Temperature dependent fission fragment distribution in the Langevin equation^{*}

WANG Kun(王鲲)^{1,2;1)} MA Yu-Gang(马余刚)^{1;2)} ZHENG Qing-Shan(郑青山)² CAI Xiang-Zhou(蔡翔舟)¹ FANG De-Qing(方德清)¹ FU Yao(傅瑶)¹ LU Guang-Cheng(陆广成)¹ TIAN Wen-Dong(田文栋)¹ WANG Hong-Wei(王宏伟)¹

 (Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China)
(The Center for Drug Clinical Research, Shanghai University of Traditional Chinese Medicine, Shanghai 201203, China)

Abstract The temperature dependent width of the fission fragment distributions was simulated in the Langevin equation by taking two-parameter exponential form of the fission fragment mass variance at scission point for each fission event. The result can reproduce experimental data well, and it permits to make reliable estimate for unmeasured product yields near symmetry fission.

Key words fission fragments, mass distribution, Langevin equation

PACS 25.85.-w, 24.60.Ky, 21.10.Gv

The understanding of mass distributions of fissioning nuclei can provide information of the reaction mechanism as well as the dynamics of the fission process. The mechanism of fission leading to the observed broad mass distribution of the fission fragments is still one of the open problems of fission physics research. It is necessary to study the dependence of the mass distribution on parameters such as fission time, temperature and fissility, and to evaluate some systematics from it. In this work, we have carried out a systematic study of the variance or standard deviation of fission fragment mass distribution as a function of the temperature of fissioning nuclei.

Typically the process of fission can be divided into two phases. The fissioning system must first overcome the saddle point (the peak of the fission barrier), then enter the irreversible path towards scission. In the second stage, the properties of the scission configuration are determined during the long descent from the saddle to the scission configuration. The process of fission can be described in terms of collective motion using the transport theory^[1-3]. The dynamics</sup> of the collective degrees of freedom is typically described using the Langevin or Fokker-Planck equation. In this work, we investigate a one-dimensional Langevin equation model with mass-symmetry dominated fission. When the system reaches the scission point, we can get the temperature of the fissioning nuclei. We therefore expect possible to discuss the relation between system temperature distribution and fission fragment mass distribution. In the present work, we deal with a Combine Dynamical and Statistical Model (CDSM) which is an overdamped Langevin equation coupled with a Monte Carlo procedure allowing for the discrete emission of light particles. It switches over to statistical model when the dynamical description reaches a quasi-stationary regime. We first specify the entrance channel through which a compound nucleus is formed, ie. the target and projectile is complete fusion. For each trajectory simulating the fission motion, an angular momentum $L = \hbar l$ is sampled from the spin distribution

Received 3 September 2008

^{*} Supported by Knowledge Innovation Program of Chinese Academy of Sciences (55010701), National Support Plan of Science and Technology of China (2006BAI08B04-7), 973 Program (2007CB815004), Shanghai Development Foundation from Science and Technology (06JC14082), National Natural Science Foundation of China (10775167)

¹⁾ E-mail: wangkun@sinap.ac.cn

²⁾ E-mail: ygma@sinap.ac.cn

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$$\sigma(l) = \frac{2\pi}{k^2} \frac{2l+1}{1 + \exp[(l-l_c)/\delta_l]},$$
 (1)

where the parameters l_c and δl is the critical angular momentum and diffusion coefficient, respectively. A parametrization for l_c and δl is used which reproduces to a certain extent the dynamical results of the surface friction model for fusion of two nuclei forming the compound system ^[4].

Trajectory with the particular angular momentum L is started at the ground state position $q_{\rm gs}$ of the entropy $S(q_{\rm gs}, E_{\rm tot}^*, A, Z, L)$. The quantity q is the dimensionless fission coordinate defined as half of the distance between the centers of masses of the future fission fragments normalized to the radius of the compound system, which is characterized by its mass and charge numbers, A and Z. The total initial excitation energy $E_{\rm tot}^*$ is given by $E_{\rm tot}^* = E_{\rm lab}A_{\rm T}/(A_{\rm T} + A_{\rm P}) + Q$ where $A_{\rm T}$ and $A_{\rm P}$ represents the mass of target and projectile, respectively, and Q is the fusion Q-value calculated by $Q = M_{\rm T} + M_{\rm P} - M_{\rm CN}^{\rm LD}$. $M_{\rm T}$ and $M_{\rm P}$ are the masses of projectile and target, respectively, and $M_{\rm CN}^{\rm LD}$ is the compound nucleus masses.

