

Intense DC beam nonlinear transport-analysis & simulation*

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Abstract The intense dc beam nonlinear transport was analyzed with the Lie algebraic method, and the particle trajectories of the second order approximation were obtained. Based on the theoretical analysis a computer code was designed. To get self-consistent solutions, iteration procedures were used in the code. As an example, we calculated a beam line (drift-electrostatic quadrupole doublet-drift). The results agree to the results calculated by using the PIC method.

Key words intense dc beam, nonlinear trajectory, Lie algebraic method

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1 Introduction

Intense dc beams are widely used in low energy beam transport systems and ion implanters, etc. But, the existing codes, such as TRACE-3D^[1], PARMILA^[2] etc, do not calculate the intense dc beam transport. Therefore, we have developed a computer code which calculates intense dc beam transport based on the Lie algebraic method. To prove the correctness of the code, we have also designed a code using the PIC method. In this paper, we present, as an example, the analysis for the dc beam nonlinear transport in electrostatic quadrupoles (EQ) with the Lie map up to the second order approximation. In the code, each optical element is divided into small segment, and the beam transport calculations are carried out by iteration procedures. The particle distribution in the 4D phase space is K-V type.

2 Hamiltonian

When a charged particle moves in the electromagnetic field, the Hamiltonian with time t as an inde-

pendent variable is

$$H_t = [m_0^2 c^4 + c^2(p_x - qA_x)^2 + c^2(p_y - qA_y)^2 + c^2(p_z - qA_z)^2]^{\frac{1}{2}} + q\phi, \quad (1)$$

where m is the particle mass, q the charge, c the light velocity, (p_x, p_y, p_z) the canonical momentum, (A_x, A_y, A_z) the magnetic potential, and ϕ the electric potential. ϕ contains two parts: $\phi = \phi_{\text{ext}} + \phi_{\text{sc}}$, where ϕ_{ext} is the external potential, and ϕ_{sc} is the space charge potential. For the EQ, ϕ_{ext} is

$$\phi_{\text{ext}} = \frac{2V}{r^2}(x^2 - y^2), \quad (2)$$

where V is the applied voltage on the pole surface, r is the inner radius, x and y are the coordinates in the transverse plane. For the K-V distribution, ϕ_{sc} is

$$\phi_{\text{sc}} = -\frac{I}{4\pi\epsilon_0 vXY} \left[x^2 + y^2 - \frac{X-Y}{X+Y}(x^2 - y^2) \right], \quad (3)$$

where I is the beam current, ϵ_0 is the dielectric constant, v is the particle velocity, X and Y are the two semi-axes of the beam ellipse. Now, define the new canonical coordinates: $\varsigma = (x, x', y, y\tau, p_\tau)$, where

$$x' = p_x/p_z, \quad y' = p_y/p_z, \quad \tau = T - z/\beta_0,$$

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$$p_\tau = p_T - p_T^0,$$

here $T = ct$, $\beta_0 = c/v_0$ (v_0 is the velocity of the reference particle); $p_T = p_t/(p_0c)$, p_0 the momentum of the reference particle; $p_t = -H_t$; and p_T^0 the value of p_T for the reference particle. The corresponding new Hamiltonian is

$$H = - \left[\left(p_\tau + p_T^0 + \frac{q\phi}{p_0c} \right)^2 - \frac{1}{\beta_0^2 \gamma^2} - (x' - qA_x/p_0)^2 \right]^{\frac{1}{2}} - (y' - qA_y/p_0)^2 - qA_z/p_0 - (p_\tau + p_T^0)/\beta_0. \quad (4)$$

