Temporal variation of fundamental constants in ^{229}Th transition*

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Abstract If is there enhanced sensitivity to variations of fine structure constant, nucleon mass and meson masses in the transition between the low energy and long-lived nuclear isomer and ground states? To answer this open problem, we investigate the transition between the long-lived 7.6±0.5 eV isomer and ground states in ^{229}Th based on the formulae derived from both the Nilsson model and Feynman-Hellmann theorem. Consistent conclusions are drawn by these two method. The sensitivity to relative variation of fine structure constant could be enhanced by 3—4 orders of magnitude, and to variations of nucleon mass and meson masses are enhanced by about 5—6 orders of magnitude in the ^{229}Th transition.

Key words fundamental physical constants, fine structure constant, nuclear isomer

PACS 06.20.Jr, 23.20.-g, 27.90.+6

1 Introduction

The temporal and spatial variation of fundamental constants is one of the fundamental problems in physics. Physicists have worked on this problem since 1937 when Dirac put forward the formulation of his large number hypothesis[1]. Over the last decade, it has been becoming a very popular topic (see the review[2—4] and references therein). The constraints of possible variations of fine structure constant $\alpha = e^2/\hbar c$, strong interaction parameter and masses were reported using the data in Oklo natural nuclear reactor (e.g. Refs. [5—11]), Big Bang nucleosynthesis (e.g. Refs. [12—14]), cosmic microwave background (e.g. Ref. [15]) and quasar absorption spectra (e.g. Refs. [16—19]). Experiments in laboratory is based on the comparison of different atomic clocks and of an atomic clock with ultrastable oscillators (see recent researches in Refs. [20—31]). Measurements in laboratory have the advantages of being repeatable and allowing for better control of the systematic errors. However, They have placed weaker constraints on the temporal variation of α so far. Therefore, we need to choose carefully the transition to be measured to increase the sensitivity.

The lowest-lying excited state in nuclear physics is the 7.6 ± 0.5 eV[32] isomer state in ^{229}Th. Its lifetime has been estimated to be roughly 5 hours[32]. The transition between this isomer and ground states is expected as possible candidates for measuring variation of α and strong interaction parameters in laboratory with higher accuracy[33—36]. In Ref. [33], the relative effects of variation of fine structure constant α and the strong interaction parameter m_q/Λ_{QCD} were estimated in the ^{229}Th transitions, and the author found a 5—6 orders of magnitude enhancement. Here

Received 8 July 2008
* Supported by National Natural Science Foundation of China (10705014,10535010,10775068), Jiangsu Planned Projects for Postdoctoral Research Funds (0701034B), Post-doctoral Research Foundation of Nanjin University (0204003080) and CAS Knowledge Innovation Project (KJCX2-SW-N02)
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$\Lambda_{\rm QCD}$ is the quantum chromodynamics (QCD) scale and $m_{\rm q}$ the light quark mass. However, in the recent work[38], the authors argue that there is no expectation of significantly enhanced sensitivity to a variation of the fine structure constant in ^{229}Th and accurately estimating a frequency shift due to variations of strong interaction parameters may be beyond current nuclear structure techniques.

Based on formulae derived from the Nilsson model and Feynman-Hellmann theorem, sensitivity of the variation of the transition frequency to variation of the fine structure constant is reexamined by relativistic mean field theory in the ^{229}Th transition. There is 3—4 orders of magnitude enhancement of the sensitivity to variation of the fine structure constant being obtained by both methods. Sensitivities of the variation of the transition frequency to variations of the nucleon mass, σ-, ω- and ρ-meson masses are also investigated, and 4—6 orders of magnitude enhancement is found in ^{229}Th.

2 Enhanced sensitivity to variation of fundamental constants in the ^{229}Th transition

An enhancement factor κ_i is introduced,

$$\frac{\delta\omega}{\omega}=\kappa_i\frac{\delta\lambda_i}{\lambda_i}, \tag{1}$$

where ω is the transition frequency, and λ_i the studied parameter. $\delta\omega$ is the change of the transition frequency over a time interval, and $\delta\lambda_i$ the variation of the parameter λ_i over the same period. It is obvious that the relative variation of the transition frequency is more sensitive to the relative variation of parameter when the absolute value of κ_i is bigger, and this will improve the accuracy of the present day experiments in laboratory.

Firstly, we calculate the enhancement factors κ_i of variations of the fine structure constant α, nucleon mass $m_{\rm n}$, meson masses m_σ, m_ω and m_ρ (denoted by κ_α, $\kappa_{\rm n}$, κ_σ, κ_ω and κ_ρ, respectively) based on Nilsson model. The energies of the long-lived isomer and ground states may be described by[39],

$$E=E_0+C\Lambda\Sigma+D\Lambda^2, \tag{2}$$

where Λ is the projection of the valence nucleon orbital angular momentum on the nuclear symmetry axis z, and Σ the spin projection. The total angular momentum projection is given by $\Omega=\Lambda+\Sigma$. The ground and the first excited states in ^{229}Th are $\Omega^P[Nn_z\Lambda]=5/2^+[633]$ and $3/2^+[631]$, respectively. The energy difference between these two levels is $\omega=E_e-E_g=2C-8D=7.6$ eV. The Values of C and D are taken from Ref. [40]. $2C\approx 8D\approx -1.4$ MeV is adopted because ω is about 5 orders of magnitude smaller than C and D. Then, one can obtain the relative variation of the transition frequency,

$$\frac{\delta\omega}{\omega}=\frac{\delta(2C)-\delta(8D)}{\omega}=0.18\times10^6\left(\frac{\delta D}{D}-\frac{\delta C}{C}\right). \tag{3}$$

