Evidence of self-affine multiplicity fluctuation of target residues in ^{84}Kr-AgBr interactions at 1.7 AGeV *

ZHANG Dong-Hai(张东海)1) LI Hui-Ling(李惠玲)

(Institute of Modern Physics, Shanxi Normal University, Linfen 041004, China)

Abstract Self-affine multiplicity scaling is investigated in the framework of a two-dimensional factorial moment methodology using the concept of the Hurst exponent (H). Analyzing the experimental data of target evaporated fragments emitted in ^{84}Kr-AgBr interactions at 1.7 AGeV revealed that the best power law behavior is exhibited for $H=0.3$ indicating a self-affine multiplicity fluctuation pattern. A signal of multifractality is also observed from knowledge of the anomalous fractal dimension d_q extracted from the intermittency exponent a_q of the anisotropic phase space scenario.

Key words relativistic heavy-ion collisions, target evaporated fragments, nuclear emulsion

PACS 25.75.Gz, 25.70.Mn, 29.40.Rg

1 Introduction

During the last decade, the study of large density fluctuations in high energy interactions has received much attention due to its potential to provide some information on the dynamics of multiparticle production processes. Bialas and Peschanski[1] were the first to introduce a new method named intermittency for the analysis of large fluctuations. Intermittency is signaled by the power law behavior of the Scaled Factorial Moments (SFMs) with increasing spatial resolution of the particle detection procedure. The unique feature of this moment is that it can detect and characterize non-statistical density fluctuations in particle spectra, which are intimately connected with the dynamics of particle production[2,3]. Until now, most of the investigations on intermittency and multifractality have been carried out for the production processes of high energy hadrons (pionization), while little attention has been paid to the analysis of target fragments.

During high energy nucleus-nucleus collision, the target nucleus may undergo a complete disintegration resulting in the emission of particles and nuclear fragments from the disintegrating nucleus. This is a complicated process developing both in space and time. In the light of a conventional cascade-evaporation model, slow or black (in emulsion terminology) particles ($\beta<0.3$) are evaporated from the disintegrating target nucleus and the emission characteristics of these particles is believed to be isotropic in the rest system of the target. However, the isotropy may be somewhat disturbed due to the motion of the target[4]. The cascade-evaporation model is based on the assumption that statistical equilibrium is established in the decaying system and its life time is assumed to be much longer than the time needed to redistribute its excitation energy among the different nucleons in the nucleus. However, different experimental data indicate the existence of a non-equilibrium process involved in the emission of black particles[5—10].

Most of the investigations of intermittency have been performed in one-dimensional space, though this is not at all sufficient for extracting the proper fluctuation patterns of the real three dimensional process[11, 12]. The analysis should be done in higher dimensions to reduce the error due to dimensional

---

Received 17 March 2008

* Supported by National Natural Science Foundation of China (10475054, 10675077), Major Science and Technology Foundation of Ministry of Education of China (205026), Natural Foundation of Shanxi Province, China (2008011005) and Shanxi Provincial Foundation for Returned Scholars of China (20031046)

[1] E-mail: zhangdh@dns.sxnu.edu.cn

reduction. Even in two-dimensional intermittency analysis the usual procedure was to divide the corresponding phase spaces subsequently into subcells by shrinking equally in each dimension and work done in both dimensions led to the conclusion that the pionization or target fragmentation process is self-similar. However, the phase space in high energy multiparticle production is anisotropic as indicated by Van Hove[13,14]. It may happen that the fluctuations are anisotropic and the scaling behavior is different in different directions. According to Mandelbrot[15], when a given pattern is differently scaled in different directions, it is called self-affine fractal. Up to now most of the intermittency investigations of target fragments have been performed based on the self-similar nature of the fluctuations[6—10,16—21]; only a few works report on the evidence of self-affine multiparticle production indicated by the data[22—31].

This paper reports an investigation on the nature of dynamical fluctuations which is carried out in the target fragmentation of ^{84}Kr-AgBr interactions at 1.7 *A*GeV in the framework of two-dimensional factorial moments considering the anisotropy of phase space.

2 Experimental details

Stacks of ILFORD G-5 nuclear emulsion plates horizontally exposed to a ^{84}Kr beam at 1.7 *A*GeV at Bevalac Berkeley are used in this work. The volume of the emulsion plate is 10 cm×10 cm×0.06 cm. Double scanning along the track, fast in the forward and slow in the backward direction, was carried out. Interactions which are within 30 μm from the top or bottom surface of the emulsion plates are not considered in the final analysis. Details of scanning and classification of events can be found in our previous paper[32—35].

