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Preliminary result of bunch length measurement using a modified Michelson interferometer*

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Abstract Based on the femtosecond accelerator device which was built at the Shanghai Institute of Applied Physics (SINAP), recently a modified far infrared Michelson interferometer has been developed to measure the length of electron bunches via the optical autocorrelation method. Compared with our former normal Michelson interferometer, we use a hollow retroreflector instead of a flat mirror as the reflective mirror. The experimental setup and results of the bunch length measurement will be described in this paper.

Key words femtosecond linear accelerator, bunch length, interferometer, hollow retroreflector

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1 Introduction

Recent experiments on electron pulse compression have produced femtosecond electron bunches with a high peak current and brightness. Interest in short bunches arises from the requirements of high beam quality in potential applications. High quality nuclear physics accelerators, free electron laser drive accelerators, next generation linear colliders, and fourth generation light sources all require short time duration beam pulses[1]. Simultaneously, research into the diagnostics of the short electron bunch has played an important role in the progress. Several methods are in use or under development to measure the length of short electron bunches[2]. These generally fall into two categories: frequency-domain methods and time-domain methods. Among the time-domain methods for measuring the bunch length the use of a streak camera is well known. The streak cameras have been shown to be limited to bunch lengths longer than 200 fs. Additionally, streak cameras are expensive and the measurement system is sophisticated.

Instead of time-domain methods, frequency-domain measurements using coherent transition radiation (CTR) from metallic foils have shown promise in the measurement of very short femtosecond electron pulses.

In this paper we first present a theoretical and experimental investigation on the generation of high intensity coherent transition radiation from short electron bunches, then discuss the method based on coherent transition radiation to measure the bunch length of femtosecond electron bunches, and then the improved experimental setup and results of the bunch length measurement are given. Finally, we analyze the effects of humidity in air on bunch length measurements and explain the plan for future investigations.

2 Theoretical background

2.1 Coherent transition radiation

Radiation from a relativistic electron bunch such as synchrotron radiation, transition radiation, etc. intrinsically has a broad spectrum. If the wavelength of the radiation is shorter than the electron bunch length, the phases of the radiation emitted by the electrons differ from each another, so the radiation is incoherent. On the other hand, if the wavelength is longer than the bunch length, the radiation is coherent and the intensity of the radiation is proportional

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to the square of the electron numbers per bunch. The spectral intensity emitted by a bunch of N particles is given by

$$I_{\text{tot}}(\lambda)=NI_1(\lambda)+N(N-1)I_1(\lambda)\left|f(\lambda)\right|, \tag{1}$$

where $I_1(\lambda)$ is the intensity radiated by a single electron and $f(\lambda)$ is the bunch form factor[3, 4], which is the Fourier transform of the normalized electron density distribution $S(z)$. For a relativistic bunch whose transverse dimension is small compared to the length, the form factor becomes

$$f(\lambda)=\int S(z)\exp[\mathrm{i}2\pi(\boldsymbol{n}\cdot\boldsymbol{z})\lambda]\mathrm{d}z\,, \tag{2}$$

where $\boldsymbol{n}$ is the unit vector pointing from the center of the bunch to the observation point and $\boldsymbol{z}$ is the position vector of the electron relative to the bunch center. Obviously, a measurement of the radiation spectrum will give the form factor and the electron density distribution through the Fourier transform.

2.2 Electron bunch length measurement

Measurement of electron bunch length is often done by examination of the autocorrelation of the CTR signal with a Michelson interferometer[5]. The interferometer is composed of a beam splitter, a fixed flat mirror and a movable flat mirror. The light entering the Michelson interferometer is split into two parts by the beam splitter. The two parts travel in two different directions and are reflected back by the mirrors. After reflection the two radiation pulses are combined again and transmitted into a Golay detector to measure the light intensity.

