Restoration of rotational symmetry in deformed relativistic mean-field theory*

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Abstract We report on a very recently developed three-dimensional angular momentum projected relativistic mean-field theory with point-coupling interaction (3DAMP+RMF-PC). Using this approach the same effective nucleon-nucleon interaction is adopted to describe both the single-particle and collective motions in nuclei. Collective states with good quantum angular momentum are built projecting out the intrinsic deformed mean-field states. Results for ^{24}Mg are shown as an illustrative application.

Key words mean-field, triaxial deformation, spontaneous symmetry breaking, angular momentum projection

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1 Introduction

Equivalent to Kohn-Sham theory, self-consistent mean-field (SCMF) frameworks represent a very successful approach to low energy nuclear structure studies. In these theories, the most important correlations in nuclei, including the long-range particle-particle correlation and the short-range particle-hole correlation between nucleons, can be taken into account in terms of a universal energy density functional, which provides a consistent microscopic description of a whole range of structure phenomena[1]. Both the non-relativistic SCMF theories, such as Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB) with Skyrme or Gogny forces, and the relativistic SCMF theory, such as the relativistic mean-field (RMF) with meson exchange or zero-range point-couplings, have been successfully applied to the study of many properties of nuclei at and far from the valley of β-stability.

However, the product wavefunctions used as ansatz in all these SCMF theories tend to show “spontaneous symmetry breaking”, i.e., the approximate many-body wavefunction does not obey the same symmetries as the underlying two-body Hamiltonian[2], like the translational, rotational or gauge symmetries connected with the particle number. In these cases, the mean-field description of nuclear properties will show some deficiencies, like missing correlations associated with symmetry restoration and mixing contributions to the ground-state coming from the low-lying excited states, absence of selection rules for transitions, improper description of superfluid and shape phase transitions, etc. Moreover, in the vicinity of phase transitions in finite systems it is imperative to consider at least to some degree quantum fluctuations. The canonical approach to include these effects is through the restoration of broken symmetries using projection on top of the generator coordinate method (GCM)[3, 4], which has been widely used in modern nuclear physics studies[1, 2, 5].

For example, for the description of axially symmetric mean-field nuclei, one dimensional angular momentum projection (1DAMP) is enough for the restoration of rotational symmetry. However, and

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due to its very demanding numerical nature, only in the last decade such an approach based on SCMF has been possible[6—8]. For triaxially deformed nuclei, full three dimensional angular momentum projection (3DAMP) is essential, and very recently the implementation based on non-relativistic SCMF (cranked HF or HFB) theories have been reported[9, 10].

Since in the past decades relativistic mean-field theory (RMF) has achieved great success in the description of nuclear properties, especially the strong spin-orbit coupling[11], Coester line[12], isotope shift in Pb isotopes[13], pseudo-spin symmetry[14] and spin symmetry in anti-nucleon spectra[15], it is only natural to extend it in an effort to improve its predictive power. In that spirit, we have developed full three dimensional angular momentum projection on top of point-coupling relativistic mean-field (3DAMP+RMF-PC) theory.

This manuscript is organized as follows. In Sect. 2, we briefly introduce the theoretical aspects of the 3DAMP+RMF-PC approach. In Sect. 3, we present an illustrative calculation in ^{24}Mg, and a short summary and outlook in Sect. 4.

2 3D-angular momentum projected relativistic point-coupling approach

The 3DAMP+RMF-PC approach can be divided into two different consecutive tasks: the first is the determination of the triaxially deformed mean-field ground-state; the second is the projection of the ground-state wavefunction on to good quantum angular momentum.

