It is shown that the δ boson plays an important role in this nucleus.

After the determination of the levels, the wavefunction is determined. The electric and magnetic transition properties can be obtained accordingly. The parameters in the E2 and M1 operators are close to the values used in Ref. [27], where $\alpha_0 = \beta_0 = 0.075$ eb, $\alpha_1 = \beta_1 = 0.05$ eb, $g_0 = 0\mu_N$ and $g_1 = 2.7\mu_N$. The results are summarized in Table 3.

For the even-even nuclei with $N = Z$, only the isoscalar part contributes to $B(\text{E}2)$, the isovector M1 transitions are isospin forbidden between its $T=0$ states and the isovector M1 components are relatively weak. From Table 3 we find that the transition between the $T=1$ states has a zero isovector component. We find that the transition is dominated by their E2 transition from the Table 3. It is noticed that the lowest mixed symmetry state 2_3^+ decays to the state 2_1^+ through a strong M1 transition with $B(\text{M}1; 2_3^+ \rightarrow 2_1^+) = 0.6961\mu_N^2$, and does not decay to the state 2_2^+ . The latter transition is almost forbidden. It is found that the E2 transitions with $\Delta T=0$ are isoscalar dominant. For example, the isoscalar E2 component is -13.1623 for the $2_1^+ \rightarrow 0_1^+$ transition. Moreover, distinguishing the $U(5)$ and $O(6)$ limits can be achieved from the behavior of the decay pattern of the states 1^+ . In the $U(5)$ limit, the one-boson $T(\text{M}1)$ operator cannot offer the transition from 1^+ to the ground state, but leads to a strong decay to the state 0_2^+ , which has a two-phonon character in the $U(5)$ limit. From Table 3, we find that the value of the $1_1^+ \rightarrow 2_2^+$ M1 transition with $\Delta T=1$ is larger, e.g. $B(\text{M}1; 1_1^+ \rightarrow 2_2^+) = 1.6243\mu_N^2$. This fact is an argument supporting $U(5)$ instead of $O(6)$. In addition, the $1_1^+ \rightarrow 2_3^+$ M1 transition with $\Delta T=0$

is isospin forbidden. The IBM-3 calculations provide also quadrupole moments of the 2_1^+ , 2_2^+ and 4_1^+ states, which are $Q(2_1^+) = 12.711$ efm 2 , $Q(2_2^+) = -5.448$ and $Q(4_1^+) = 19.045$ efm 2 , respectively.

As we known, nuclei lying in the middle of the $f_{7/2}$ shell are strongly deformed near the ground state. The cross-conjugate nuclei ^{44}Ti and ^{52}Fe , which do not lie in the middle of the shell, are less deformed, for instance $E_{4/2}=2.3$ and $E_{4/2}=2.8$, respectively. The rotational characteristics at low spin rapidly weakens with increasing angular momentum because of the competition between single particle and collective degrees of freedom. The observed ground-state bands of the cross-conjugate nuclei ^{44}Ti and ^{52}Fe are shown in Fig. 2. The cross-conjugate nuclei ^{44}Ti and ^{52}Fe should have equal spectra in the IBM-3 calculation. From Fig. 2, we found that the two spectra are very similar at low spin. It is noticed that the symmetry is broken at high spin, e.g., the observed 12^+ , 10^+ inversion in nucleus ^{52}Fe . The inversion was understood by describing the 12^+ state as two $f_{7/2}$ neutron and two $f_{7/2}$ proton holes aligned [24]. Recently, Gadea et al. [13] measured the γ decay of the 12^+ yrast trap for the nucleus ^{52}Fe and found that the two E4 transitions to the 8^+ states are hindered with respect to other $B(\text{E}4)$ transitions measured in the $f_{7/2}$ shell. This 12^+ state is considered to be an isomeric one, which mainly decays by β^+ decay into excited states of the daughter nucleus ^{52}Mn [52].

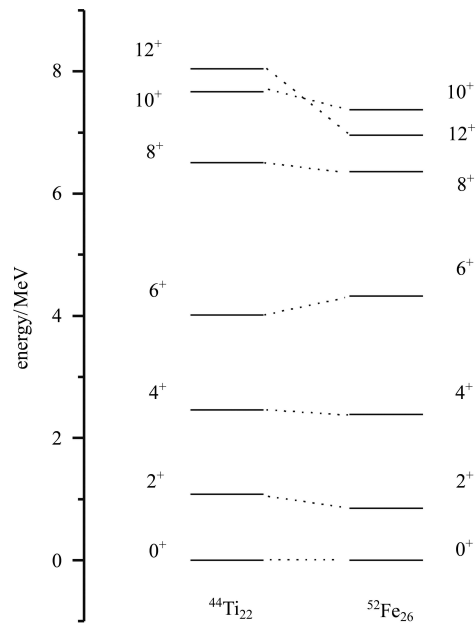


Fig. 2. Observed yrast states in the cross-conjugate nuclei ^{44}Ti and ^{52}Fe .

