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Mixed symmetry states and isospin excitation in the $N=Z$ nucleus ^{28}Si*

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Abstract The interacting boson model with isospin (IBM-3) has been used to study the isospin excitation states and mixed symmetry states at low spin for ^{28}Si. The theoretical calculations are in agreement with experimental data. The theoretical results show that the 8_1^+ energy is 14.73 MeV.

Key words IBM-3, energy level, isospin, mixed symmetry states

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1 Introduction

During recent years, many authors discussed the nuclear structure characters in this regain for $N=Z$. The research results of nuclear structure not only validated the isospin symmetry of nuclei force, but also gave new insights into neutron-proton correlation. Nuclei with $N=Z$ are expected to exhibit interesting deformation characteristics[1—2]. It is worth mentioning the work of Long G L, Falih H.Al-Khudair[3—7]. By making use of the interacting boson model with isospin (IBM-3), we study the properties at low spin for ^{28}Si nucleus.

2 The IBM-3 Hamiltonian

In IBM-3, to take into account the isospin conservation in the framework of boson models, besides proton-proton and neutron-neutron pairs, a proton-neutron pairs is also introduced. The building blocks of IBM-3 are s_π^+, s_ν^+, s_δ^+, d_π^+, d_ν^+ and d_δ^+ .The three s-boson and three d-boson form the isospin $T=1$ triplet. The dynamical symmetry group for IBM-3 is $U(18)$, which starts with $U_{\rm sd}(6)\times U_{\rm c}(3)$ and must contain $SU_{\rm T}(2)$ and $O(3)$ as subgroups because the isospin and the angular momentum are good quantum numbers. The natural chains of IBM-3 group $U(18)$, see Ref.[8]. The subgroups $U_{\rm d}(5)$, $O_{\rm sd}(6)$ and $SU_{\rm sd}(3)$ describe vibrational, γ-unstable and rotational nuclei respectively.

The isospin-invariant IBM-3 Hamiltonian can be written as[4]

$$H=\varepsilon_{\rm s}\hat{n}_{\rm s}+\varepsilon_{\rm d}\hat{n}_{\rm d}+H_2, \tag{1}$$

where

$$\begin{aligned}H_2 = & \frac{1}{2}\sum_{L_2T_2}C_{L_2T_2}[(d^\dagger d^\dagger)^{L_2T_2}\cdot({\rm dd})^{L_2T_2}]+\\ & \frac{1}{2}\sum_{T_2}B_{0T_2}[({\rm s}^\dagger{\rm s}^\dagger)^{0T_2}\cdot({\rm ss})^{0T_2}]+\\ & \sum_{T_2}A_{2T_2}[({\rm s}^\dagger{\rm d}^\dagger)^{2T_2}\cdot({\rm ds})^{2T_2}]+\\ & \frac{1}{\sqrt{2}}\sum_{T_2}D_{2T_2}[({\rm s}^\dagger{\rm d}^\dagger)^{2T_2}\cdot({\rm dd})^{2T_2}]+\\ & \frac{1}{2}\sum_{T_2}G_{0T_2}[({\rm s}^\dagger{\rm s}^\dagger)^{0T_2}\cdot({\rm dd})^{0T_2}].\end{aligned} \tag{2}$$

Where T_2 and L_2 represent the two-boson isospin and angular momentum. The parameters A, B, C, D and G are the two-body matrix elements. $A_{T_2}=\langle sd20 \mid H_2 \mid sd20\rangle$, $T_2=0,1,2$; $B_{T_2}=\langle s^20T_2 \mid H_2 \mid s^20T_2\rangle$,$G_{T_2}=\langle s^20T_2 \mid H_2 \mid d^20T_2\rangle$,$D_{T_2}=\langle sd2T_2 \mid H_2 \mid$

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$d^2 2T_2\rangle$ and $C_{L_2T_2} = \langle d^2L_2T_2 \mid H_2 \mid d^2L_2T_2\rangle$, with $T_2=0,2$, $L_2=0,2,4$; $C_{L_21} = \langle d^2L_21 \mid H_2 \mid d^2L_21\rangle$ with $L_2=1,3$. The parameters A_1, C_{11} ,C_{31} are Majorana parameters which are similar to those in the IBM-2, and they are important to shift the states with mixed symmetry with respect to the total symmetric ones.In order to analyze the symmetry structures of nucleus, we have rewritten the Hamiltonian in terms of a linear combination of the corresponding Casimir operators. In the Casimir operator form, the Hamiltonian is

$$H_{\text{Casimir}} = \lambda C_{2U_{\text{sd}}(6)} + a_{\text{T}}T(T+1) + \varepsilon C_{1U_{\text{d}}(5)} + \gamma C_{2O_{\text{sd}}(6)} + \eta C_{2SU_{\text{sd}}(3)} + \beta C_{2U_{\text{d}}(5)} + \delta C_{2O_{\text{d}}(5)} + a_{\text{L}} C_{O_{\text{d}}(3)}. \quad (3)$$