The event of ^{84}Kr-AgBr interactions was chosen by using a criterion of having at least eight heavy ionizing tracks of particles ($N_{\mathrm{h}} \geqslant 8$).

The self-affine multiplicity fluctuations of target residues in ^{84}Kr-AgBr interactions at 1.7 *A*GeV are analyzed in the framework of a two-dimensional factorial moment methodology using the concept of the Hurst exponent. The cumulative variables[36] $X_{\cos\theta}$ and X_{ϕ} are used instead of $\cos\theta$ and ϕ. Details of the analyzing method can be found in our paper[37].

3 Experimental results

We have studied the dependence of the natural logarithm of the average value of the factorial moments ($\ln\langle F_q\rangle$) on the natural logarithm of ($\delta X_{\cos\theta}\cdot\delta X_{\phi}$) for ^{84}Kr-AgBr interactions at 1.7 *A*GeV for different Hurst exponents (0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0). In each case a linear behavior could be observed in two or three regions. In order to find the partitioning condition at which the scaling behavior is best revealed, we have performed a linear fit in the first region and have estimated the χ^2 per Degrees of Freedom (DOF) for each linear fit. It is interesting that the best linear behavior is revealed at $H = 0.3$ and not at $H = 1$ for each order of moment in the data set. The plots of $\ln\langle F_2\rangle$ against $\ln(\delta X_{\cos\theta}\delta X_{\phi})$ at $H = 0.3$ and 1.0 are shown in Figs. 1 and 2, respectively. Table 1 represents the value of χ^2 per DOF and the intermittency exponent for ^{84}Kr-AgBr interactions for different values of H and order of moments. From the table it is seen that χ^2 per DOF is smaller for $H = 0.3$ for different orders of moments, which is also clearly shown in Figs. 1 and 2, respectively.

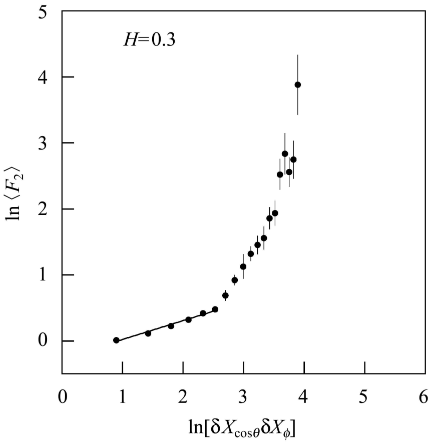

Fig. 1. Plot of $\ln\langle F_2\rangle$ against $\ln(\delta X_{\cos\theta}\delta X_{\phi})$ at $H = 0.3$ in the case of ^{84}Kr-AgBr interactions at 1.7 *A*GeV.

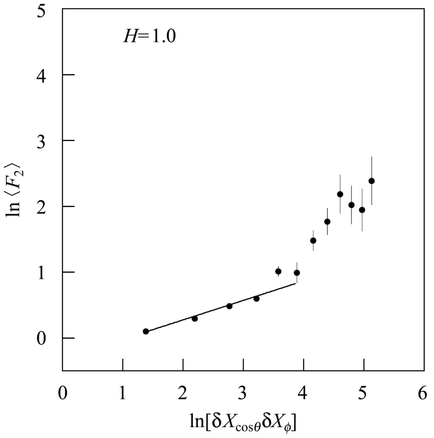

Fig. 2. Plot of $\ln\langle F_2\rangle$ against $\ln(\delta X_{\cos\theta}\delta X_{\phi})$ at $H = 1.0$ in the case of ^{84}Kr-AgBr interactions at 1.7 *A*GeV.

Table 1. The values of χ^2 per DOF and intermittency exponents for particular values of Hurst exponent H in ^{84}Kr-AgBr interactions at 1.7 AGeV.

value of H	$q=2$		$q=3$	
	χ^2 per DOF	intermittency exponents (a_q)	χ^2 per DOF	intermittency exponents (a_q)
0.3	1.208	0.285±0.017	2.509	0.786±0.071
0.4	1.865	0.277±0.020	2.018	0.754±0.068
0.5	1.492	0.291±0.021	1.560	0.820±0.082
0.6	3.849	0.324±0.019	1.387	0.835±0.084
0.7	1.270	0.286±0.022	1.591	0.778±0.097
0.8	2.875	0.296±0.019	3.402	0.815±0.093
0.9	2.194	0.311±0.018	1.015	0.955±0.091
1.0	4.325	0.296±0.015	2.671	0.857±0.079

So the dynamical fluctuation pattern in ^{84}Kr-AgBr interactions is not self-similar but self-affine.