Table 3. E2 and M1 transition properties analyzed with IBM-3. Columns 2–6 are the reduced matrix elements for the various terms in the E2 transition operator. Column 7 is the $B(E2)$ value ($e^2\text{fm}^4$). Columns 8 and 9 are the reduced matrix elements for the isovector and the all M1 transition operator. Column 10 is the $B(M1)$ value (μ_N^2).

$J_i^+ \rightarrow J_f^+$	$T_{sd}^0(E2)$	$T_{dd}^0(E2)$	$T_{sd}^1(E2)$	$T_{dd}^1(E2)$	$T(E2)$	$B(E2)$	$T_{dd}^1(M1)$	$T(M1)$	$B(M1)$
$2_1^+ \rightarrow 0_1^+$	-13.1623				-0.2372	112.5			
$2_2^+ \rightarrow 2_1^+$	1.4142				0.1061	112.5			
$2_2^+ \rightarrow 0_2^+$		2.00			0.15	45			
$2_3^+ \rightarrow 0_2^+$			-1.1547		-0.0577	6.7			
$2_3^+ \rightarrow 0_1^+$			-2.582		-0.1291	33.3			
$2_3^+ \rightarrow 2_1^+$				0.8165	0.0408	16.7	0.6325	0.1708	0.6961
$2_3^+ \rightarrow 2_2^+$			-1.1547		0.0577	33.3			
$2_4^+ \rightarrow 2_3^+$				-0.0577	-0.0289	8.3	-0.04472	-1.2075	0.3481
$0_2^+ \rightarrow 2_1^+$	0.6325				0.0474	112.5			
$3_1^+ \rightarrow 2_1^+$			1.3663		0.0683	33.3			
$3_1^+ \rightarrow 2_2^+$				1.1041	0.0552	21.8	0.8	2.16	0.7956
$3_1^+ \rightarrow 2_3^+$	1.6733				0.1255	112.5			
$3_1^+ \rightarrow 4_1^+$				0.9759	0.0488	30.06	0.5164	1.3943	0.5967
$4_1^+ \rightarrow 2_1^+$	-1.8974				-0.1423	112.5			
$4_1^+ \rightarrow 2_2^+$		-0.7667			-0.0575	18.4			
$4_2^+ \rightarrow 2_3^+$			1.0954		0.0548	16.7			
$0_3^+ \rightarrow 2_3^+$			0.8165		0.0408	83.3			
$1_1^+ \rightarrow 0_2^+$							1.2649	3.4153	0.9282
$1_1^+ \rightarrow 2_1^+$			-0.8944		-0.0447	33.3			
$1_1^+ \rightarrow 2_2^+$				-0.6325	-0.0316	16.7	-0.7483	-2.021	1.6243
$1_1^+ \rightarrow 2_3^+$	1.0954				0.0822	112.5			

4 Conclusion

Using the interacting boson model with isospin, we have calculated the isospin excitation bands at low spin, the electromagnetic transitions and the mixed symmetry structure of the cross-conjugate nuclei ^{44}Ti and ^{52}Fe . The calculated levels and the electromagnetic properties are in agreement with the available data. The E2 and M1 matrix elements for the low-lying states have been investigated. The present calculations also give the structures of the isospin and

mixed symmetry states for the cross-conjugate nuclei ^{44}Ti and ^{52}Fe . The states with $T=1$ 2_3^+ are the lowest mixed symmetry states. The states 1_1^+ are the isospin excitation states with $T=1$. The excitation energy of the second states 0_2^+ and 2_2^+ with $T=0$ in the nucleus ^{52}Fe are identified as 1.916 MeV and 2.176 MeV, respectively. It will be desirable to confirm these model predictions in future experiments.

The authors are greatly indebted to Prof. G. L. Long for his continuing interest in this work and his helpful discussions.