Where λ parameter can be used to determine the position of the mixed symmetry states. The parameters in the Hamiltonian can be determined by fitting to the experimental spectra. The low-lying levels of ^{28}Si can be described by the following Hamiltonians,

$$H_{\text{Casimir}} = -0.06C_{2U_{\text{sd}}(6)} + 2.5T(T+1) + 0.2C_{1U_{\text{d}}(5)} - 0.17C_{2O_{\text{sd}}(6)} + +0.28C_{2O_{\text{d}}(5)} + 0.09C_{O_{\text{d}}(3)}. \quad (4)$$

3 Excitation energy

^{28}Si this nucleus is one with $Z=N$, By making use of IBM-3, we assume ^{16}O as the core. The nucleus ^{28}Si has $N_\pi = N_\nu = 3$ bosons. The parameters in Hamiltonian, which are determined by a best fit to experimental levels are listed in Table 1.

Table 1. The parameters of Hamiltonian of nuclei ^{28}Si. The $\varepsilon_{d\rho} - \varepsilon_{s\rho}$=1.86 MeV, where $\rho=\pi,\nu$.

$A_i(i=0,1,2)$	-10.46	-4.54	4.54
$C_{i0}(i=0,2,4)$	-12.08	-10.44	-9.18
$C_{i2}(i=0,2,4)$	2.92	4.56	5.82
$C_{i1}(i=1,3)$	-6.00	-5.10	
$B_i(i=0,2)$	-10.12	4.88	
$D_i(i=0,2)$	0.00	0.00	
$G_i(i=0,2)$	-0.76	-0.76	

The calculated and experimental energy levels[9] are exhibited in Fig. 1. In this figure the energy levels have been arranged into groups according to the isospin and the dynamical symmetry structure. As it can be seen ,our results are in agreement well with the experimental data. Theoretical result is $E(8_1^+) = 14.73$ MeV. The calculation results of Ref. [10] is $E(8_1^+)_{\text{O}} = 14.30 \pm 0.64$ MeV. Our results is same as the results of Ref. [10].

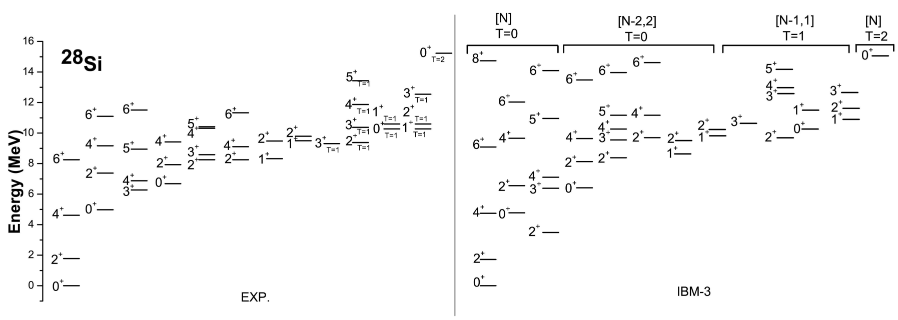

Fig. 1. Comparison between lowest excitation energy bands ($T=T_Z, T_Z+1, T_Z+2$) of the IBM-3 calculation and experimental excitation energies of ^{28}Si.

4 Mixed symmetry state

Mixed symmetry states occur when the motions of the proton and neutrons are not in phase. In the isospin-invariant IBM-3 Hamiltonian, the C_{11}, C_{31} and A_1 are Majorana parameters and the variation of which greatly affects the mixed symmetry states without much affecting the energy of the full symmetric states. We have varied each of Majorana parameters around the best-fit values, and shown the variations of the energy of these states with the parameters in Figs. 2 and 3.

Figures 2, 3 show that the 1_1^+, 2_4^+, 3_2^+ and 4_3^+

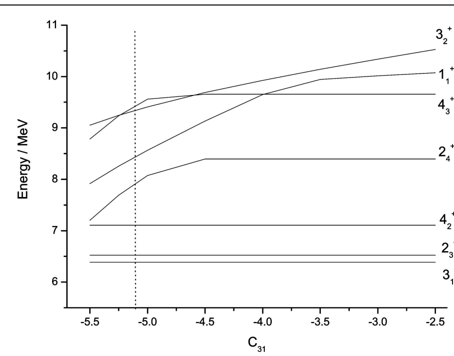

Fig. 2. Variation in level energy of ^{28}Si as a function of C_{31}; all other parameters were kept at their best-fit values.