The dynamical part of CDSM model is described by the Langevin equation which is driven by the free energy F. The free energy is related to the level density parameter $a(q)^{[5]}$ and fission potential, $F(q,T) = V(q) - a(q)T^2$.

The overdumped Langevin equation reads

$$\frac{\mathrm{d}q}{\mathrm{d}t} = -\frac{1}{M\beta(q)} \left(\frac{\partial F(q,T)_{\mathrm{T}}}{\partial q}\right) + \sqrt{D(q)}\Gamma(t) \qquad (2)$$

where $\Gamma(t)$ is a time-dependent stochastic variable with Gaussian distribution. Its average and its correlation function are $\langle \Gamma(t) \rangle = 0$, $\langle \Gamma(t)\Gamma(t') \rangle = 2\delta_{\varepsilon}(t-t')$. The fluctuation strength coefficient D(q) is according to the fluctuation-dissipation theorem expressed as $D(q) = \frac{T}{M}\beta(q)$, where β is the reduced friction parameter which is the only parameter of this model and M is the total mass.

The potential energy V(A, Z, L, q) is obtained from the finite-range liquid drop model^[6]

$$\begin{split} V(A,Z,L,q) = & a_2 [1-k(\frac{N-Z}{A})^2] A^{2/3} [B_{\rm s}(q)-1] + \\ & c_3 \frac{Z^2}{A^{1/3}} [B_{\rm c}(q)-1] + c_{\rm r} L^2 A^{-5/3} B_{\rm r}(q), \end{split}$$

where $B_s(q)$, $B_c(q)$ and $B_r(q)$ means surface, Coulomb and rotational energy terms, respectively, which depend on the deformation coordinate q. In our calculation we take them according to Ref. [1].

The fission process of the Langevin equation is propagated using an interpretation of Smoluchowski^[7]. In our calculation we adopt onebody dissipation (OBD) friction form factor $\beta(q)^{[8]}$ with a reduction of wall term. Here we use an analytical fit formula which was developed in Ref. [9],

$$\beta_{\rm OBD}(q) = \begin{cases} 15/q^{0.43} + 1 - 10.5q^{0.9} + q^2 & \text{if } q > 0.38\\ 32 - 32.21q & \text{if } q < 0.38 \end{cases}.$$

In the dynamical part of the model light particles (n, α, p, d) emission and giant dipole γ are calculated at each Langevin time step τ , the widths for particle and giant dipole γ decay are given by the parametrization due to Blann^[10] and Lynn^[11], respectively.

Within the framework of the Langevin simulation we chose fission events which happen on dynamic channel (we give up the events which happened in statistic part of CDSM model). Scission configuration in the model are parameterized by two separate fragments facing each other and being aligned with their axes of elongations on a common axis. Considering that the variance $\sigma_{\rm A}^2$ of fission fragment mass distribution is related to the scission configuration of the fissioning nucleus and it increases follow a single global exponential curve with fragment temperature in recent Ref. [12], we assume that for each fission event at scission point the fission fragment is a Gaussian distribution with the variance of mass distribution obeys this two-parameter exponential equation of the form $\sigma_{\rm A}^2 = E e^{cT_{\rm f}}$, where $E = 1.2531 \pm 0.0364$ and $c = 3.6000 \pm 0.1774$ is taken from Ref. [12], $T_{\rm f}$ is the temperature of fissioning nuclei.

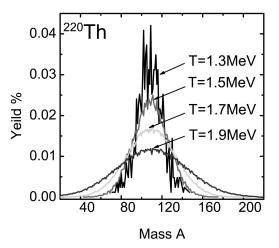


Fig. 1. Mass distribution with various variance in different temperature for ²²⁰Th.