References

- 1 Jenkins D G, Kelsall N S, Lister C J et al. *Phys. Rev. C*, 2002, **65**: 064307
- 2 Rudolph D et al. *Phys. Rev. C*, 2004, **69**: 034309
- 3 O'Leary C D, Bentley M A, Appelbe D E et al. *Phys. Lett. B*, 1999, **459**: 73
- 4 O'Leary C D, Bentley M A, Lenzi S M et al. *Phys. Lett. B*, 2002, **525**: 49
- 5 Möller O, Jessen K, Dewald A et al. *Phys. Rev. C*, 2003, **67**: 011301
- 6 Von Brentano P, Lisetskiy A F, Schneider I et al. *Prog. Part. Nucl. Phys.*, 2000, **44**: 29
- 7 Von Brentano P, Lisetskiy A F, Friessner C et al. *Prog. Part. Nucl. Phys.*, 2001, **46**: 197
- 8 Wojciech Satula, Ramon Wyss. *Phys. Rev. Lett.*, 2001, **87**: 052504; **86**: 4488
- 9 Fischer S M, Balamuth D P, Hausladen P A et al. *Phys. Rev. Lett.*, 2000, **84**: 4064; 2001, **87**: 132501
- 10 Van Isacker P, Warner D D, Frank A. *Phys. Rev. Lett.*, 2005, **94**: 162502
- 11 Cameron J A, J Jonkman, Svensson C E et al. *Phys. Lett. B*, 1996, **387**: 266
- 12 Terasaki J, Wyss R, Heenen P-H. *Phys. Lett. B*, 1998, **437**: 1
- 13 Gadea A, Lenzi S M, Napoli D R et al. *Phys. Lett. B*, 2005, **619**: 88
- 14 Kelsall N S, Wadsworth R, Wilson S N et al. *Phys. Rev. C*, 2001, **64**: 024309
- 15 Patra S K, Raj B K, Mehta M S et al. *Phys. Rev. C*, 2002, **65**: 054323
- 16 Brandolini F, Ur C A. *Phys. Rev. C*, 2005, **71**: 054316
- 17 Fischer S M, Lister C J, Balamuth D P. *Phys. Rev. C*, 2003, **67**: 064318
- 18 Lenzi S M, Maqueda E E, von Brentano P. *Eur. Phys. J. A*, 2006, **27**: 341
- 19 LONG Gui-Lu. *Chinese J. Nucl. Phys.*, 1994, **16**: 331
- 20 Al-Khudair F H, LI Yan-Song, LONG Gui-Lu. *J. Phys. G*, 2004, **30**: 1287
- 21 Al-Khudair F H, LI Yan-Song, LONG Gui-Lu. *HEP & NP*, 2004, **28**: 370 (in Chinese)
- 22 Al-Khudair F H, LONG Gui-Lu. *Chin. Phys.*, 2004, **13**: 1230
- 23 ZHANG Jin-Fu, BAI Hong-Bo. *Chin. Phys.*, 2004, **13**: 1843
- 24 Ur C A, Lenzi S M, Bucurescu D et al. *Prog. Part. Nucl. Phys.*, 1997, **38**: 223
- 25 Kota V K B. *Ann. Phys.*, 1998, **265**: 101
- 26 Sahu R, Kota V K B. *Eur. Phys. J. A*, 2005, **24**: 5
- 27 Al-Khudair F H, LI Yan-Song, LONG Gui-Lu. *Phys. Rev. C*, 2007, **75**: 054316
- 28 ZHANG Jin-Fu, LÜ Li-Jun, BAI Hong-Bo et al. *Sci. Chin. G*, 2008, **51**: 1845
- 29 LÜ Li-Jun, Al-Khudair F H, ZHANG Jin-Fu et al. *Chin. Phys. C*, 2009, **33**(Suppl): 46
- 30 BAI Hong-Bo, DONG Hong-Fei, ZHANG Jin-Fu et al. *Chin. Phys. C*, 2009, **33**(Suppl): 40
- 31 LÜ Li-Jun, BAI Hong-Bo, ZHANG Jin-Fu, *Chin. Phys. C*, 2008, **32**: 177
- 32 BAI Hong-Bo, ZHANG Jin-Fu, LÜ Li-Jun et al. *Commun Theor. Phys.*, 2007, **48**: 1067
- 33 SUN Yang. *Chinese Science Bulletin*, 2009, **54**: 4594
- 34 XU Hu-Shan, TU Xiao-Lin, YUAN You-Jin et al. *Chinese Science Bulletin*, 2009, **54**: 4749
- 35 SUN Yang, ZHANG Jing-Ye, LONG Gui-Lu et al. *Chinese Science Bulletin*, 2009, **54**: 358
- 36 QI Chong, DU Ren-Zhong, GAO Yang et al. *Science in China G*, 2009, **52**: 1464
- 37 LI Jian, ZHANG Ying, YAO Jiang-Ming et al. *Science in China G*, 2009, **52**: 1586
- 38 Al-Khudair F H. *Chin. Phys. C (HEP & NP)*, 2009, **33**: 538
- 39 Joseph N, Ginocchio. *Phys. Rev. Lett.*, 1996, **77**: 28
- 40 Satula W, Wyss R. *Phys. Lett. B*, 1996, **393**: 1
- 41 Iachello F. Arima A. *The Interacting Boson Model*. Cambridge: Cambridge University Press, 1987
- 42 Elliott J P, White A P. *Phys. Lett. B*, 1980, **97**: 169
- 43 Elliott J P, Evans J A. *Phys. Lett. B*, 1981, **101**: 216
- 44 HAN Q Z, SUN H Z, LI G H. *Phys. Rev. C*, 1987, **35**: 786
- 45 Sugita M. *Phys. Lett. B*, 1997, **394**: 235
- 46 Evans J A, LONG Gui-Lu, Elliott J P. *Nucl. Phys. A*, 1993, **561**: 201
- 47 Evans J A, Elliott J P, Lac V S, LONG Gui-Lu. *Nucl. Phys. A*, 1995, **593**: 85
- 48 Lac V S, Elliott J P, Evans J A. *Phys. Lett. B*, 1997, **394**: 231
- 49 Ur C A et al. *Phys. Rev. C*, 1998, **58**: 3163
- 50 CHU S Y, Nordberg H, Firestone R B et al. *Isotopes Explorer 2.00*, 1998
- 51 LONG Gui-Lu, LI Yan-Song, TU C C et al. *Commun. Theor. Phys.*, 2002, **37**: 75
- 52 Geesaman D F et al. *Phys. Rev. C*, 1979, **19**: 1938