The starting point of RMF-PC approach is the following Lagrangian density[16],

$$\begin{aligned}\mathcal{L} = {} & \bar{\psi}(\mathrm{i}\gamma_\mu\partial^\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\frac{1-\tau_3}{2}\bar{\psi}\gamma^\mu\psi A_\mu - \\ & \frac{1}{2}\alpha_{\mathrm{S}}(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_{\mathrm{V}}(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) - \\ & \frac{1}{2}\alpha_{\mathrm{TS}}(\bar{\psi}\boldsymbol{\tau}\psi)\cdot(\bar{\psi}\boldsymbol{\tau}\psi) - \frac{1}{2}\alpha_{\mathrm{TV}}(\bar{\psi}\boldsymbol{\tau}\gamma_\mu\psi)\cdot(\bar{\psi}\boldsymbol{\tau}\gamma^\mu\psi) - \\ & \frac{1}{3}\beta_{\mathrm{S}}(\bar{\psi}\psi)^3 - \frac{1}{4}\gamma_{\mathrm{S}}(\bar{\psi}\psi)^4 - \frac{1}{4}\gamma_{\mathrm{V}}[(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)]^2 - \\ & \frac{1}{2}\delta_{\mathrm{S}}\partial_\nu(\bar{\psi}\psi)\partial^\nu(\bar{\psi}\psi) - \frac{1}{2}\delta_{\mathrm{V}}\partial_\nu(\bar{\psi}\gamma_\mu\psi)\partial^\nu(\bar{\psi}\gamma^\mu\psi) - \\ & \frac{\delta_{\mathrm{TS}}}{2}\partial_\nu(\bar{\psi}\boldsymbol{\tau}\psi)\partial^\nu(\bar{\psi}\boldsymbol{\tau}\psi) - \\ & \frac{\delta_{\mathrm{TV}}}{2}\partial_\nu(\bar{\psi}\boldsymbol{\tau}\gamma_\mu\psi)\partial^\nu(\bar{\psi}\boldsymbol{\tau}\gamma^\mu\psi). \end{aligned} \tag{1}$$

The Lagrangian density contains eleven coupling constants α_{S}, α_{V}, α_{TS}, α_{TV}, β_{S}, γ_{S}, γ_{V}, δ_{S}, δ_{V}, δ_{TS} and δ_{TV}. The four Greek letters refer to the kind of contact interaction: α for linear four-fermion terms, β and γ for third- and fourth-order terms respectively, and δ for derivative couplings. The Latin subscripts indicate the symmetries associated to the coupling: S stands for scalar, V for vector, and T for isovector. The parameter set adopted in this work is the widely used PC-F1[16], which does not include the poorly-constrained scalar isoscalar channel.

From the Lagrangian density (1), and using the classical time-dependent variational principle, one can obtain the fermion and boson equations of motion. Their static solution under the no-sea approximation provides the ground-state wavefunction of the nuclear system. Pairing correlations are taken into account using a BCS-like monopole interaction.

This procedure often gives a deformed solution $|\Phi(q)\rangle$, with mass quadrupole moment q, for the nuclear system ground-state when the spherical symmetry constrain is relaxed. In order to obtain an energy spectrum that can be compared with experimental data, one has to go beyond mean-field theory.

Therefore, we have implemented full three-dimensional angular momentum projection for triaxially deformed intrinsic mean-field ground-states. Regarding the D_2 symmetry of a triaxial shape, one can construct a wavefunction $|\Psi^{JM}_{\alpha,q}\rangle$ eigenstate of the angular momentum operator $\hat{J}$ and its projection $\hat{J}_z$ as[17],

$$|\Psi^{JM}_{\alpha,q}\rangle = \sum_{K\geqslant 0}\frac{f^{JK}_\alpha(q)}{1+\delta_{\mathrm{K0}}}|JMK+,q\rangle \tag{2}$$

where $\alpha=1,2,\cdots$, labels consecutive collective excitation states. The angular momentum projected state $|JMK+,q\rangle$ is given by

$$|JMK+,q\rangle = [\hat{P}^J_{MK} + (-1)^J\hat{P}^J_{M-K}]|\Phi(q)\rangle, \tag{3}$$

where the projection operator $\hat{P}^J_{MK}$ is defined by

$$\hat{P}^J_{MK} = \frac{2J+1}{8\pi^2}\int \mathrm{d}\Omega D^{J*}_{MK}(\Omega)\hat{R}(\Omega),\ \ \mathrm{d}\Omega = \mathrm{d}\phi\sin\theta\mathrm{d}\theta\mathrm{d}\psi, \tag{4}$$

with Ω being the set of three Euler angles (ϕ,θ,ψ). $D^J_{MK}(\Omega)$ is the Wigner D-function with the rotational operator chosen as $\hat{R}(\Omega)=\mathrm{e}^{\mathrm{i}\phi\hat{J}_z}\mathrm{e}^{\mathrm{i}\theta\hat{J}_y}\mathrm{e}^{\mathrm{i}\psi\hat{J}_z}$.