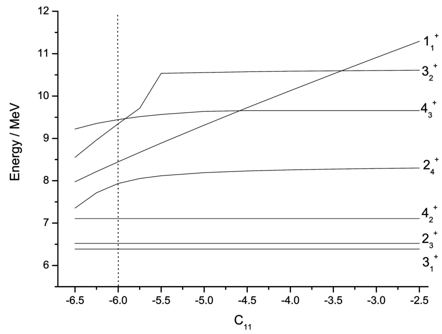

Fig. 3. Variation in level energy of ^{28}Si as a function of C_{11} ; all other parameters were kept at their best-fit values.

states are the mixed symmetry states. The main components of the wave function for these states are

$$|1_1^+\rangle = 0.3815\,|d_\nu^1 d_\pi^1 d_\delta^1 s_\delta^3\rangle + 0.2205\,|d_\nu^1 d_\pi^1 d_\delta^3 s_\delta^1\rangle + 0.3115\{|d_\nu^1 s_\nu^2 d_\pi^2 s_\pi^1\rangle - |d_\nu^2 s_\nu^1 d_\pi^1 s_\pi^2\rangle + |s_\nu^2 d_\pi^1 s_\pi^1 d_\delta^2\rangle + |d_\nu^1 s_\nu^1 s_\pi^2 d_\delta^2\rangle - |d_\nu^1 s_\nu^1 d_\pi^1 s_\pi^1 d_\delta^1 s_\delta^1\rangle\} + \cdots ,$$

$$|2_4^+\rangle = 0.3014\,|d_\nu^1 d_\pi^1 d_\delta^1 s_\delta^3\rangle + 0.2091\,|d_\nu^1 d_\pi^1 d_\delta^3 s_\delta^1\rangle - 0.2534\{|s_\nu^2 d_\pi^1 s_\pi^1 d_\delta^2\rangle + |d_\nu^1 s_\nu^1 s_\pi^2 d_\delta^2\rangle\} - 0.1964\{|d_\nu^2 s_\nu^1 d_\pi^1 s_\pi^2\rangle + |d_\nu^1 s_\nu^2 d_\pi^2 s_\pi^1\rangle + |d_\nu^1 s_\nu^1 d_\pi^1 s_\pi^1 d_\delta^1 s_\delta^1\rangle\} + \cdots ,$$

$$|3_2^+\rangle = 0.3224\,|d_\nu^1 d_\pi^1 d_\delta^1 s_\delta^3\rangle + 0.2632\{|d_\nu^1 s_\nu^2 d_\pi^2 s_\pi^1\rangle - |d_\nu^2 s_\nu^1 d_\pi^1 s_\pi^2\rangle + |s_\nu^2 d_\pi^1 s_\pi^1 d_\delta^2\rangle + d_\nu^1 s_\nu^1 s_\pi^2 d_\delta^2\rangle - |d_\nu^1 s_\nu^1 d_\pi^1 s_\pi^1 d_\delta^1 s_\delta^1\rangle\} + 0.1864\,|d_\nu^1 d_\pi^1 d_\delta^3 s_\delta^1\rangle + \cdots ,$$

$$|4_3^+\rangle = -0.3042\,|s_\nu^2 s_\pi^2 d_\delta^2\rangle + 0.2634\,|d_\nu^1 d_\pi^1 s_\delta^4\rangle + 0.2522\,|d_\nu^1 d_\pi^1 d_\delta^2 s_\delta^2\rangle - 0.2371\,|d_\nu^1 s_\nu^1 d_\pi^1 s_\pi^1 d_\delta^2\rangle - 0.2282\,|d_\nu^1 d_\pi^1 s_\nu^1 s_\pi^1 s_\delta^2\rangle + 0.2151\,|d_\nu^1 s_\nu^2 d_\pi^1 s_\pi^2\rangle + \cdots .$$

From above wavefunction expressions, we can see that the every state contains a δ boson component. δ boson is very important. "……" represents some smaller component.

5 Conclusion

By using the interacting boson model with isospin (IBM-3), we have calculated the isospin excitation bands at low spin and mixed symmetry structure of ^{28}Si. The IBM-3 calculated results agree very well with the available experimental data. The results conclude that the IBM-3 description of the low-lying levels in the ^{28}Si nucleus is satisfactory. The present calculations also give the structures of the isospin and mixed symmetry states for ^{28}Si nucleus. The 1_1^+, 2_4^+, 3_2^+ and 4_3^+ states are the mixed symmetry states. Theoretical result is $E(8_1^+)$=14.73 MeV and it will be confirmed in future experiment.

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