The interferogram is obtained by measuring the detector signal as a function of the path difference in the two arms. The measured energy of the recombined radiation pulses are the radiation pulses from the fixed mirror, $E_{\text{fix}}=TRE(t)$, and the radiation from the movable mirror delayed in the time δ/c, $E_{\text{move}}=RTE(t+\delta/c)$. Here $R=R(\omega)$ and $T=T(\omega)$ are the reflection and transmission coefficients of the beam splitter. The intensity measured at the detector can be expressed as

$$\begin{aligned}I(\delta)\propto\int_{-\infty}^{+\infty}\left|TRE(t)+RTE\left(t+\frac{\delta}{c}\right)\right|\mathrm{d}t=\\ 2\int\left|RT\right|^2E(t)E^*\left(t+\frac{\delta}{c}\right)\mathrm{d}t+2\left|RT\right|^2\int_{-\infty}^{+\infty}\left|E(t)\mathrm{d}t\right|^2,\end{aligned} \tag{3}$$

where δ is the optical path difference, c is the speed of light. Alternatively, a similar expression can be obtained in the frequency domain by adding an extra phase difference $\mathrm{e}^{-\mathrm{i}\omega\delta/c}$ to the radiation from the movable arm at angular frequency $\omega=2\pi f$. Thus, the total energy measured at the detector is expressed as

$$\begin{aligned}I(\delta)\propto\int_{-\infty}^{+\infty}\left|TR\widetilde{E}(\omega)+RT\widetilde{E}(\omega)\mathrm{e}^{-\mathrm{i}\omega\delta/c}\right|^2\mathrm{d}\omega=\\ 2RE\int_{-\infty}^{+\infty}\left|RT\right|^2\left|\widetilde{E}(\omega)\right|^2\mathrm{e}^{-\mathrm{i}\omega\delta/c}\mathrm{d}\omega+\\ 2\int_{-\infty}^{+\infty}\left|RT\right|^2\left|\widetilde{E}(\omega)\right|^2\mathrm{d}\omega,\end{aligned} \tag{4}$$

and Eqs. (3) and (4) are related by the Fourier transform

$$\widetilde{E}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}E(t)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}t. \tag{5}$$

The baseline is defined as the intensity at $\delta\rightarrow\pm\infty$, where the two pulses are totally separated, hence, we have

$$I_\infty\propto\begin{cases}2\left|RT\right|^2\int_{-\infty}^{+\infty}\left|E(t)\right|^2\mathrm{d}t & \text{time domain}\\ 2\int_{-\infty}^{+\infty}\left|RT\right|^2\left|\widetilde{E}(\omega)\right|^2\mathrm{d}\omega & \text{frequency domain}\end{cases}. \tag{6}$$

By definition, the interferogram can be written as

$$S(\delta)=I(\delta)\text{–}I_\infty\propto\begin{cases}2\left|RT\right|^2\mathrm{Re}\int_{-\infty}^{+\infty}E(t)E^*\left(t+\frac{\delta}{c}\right)\mathrm{d}t\\ \quad\text{time domain}\\ 2\mathrm{Re}\int_{-\infty}^{+\infty}\left|RT\right|^2\left|\widetilde{E}(\omega)\right|^2\mathrm{e}^{-\mathrm{i}\omega\delta/c}\mathrm{d}\omega\\ \quad\text{frequency domain}\end{cases}. \tag{7}$$

Solving for $\left|\widetilde{E}(\omega)\right|^2$ in Eq. (7) yields

$$\left|\widetilde{E}(\omega)\right|^2\propto\frac{1}{4\pi c\left|RT\right|^2}\int_{-\infty}^{+\infty}S(\delta)\mathrm{e}^{\mathrm{i}\omega\delta/c}\mathrm{d}\delta, \tag{8}$$

where $\left|\widetilde{E}(\omega)\right|^2=\left|\widetilde{E}(-\omega)\right|^2$. Using Eq. (1) and the relation $I_{\text{total}}(\lambda)\propto\left|\widetilde{E}(2\pi c/\lambda)\right|^2$ the bunch form factor can be obtained from

$$\begin{aligned}f(\lambda;\hat{n})\ \propto\ \frac{1}{N-1}\left[\frac{1}{4\pi c\left|RT\right|^2NI_{\mathrm{e}}(\lambda)}\times\right.\\ \left.\int_{-\infty}^{+\infty}S(\delta)\mathrm{e}^{\mathrm{i}2\pi\lambda\delta/c}\mathrm{d}\delta-1\right],\end{aligned} \tag{9}$$

hence, the interferogram contains the frequency spectrum of coherent transition radiation and can be used to derive the bunch length.