For the EQ, $A_x=0$, $A_y=0$, $A_z=0$. Expanding the Hamiltonian to the third order, one obtains

$$H = \sum_{n=0}^{\infty} H_n, \quad (5)$$

where

$$\begin{aligned} H_0 &= 1/(\beta_0 \gamma_0), \quad H_1 = 0, \\ H_2 &= \frac{1}{2} s_x^2 x^2 + \frac{1}{2} p_x^2 - \frac{1}{2} s_y^2 y^2 + \frac{1}{2} p_y^2 + \frac{p_\tau^2}{2\beta_0^2 \gamma_0^2}, \\ H_3 &= \frac{x^2 p_\tau s_x^2}{2\beta_0 \gamma_0^2} + \frac{p_x^2 p_\tau}{2\beta_0} - \frac{y^2 p_\tau s_y^2}{2\beta_0 \gamma_0^2} + \frac{p_y^2 p_\tau}{2\beta_0} + \frac{p_\tau^3}{2\beta_0^3 \gamma_0^2}, \end{aligned} \quad (6)$$

and

$$\begin{aligned} s_x^2 &= \frac{2qV}{mc^2 r^2 \beta_0^2 \gamma_0} - \frac{qI}{X(X+Y)mc^3 \beta_0^3 \gamma_0 \pi \epsilon_0}, \\ s_y^2 &= \frac{2qV}{mc^2 r^2 \beta_0^2 \gamma_0} + \frac{qI}{Y(X+Y)mc^3 \beta_0^3 \gamma_0 \pi \epsilon_0}. \end{aligned} \quad (7)$$

3 Lie algebraic method

In the phase space ζ , the final coordinate ζ_f and the initial coordinate ζ_0 can be connected by a map \mathcal{M} :

$$\zeta_f = \mathcal{M}\zeta_0. \quad (8)$$

According to Ref. [3], \mathcal{M} is

$$\mathcal{M} = \cdots \mathcal{M}_3 \mathcal{M}_2, \quad (9)$$

where

$$\mathcal{M}_2 = \exp(:f_2:), \quad \mathcal{M}_3 = \exp(:f_3:), \quad \cdots \quad (10)$$

and

$$f_2 = -\mathcal{L}H_2, \quad (11)$$

$$f_3 = - \int_0^\ell h_3^{\text{int}}(\zeta_0, z) dz, \quad (12)$$

$$h_3^{\text{int}}(\zeta_0, z) = \mathcal{M}_2 H_3. \quad (13)$$

The first order solutions of the orbit are

$$\zeta_1 = f_2 \zeta. \quad (14)$$

The second terms of the orbit are

$$\zeta_2 = f_3 \zeta_1. \quad (15)$$

4 Particle trajectory in EQ

Using Eq. (14), we have

$$\begin{pmatrix} \cos(s_x \mathcal{L}) & \frac{1}{s_x} \sin(s_x \mathcal{L}) & 0 & 0 & 0 & 0 \\ -s_x \sin(s_x \mathcal{L}) & \cos(s_x \mathcal{L}) & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh(s_y \mathcal{L}) & \frac{1}{s_y} \sinh(s_y \mathcal{L}) & 0 & 0 \\ 0 & 0 & s_y \sin(s_y \mathcal{L}) & \cosh(s_y \mathcal{L}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (16)$$

Using Eq. (15), we have:

$$\begin{aligned} x_2 &= -x \cdot p_\tau \frac{\mathcal{L}(1+\gamma_0^2)s_x}{2\beta_0^2 \gamma_0^2} \sin(s_x \mathcal{L}) + \\ &\quad \frac{x' p_\tau}{2} \left[\frac{\mathcal{L}(1+\gamma_0^2) \cos(s_x \mathcal{L})}{\beta_0 \gamma_0^2} + \frac{\beta_0 \sin(s_x \mathcal{L})}{s_x} \right], \\ x'_2 &= \frac{x \cdot p_\tau s_x}{2\beta_0} \left[\frac{\mathcal{L}(1+\gamma_0^2)s_x \cos(s_x \mathcal{L})}{2\gamma_0^2} + \beta_0^2 \sin(s_x \mathcal{L}) \right] + \\ &\quad \frac{x' p_\tau \mathcal{L}(1+\gamma_0^2)s_x \sin(s_x \mathcal{L})}{2\beta_0 \gamma_0^2}, \end{aligned}$$

$$y_2 = y \cdot p_\tau \frac{\mathcal{L}(1+\gamma_0^2)s_y}{2\beta_0^2 \gamma_0^2} \sinh(s_y \mathcal{L}) +$$