The power-law behavior of the scaled factorial moments indeed implies the existence of some kind of fractal pattern[38] in the dynamics of the particles produced in their final state. Therefore, it is natural to study the fractal nature of the target fragmentation process in ^{84}Kr-AgBr interactions under the self-affine scaling scenario.

In order to study the dependence of the anomalous fractal dimensions d_q ($d_q=a_q/(q-1)$) on the order of the moments q under the self-affine scaling scenario, the d_q values have been calculated at $H=0.3$. The variation of d_q with the order q is shown in Fig. 3. From the plot it is seen that d_q is linearly dependent on the order q, which suggests the presence of multifractality of emission of target evaporated fragments in ^{84}Kr-AgBr interactions.

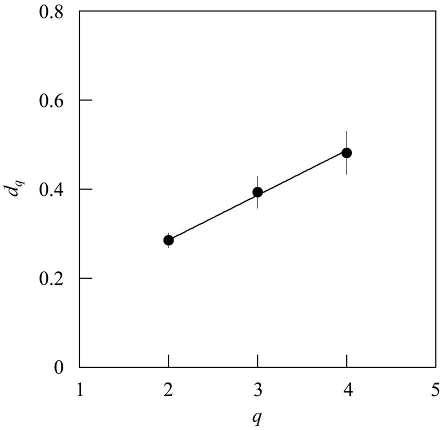

Fig. 3. Plot of d_q against q at $H=0.3$ in ^{84}Kr-AgBr interactions at 1.7 AGeV.

Finally we will discuss the non-thermal phase transition under the self-affine scaling scenario. According to Peschanski[39] the signals of non-thermal phase transition can be studied with help of the parameter $\lambda_q=(a_q+1)/q$. The condition for the existence of two different phases is that the function λ_q should have a minimum at some value $q=q_\mathrm{c}$. In the region $q<q_\mathrm{c}$ a larger number of small fluctuations occur and the region $q>q_\mathrm{c}$ is governed by a small number of very large fluctuations[40]. This situation resembles a mixture of a liquid of many small number fluctuations and a dust consisting of few grains of very large density. The minimum of the function λ_q may be a manifestation of the fact that the liquid and the dust phase coexist. If the system is tested by a moment of rank $q<q_\mathrm{c}$, one sees only the liquid phase and if it is tested by a moment of $q>q_\mathrm{c}$, one sees only the dust phase. Fig. 4 presents the dependence of λ_q on the order q. From the plot it is seen that a minimum of λ_q is obtained at $q=3$ which suggests that a non-thermal type phase transition might exist.

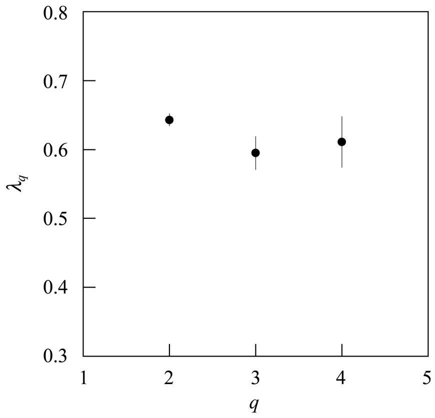

Fig. 4. Plot of λ_q against q at $H=0.3$ in ^{84}Kr-AgBr interactions at 1.7 AGeV.

4 Conclusions

From the present study of the 1.7 AGeV ^{84}Kr-AgBr interactions it may be concluded that the effect of intermittency is observed and the best power law behavior is exhibited at $H=0.3$ which suggests that the dynamical fluctuation pattern in ^{84}Kr-AgBr interactions is not self-similar but self-affine. The anomalous fractal dimensions of the intermittency are found to increase with the increase of the order of moments, which suggests the presence of multifractality of the emission of target fragments in ^{84}Kr-AgBr interactions. A minimum value of λ_q is observed at $q=3$, which suggested that there exists a non-thermal type of phase transition.

We are grateful to Professor I. Otterlund of Lund University in Sweden for supplying the emulsion plates.