The expansion coefficients $f^{JK}_\alpha(q)$ in Eq. (2) are determined by requiring the expectation value of the Hamiltonian, applied to $|\Psi^{JM}_{\alpha,q}\rangle$, to be stationary with respect to $f^{JK*}_\alpha(q)$,

$$\sum_{K'\geqslant 0}\left\{\mathcal{H}^J_{KK'}(q;q) - E^J_\alpha\mathcal{N}^J_{KK'}(q;q)\right\}f^{JK'}_\alpha(q) = 0, \tag{5}$$

where the overlap kernels $\mathcal{O}^J_{KK'}(q;q)$ are determined by $(\mathcal{O}=\mathcal{N},\mathcal{H})$,

$$\mathcal{O}^J_{KK'}(q;q)=\Delta_{KK'}\left[O^J_{KK'}(q;q)+(-1)^{2J}O^J_{-K-K'}(q;q)+(-1)^JO^J_{K-K'}(q;q)+(-1)^JO^J_{-KK'}(q;q)\right], \quad (6)$$

with $\hat{O}=1,\hat{H}$, and $\Delta_{KK'}=1/[(1+\delta_{K0})(1+\delta_{K'0})]$,

$$O^J_{KK'}(q;q)=\frac{2J+1}{8\pi^2}\int \mathrm{d}\Omega\langle\Phi(q)|\hat{O}\hat{R}(\Omega)|\Phi(q)\rangle D^{J*}_{KK'}(\Omega). \quad (7)$$

The overlaps $\langle\hat{O}\hat{R}(\Omega)\rangle$ of the kernels $O^J_{KK'}(q;q)$ are determined with the help of the generalized Wick's theorem[18] or Onishi formulae[19]. More details about the formalism can be found in Ref. [20].

3 Results and discussion

Figure 1 shows the projected norm kernel $\mathcal{N}^J_{00}(q,q)$ for the four lowest angular momenta as functions of the mass quadrupole moment q in ^{24}Mg. As expected, one finds that the spherical mean-field ground-state has a $J=0$ component only. However, several states with non-zero angular momentum can be projected out from intrinsic deformed mean-field states. It also shows that the larger the deformation of the intrinsic state, the higher angular momentum J of the dominant component $|JM\rangle$.

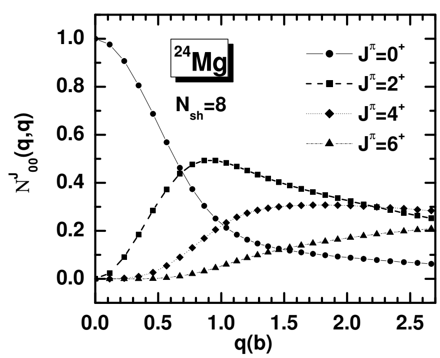

Fig. 1. Norm kernel $\mathcal{N}^J_{00}(q,q)$ (6) as a function of the mass quadrupole moment q.

Figure 2 shows the potential energy curves of collective states with the four lowest angular momenta in ^{24}Mg, as functions of the deformation $\beta(\gamma=0)$. The corresponding mean-field potential energy curve is given as well. The energy gap (~5 MeV) between the 0^+ curve and mean-field curve is due to the restoration of rotational symmetry. For the spherical configuration, $\beta=0$, only the $J=0$ component contributes (as depicted in Fig. 1), and thus there is no energy gain. The increased spread for the curves when currents were not included in the projection clearly show their effect on the nuclear moment of inertia.

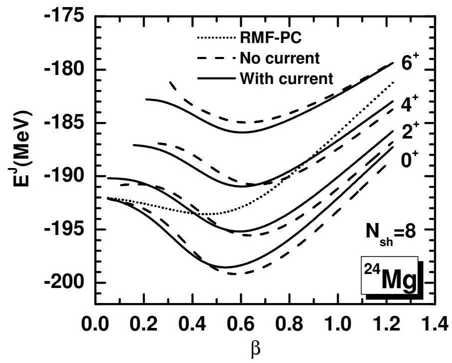

Fig. 2. Potential energy curves of the projected $J^\pi=0^+,2^+,4^+,6^+$ states in ^{24}Mg, as a function of the deformation $\beta(\gamma=0)$, with (solid lines) and without (dashed lines) the inclusion of currents. The mean-field potential energy surface (dash-dotted line) is given as well.

4 Summary and concluding remarks

In conclusion, a new code for three dimensional angular momentum projection has been developed on top of RMF-PC, in which pairing correlations are taken into account by BCS theory. Collective eigenstates of $\hat{J}$ and $\hat{J}_z$ are built via projection of intrinsic deformed mean-field states. Results for ^{24}Mg, where only axial deformation has been considered, have been presented as an illustrative application. Investigations in triaxial nuclei are currently under progress.

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