$$\frac{y' p_\tau}{2} \left[\frac{\mathcal{L}(1+\gamma_0^2) \cosh(s_y \mathcal{L})}{\beta_0 \gamma_0^2} + \frac{\beta_0 \sinh(s_y \mathcal{L})}{s_y} \right],$$

$$\begin{aligned} x'_2 &= \frac{y \cdot p_\tau s_y}{2\beta_0} \left[\frac{\mathcal{L}(1+\gamma_0^2)s_y \cosh(s_y \mathcal{L})}{2\gamma_0^2} - \beta_0^2 \sinh(s_y \mathcal{L}) \right] + \\ &\quad \frac{y' p_\tau \mathcal{L}(1+\gamma_0^2)s_y \sinh(s_y \mathcal{L})}{2\beta_0 \gamma_0^2}, \end{aligned}$$

$$\tau_2 = 0,$$

$$p_{\tau 2} = 0. \quad (17)$$

5 Simulation example

The analytical results have been written into program LEADS^[4]. Using this code we computed the dc beam transport in the system consisting of drift – EQ doublet-drift (see Fig. 1). The length of the electrodes is 15.5 cm; the distance between the electrodes is 5 cm; the drift spaces before and after the EQ doublet are 200 cm; the beam transverse dimensions X and Y are all 1 mm; the beam maximum angles in the two transverse directions are all 6 mrad; the particle number is 5000. First, we put zero current beams into the system. In this code running, EQ voltages

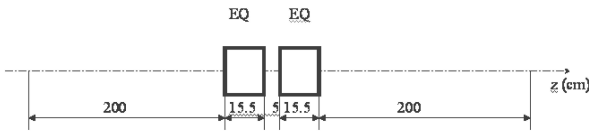


Fig. 1. Layout of the system.

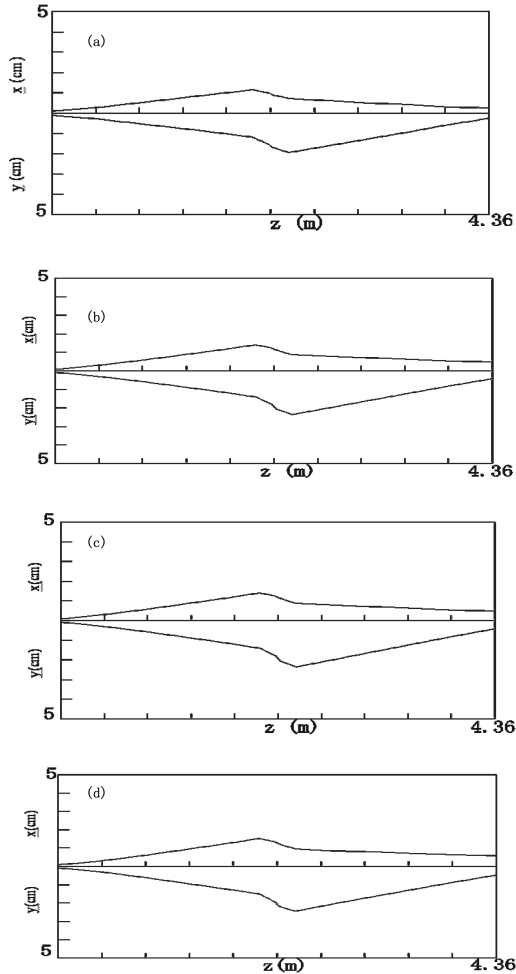


Fig. 2. Beam envelopes by Lie map. (a) $I = 0$ mA; (b) $I = 10$ mA; (c) $I = 20$ mA; (d) $I = 30$ mA.

were optimized. Using the optimized parameters, we calculated the 400 keV proton beam transport with the beam current of 10 mA, 20 mA and 30 mA respectively. The calculated beam envelopes in the cases of 0 mA, 10 mA, 20 mA and 30 mA are shown in Fig. 2(a)–(d).

The particle distributions in the x - y in the transverse section at the beginning and at the end of the system are shown in Fig. 3(a) and (b), from which we can see that the initial K-V distribution (uniform in the x - y plane) is still K-V distribution at the system end.