For a bunch with Gaussian longitudinal distribution,

$$f(z)=\frac{1}{\sqrt{2\pi}\sigma_z}\mathrm{e}^{-\pi^2/2\sigma_z^2}\,, \tag{10}$$

the interferogram becomes

$$S(\delta) \propto \int_{-\infty}^{+\infty} f(z)f^*(z+\delta)\mathrm{d}z = \frac{1}{2\sqrt{\pi}\sigma_z}\mathrm{e}^{-\delta^2/4\sigma_z^2} \quad (11)$$

and the FWHM of this Gaussian interferogram is $4\sqrt{\ln 2}\sigma_z$. Therefore, the equivalent bunch length for a Gaussian bunch distribution is $\sqrt{\pi/\ln 2} \approx 0.7527$ times the interferogram FWHM.

3 Description of the equipment

The present experiment was performed at the Femtosecond Accelerator in the THz Research Center of SINAP, which mainly consists of a thermionic RF gun, an α magnet, and a SLAC (Stanford Linear Accelerator Center) type accelerating tube. The α magnet is used to compress the bunches produced by the thermionic RF gun. Then the electron beam is transported through the gun-to-linac beam line and finally accelerated up to 20—30 MeV by a SLAC type tube. Coherent THz radiation with high brightness will be emitted when super-short bunches pass through the aluminum foil[6].

It is well known that the resolution of a Michelson interferometer is mainly determined by the maximum optical path difference of the two parts of coherent light. However, this is only true if the planes of the mirrors remain in good alignment throughout the entire scan and if the light that passes through the interferometer is sufficiently collimated. However, the moving flat mirror tends to tilt or wobble as it is retarded and, as such, will not always be perpendicular to the incident beam. This causes the light originating from the reflection of the movable mirror to deviate off the optical axis of the detector, as shown in Fig. 1.

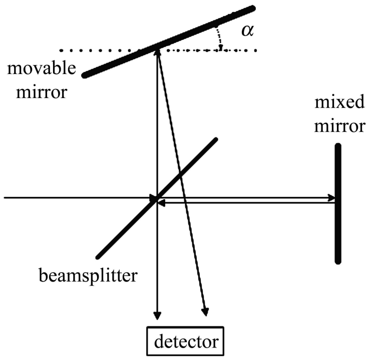

Fig. 1. Schematic diagram of the Michelson optics showing how tilting the moving mirror causes the recombinant beams to diverge from the optical axis.

In order to calculate the maximum allowable mirror tilt, we introduce first the modulation efficiency in the case of the circular shape of the light spot on the mirrors, which could be written as

$$\eta(m) = 2\cdot[J_1(a)/a], \quad (12)$$

where $\eta(m)$ is the modulation efficiency, $J_1(a)$ is the first order Bessel function, with a given by

$$a = 4\pi\sigma\alpha r\,. \quad (13)$$

σ is the wave number of interest (cm^{-1}), α is the tilt angle (radians) and r is the radius of the light spot (cm).

As a general rule, a satisfactory modulation efficiency must satisfy[7]

$$\eta(m) \geqslant 0.9\,, \quad (14)$$

$$2\cdot[J_1(a)/a] \geqslant 0.9\,, \quad (15)$$

According to Cohen[8], Eq. (12) can be approximated by

$$\frac{2J_1(4\pi\sigma r\alpha)}{4\pi\sigma r\alpha} \approx 1 - A\sigma^2\alpha^2\,, \quad (16)$$

where $A = 2\pi^2 r^2$. Assuming $\sigma = 100\ \mathrm{cm}^{-1}$, $r = 2.5$ cm requires that the allowable mirror tilt be kept at a value of $\alpha \leqslant 2.85\times10^{-4}$ rad, or $\alpha \leqslant 58.7$ arc seconds. According to the above analysis, α must therefore be less than 58.7 arc seconds throughout the entire scan.