It has wider fission channel to be chosen if the system reaches scission point with higher temperature. In Fig. 1, the mass distribution with various variance (σ_A^2) at different temperature of fissioning nuclei was shown. The resultant fission fragments are Gaussian-like mass distributions. We use CDSM model to calculate the σ_A for some channels and compare with some experimental data,^[13-19] which was displayed in Fig. 2. The different symbols present the results from different channels: ⁴He +²⁰⁹ Bi (diamond), ¹¹B + ²³²Th (Hexagon), ¹⁶O+²³⁸U (FiveStar), ¹⁹F + ²³²Th (lefttriangle), ²⁰Ne+ ¹⁸¹Ta (square), ²⁰Ne+²⁰⁹Bi (Pentagon), ¹⁸O+¹⁹⁷Au (triangle), ¹⁸O + ²³⁸U (righttriangle), ²⁰Ne + ²³²Th (circle). It looks that the model has a good agreement with the data for these reactions.

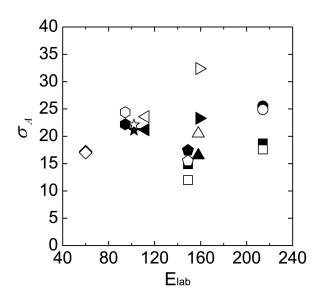


Fig. 2. Comparison the calculation results of σ_A by this work (empty symbol) with the experimental data (full symbol).

In order to investigate the influence of the analytical fit formula of reduced friction parameter on the width of mass distribution, we use a constant number $\beta_0 = 2, 4, 6, 8$ and 10 instead of one-body dissipation β_{obd} as the reduced friction parameter. We plot standard deviation σ_A as a function of β_0 . In Fig. 3, it shows standard deviation has a weak increasing trend with the increasing of the reduced friction parameter.

The influence of the projectile angular momentum on the standard deviation of mass distribution is shown in upper panel of Fig. 4. The σ_A does not change much in l < 35 which reflects the temperature does not change much in this range. With the increase of the angular momentum it leads to the lower barrier and the time that nucleus undergoes fission becomes more and more short without evaporating large number of light particles. So the dependence of σ_A on angular momentum should be related with the fission time too. In bottom panel, it obviously shows the opposite tendency.

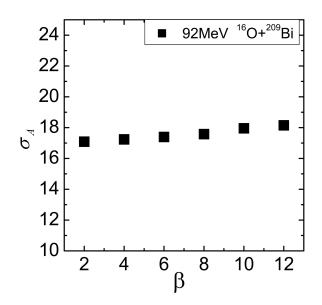


Fig. 3. The standard deviation σ_A of mass distribution of 92 MeV ${}^{16}\text{O} + {}^{209}\text{Bi}$ system as a function of reduced friction parameter.

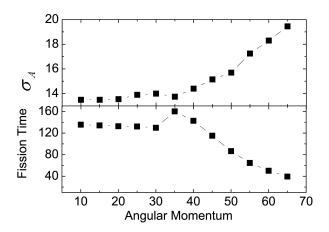


Fig. 4. The σ_A of mass distribution and fission time as a function of angular momentum for 220 Th.

In summary, we applied a theoretical model to describe dynamical process of compound nuclear fission with the statistical light particles and photons emission. In order to treat the fission fragments, we assume the fissioning nucleus has a Gaussian fission probability with the finite width σ and its centroid corresponds to the symmetric fission so that it splits into two fission fragments. The simulation illustrates that the fissioning fragment temperature dependent distribution can fit some data satisfactorily, and more interestingly, we found that the variance of mass distribution is sensitive to the fission time strongly. When fission time is long, the variance of fission fragments becomes small. In contrary, when fission time is short, the variance becomes large. In addition, it is found that variance of mass distribution is weakly dependent on reduced friction parameter. It somehow reflects the temperature of scission point dependence of friction parameter. In general, our work permits to make reliable estimates for unmeasured product yields near symmetry fission. The analysis of the fission fragments distribution appears to be a sensitive tool to investigate the fission dynamical information.

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