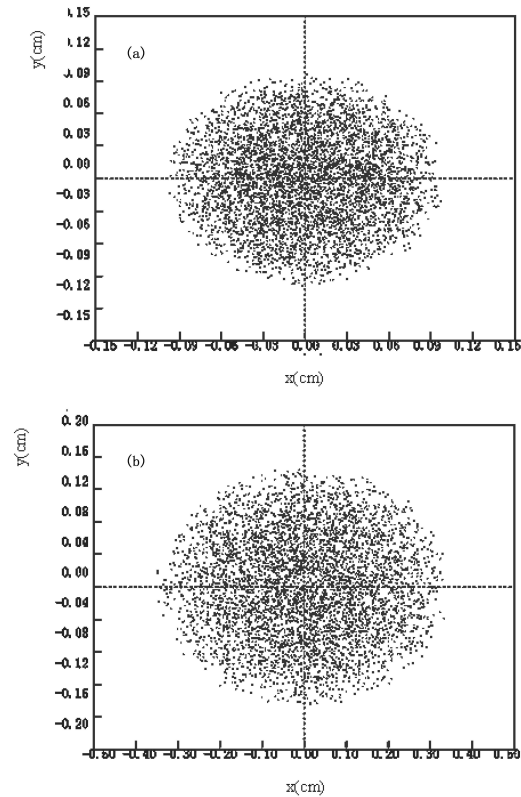


Fig. 3. Beam distribution in x - y plane. (a) Particle distribution at the beginning; (b) Particle distribution at the end.

6 Comparison with the PIC calculation

To prove the correctness of the above results, we also designed a PIC (particle-in-cell) code and calculated the same problems with it. The results show that they agree with each other very well.

In general, to simulate the particle motion in the applied field and beam self field with the PIC method, the following steps will be taken:

(1) Discrete a rectangular area covering all the particles of the beam into a mesh;

- (2) Assign the particle to the mesh nodes;
- (3) Solve the Poisson's equation on the mesh;
- (4) Compute the fields on the mesh with finite difference method;
- (5) Interpolate the field on the particle from the fields on the mesh nodes;
- (6) Calculate the particle motion.

With the PIC code, we calculated the same system as shown in Fig. 1. The beam currents are also 10 mA, 20 mA and 30 mA respectively. Fig. 4 shows the beam envelopes of the three beams. Comparing Fig. 3 with Fig. 4, we can see that the results obtained from the Lie map and from the PIC method are very close.

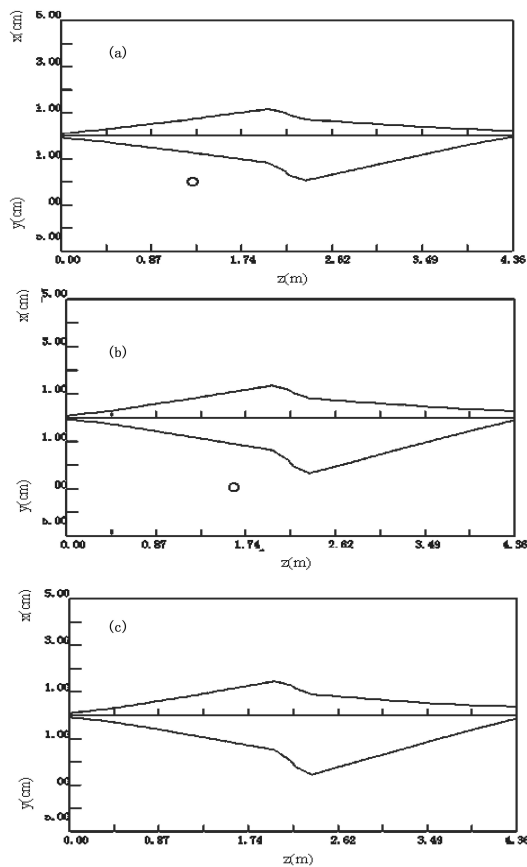


Fig. 4. Beam envelopes by PIC method. (a) $I = 10$ mA; (b) $I = 20$ mA; (c) $I = 30$ mA.

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