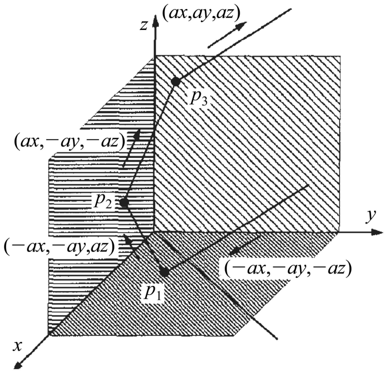

Fig. 2. The retroreflection property of the hollow retroreflector.

To overcome the effect of tilt in our former Michelson interferometer, our solution is to replace the flat mirror by a hollow retroreflector. A hollow retroreflector is a device made up of three mutually orthogonal reflective mirrors. For our experiment the hollow retroreflector was made by the Edmund corporation (NT46-189); because gold has a good reflectivity in the THz region, all the three mirror are metal coated. The most important advantage of the hollow retroreflector is the fact that it can return the light along a path that is parallel to that of the incident light. As a result, the required accuracy of alignment is 1 or 2 orders of magnitude less than that of the flat mirror. The interferometric accuracy is determined by the hollow retroreflector itself (NT46-189's maximun beam deviation is 5 arc seconds) and the position of the moving hollow retroreflector. A picture of the retroreflection property of the hollow retroreflector is shown in Fig. 2.

4 Measurement result and analysis

The interferogram obtained from a bunch length measurement is shown in Fig. 3. Because of the impacts of the beam splitter efficiency, the measured *FWHM* of the interferogram appears to be narrower than the real value[9]. Therefore a correction became necessary, which we obtained by investigating the beam splitter effects on the power spectrum. For a Gaussian bunch the relationship between the corrected *FWHM* and the measured *FWHM* is shown in Fig. 4. The measurement of 224 μm *FWHM* indicates a Gaussian bunch length of 74 μm or 248 fs after applying corrections due to beam splitter interferences.

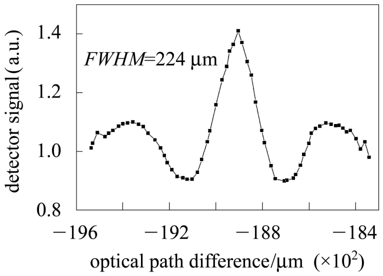

Fig. 3. Interferogram from a bunch length measurement.

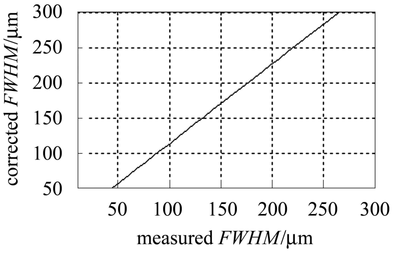

Fig. 4. The relationship between corrected *FWHM* and measured *FWHM* for a Gaussian bunch.

5 Future work

A system to produce and characterize femtosecond length bunches has been developed and described. We carried out CTR Michelson interferometry for femtosecond electron bunch diagnostics by a preliminary improved Michelson interferometer. We achieved a bunch length evaluation of about 248 fs at *FWHM* in the CTR interferometry. However, THz light is strongly absorbed in an atmospheric environment, which will affect the accuracy of the measurement. According to Birch[10], although water absorption will not affect the overall shape of the spectrum, it will broaden the bunch length. This broadening can be explained by dispersion due to water vapor in humid air. Because the refractive index of humid air is not constant over the THz range, different frequencies propagate with different velocities. Consequently, the radiation pulse spreads when it travels through the air. We calculated the interferograms taken in a vacuum and in humid air. Interferograms for both cases are shown in Fig. 5, displaying the increase of the interferogram width for the measurement in air. So for better precision, it is necessary to place the interferometer in a vacuum.

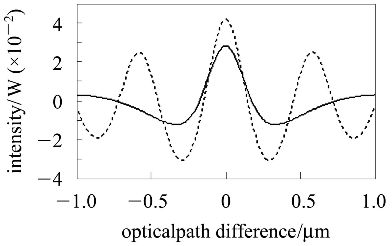

Fig. 5. Comparison of the interferograms taken in a vacuum (solid) and in humid air (dashed-line) ($E = 20$ MeV, $Q = 0.05$ nC, $\sigma_z = 200$ fs).

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