References

1 Bialas A, Peschanski R. Nucl. Phys. B, 1986, **273**: 703
2 Bialas A, Peschanski R. Nucl. Phys. B, 1988, **308**: 857
3 De Wolf E A, Dremin I M, Kittel W. Phys. Rept., 1996, **270**: 1
4 Powell C F, Fowler P H, Perkins D H. The Study of Elementary Particles by Photographic Method. Oxford: Pergamon, 1959. 452
5 Ghosh D, Ghosh P, Ghosh A, Roy J. J. Phys. G, 1994, **20**: 1077
6 Ghosh D, Deb A, Bhattacharyya S, Ghosh J, Das R, Mukherjee S. Intern. J. Mod. Phys. E, 2003, **12**: 407
7 Ghosh D, Deb A, Bhattacharyya S, Ghosh J. Intern. J. Mod. Phys. E, 2004, **13**: 737
8 Ghosh D, Deb A, Chattopadhyay R et al. Phys. Rev. C, 1998, **58**: 3553
9 Ghosh D, Chattopadhyay R, Sarkar S et al. Z. Phys. C, 1997, **73**: 269
10 ZHANG Dong-Hai, LI Xue-Qin, JIA Hui-Ming, HE Chun-Le, LIU Fang, ZHAO Hui-Hua, LI Zhen-Yu, LI Jun-Sheng. Chin. Phys., 2007, **16**: 2683
11 Ochs W. Phys. Lett. B, 1990, **247**: 101
12 Ochs W. Z. Phys. C, 1991, **50**: 339
13 Van Hove L. Phys. Lett. B, 1968, **28**: 429
14 Van Hove L. Nucl. Phys. B, 1969, **9**: 331
15 Mandebrot B B. In Dynamics of Fractal Surfaces. Edited Family E, Vicsek T. Singapore: World Scientific, 1991
16 Bhattacharjee B. Nucl. Phys. A, 2005, **748**: 641
17 Haq M M, Islam S, Hasan R. Intern. J. Mod. Phys. E, 2006, **15**: 685
18 Ghosh D, Ghosh P, Ghosh A, Roy J. Phys. Rev. C, 1994, **49**: R1747
19 Ghosh D, Deb A, Bhattacharyya S, Ghosh J. Europhys. Lett., 2003, **63**: 805
20 Sarkar S, Goswami T D, Ghosh D, Deb A. Czech. J. Phys., 2003, **53**: 133
21 Ghosh D, Deb A, Sahoo S R et al. Czech. J. Phys., 2002, **52**: 789
22 WU Yuan-Fang, LIU Lian-Shou. Phys. Rev. Lett., 1993, **70**: 3197
23 Agababyan N M et al. Phys. Lett. B, 1998, **431**: 451
24 WANG Shao-Shun, WANG Zhao-Min, WU Chong. Phys. Lett. B, 1997, **410**: 323
25 WANG Shao-Shun, WU Chong. Chin. Phys. Lett., 2001, **18**: 18
26 Ghosh D, Deb A, Chattopadhyay K D et al. Intern. J. Mod. Phys. E, 2004, **13**: 1179
27 Ghosh D, Deb A, Mandal P et al. Phys. Rev. C, 2004, **69**: 017901
28 Ghosh D, Deb A, Patra K K, Ghosh J. Phys. Rev. C, 2002, **66**: 047901
29 Ghosh D, Deb A, Mandal P et al. Eur. Phys. J. A, 2002, **14**: 77
30 Ghosh D, Deb A, Bhattacharyya S et al. J. Phys. G, 2003, **29**: 983
31 Ghosh D, Deb A, Bhattacharyya S, Ghosh J. Nucl. Phys. A, 2003, **720**: 419
32 ZHANG Dong-Hai, LIU Fang, HE Chun-Le, ZHAO Hui-Hua, JIA Hui-Ming, LI Xue-Qin, LI Zhen-Yu, LI Jun-Sheng. Chin. Phys., 2006, **15**: 2564
33 ZHANG Dong-Hai, ZHAO Hui-Hua, LIU Fang, HE Chun-Le, JIA Hui-Ming, LI Xue-Qin, LI Zhen-Yu, LI Jun-Sheng. Chin. Phys., 2006, **15**: 1987
34 SONG Fu, ZHANG Dong-Hai, LI Jun-Sheng. Chin. Phys., 2005, **14**: 942
35 ZHANG Dong-Hai, LI Zhen-Yu, LI Hui-Ling, LI Jun-Sheng. Chin. Phys., 2005, **14**: 2451
36 Bialas A, Gozdzicki M. Phys. Lett. B, 1990, **252**: 483
37 ZHANG Dong-Hai, LI Hui-Ling. Chin. Phys. C, to be published
38 Hua R C. Phys. Rev. D, 1990, **41**: 1456
39 Peschanski R. Nucl. Phys. B, 1989, **327**: 144
40 Bialas A, Zaleswki K. Phys. Lett. B, 1990, **238** 413