The spin-orbit interaction is inversely proportional to the nucleon mass m_n squared, $C\propto 1/m_n^2$[33]. Then,

$$\frac{\delta C}{C}=-2\frac{\delta m_n}{m_n}. \tag{4}$$

The orbit-orbit interaction constant D was assumed to depend linearly on the mean field potential V_0[34], $D\approx \text{const}\times V_0$, and then,

$$\frac{\delta D}{D}\approx\frac{\delta V_0}{V_0}. \tag{5}$$

We perform the RMF calculations following the method described in Ref. [34]. Then the contributions from the variation of nucleon mass, meson masses and the fine structure constant α are calculated for neutron and for proton, respectively,

$$\frac{\delta\omega}{\omega}\approx 10^4\left(0.38\frac{\delta\alpha}{\alpha}-234.00\frac{\delta m_\sigma}{m_\sigma}+237.00\frac{\delta m_\omega}{m_\omega}+3.96\frac{\delta m_\rho}{m_\rho}+66.78\frac{\delta m_{\rm n}}{m_{\rm n}}\right), \tag{6a}$$

$$\frac{\delta\omega}{\omega}\approx 10^4\left(9.18\frac{\delta\alpha}{\alpha}-299.52\frac{\delta m_\sigma}{m_\sigma}+306.72\frac{\delta m_\omega}{m_\omega}-3.60\frac{\delta m_\rho}{m_\rho}+76.68\frac{\delta m_{\rm n}}{m_{\rm n}}\right), \tag{6b}$$

we can see that the enhancement factors $\kappa_{\rm n}$, $\kappa_\sigma(\kappa_\omega)$ and κ_ρ are about 5, 6 and 4 orders of magnitude, respectively. The enhancement factor κ_α is about 2 orders of magnitude larger for an excited state with single proton configuration than that with a single neutron configuration. For ^{229}Th, both the isomer and ground states are neutron excited state. Therefore, the enhancement of the sensitivity to α variation is 3 orders of magnitude. This is because the fine structure constant α is a measure of the strength of the electromagnetic force, and neutron has no net charge. On the other hand, with nuclear strong interaction and the Coulomb force being solved self-consistently, the relation between the strong interaction and the Coulomb force is dealt with consistently in RMF theory. Neutron will be affected by Coulomb force indirectly. Thus, the enhancement factor κ_α for neutron is not equal to zero[34].

Then, using the general results obtained by Feynman-Hellmann theorem, we investigate the enhancement factor κ_α again as follows[36]. The energy of the transition between the ground state ($|\text{g.s.}\rangle$) and an excited state ($|f\rangle$) is determined by,

$$\omega \equiv \langle f|H|f\rangle - \langle \text{g.s.}|H|\text{g.s.}\rangle \equiv \langle\langle H\rangle\rangle \,, \tag{7}$$

where H is the nuclear Hamiltonian. $\langle\langle O\rangle\rangle$ denotes the difference of the expectation value of the operator O between two states. By using the Feynman-Hellmann theorem, the temporal variation of transition frequency reads[38],

$$\dot{\omega} = \sum_i \left(\langle f|\frac{\partial H}{\partial \lambda_i}|f\rangle - \langle \text{g.s.}|\frac{\partial H}{\partial \lambda_i}|\text{g.s.}\rangle \right) \dot{\lambda}_i, \tag{8}$$

with λ_i are the dynamical constants embedded in H. The nuclear Hamiltonian are expressed explicitly as $H = H_\text{n} + V_\text{c}$ where H_n includes the kinetic energy T and the nuclear strong interaction V_n. V_C is the Coulomb force which depends on the fine structure constant linearly. Then the relative variation of the transition frequency is[38, 41],

$$\frac{\dot{\omega}}{\omega} = \left[\frac{\langle\langle V_\text{C}\rangle\rangle}{\omega} \right] \frac{\dot{\alpha}}{\alpha}. \tag{9}$$

How big of the enhancement factor $\kappa_\alpha = \langle\langle V_C\rangle\rangle/\omega$ is in a nuclear transition depends on the Coulomb energy difference of the two states. We calculate Coulomb energies of the isomer and ground state by the RMF theory for ^{229}Th. The Coulomb energy difference is $\langle\langle V_\text{C}\rangle\rangle \approx -0.698$ MeV. It results in $\kappa_\alpha = \langle\langle V_\text{C}\rangle\rangle/\omega \approx -9.184\times10^4$. The enhancement of the sensitivity to α variation is about 4 orders of magnitude, which is close to that have been obtained by Eq. (6). Further investigation based on Eq.(8) shows that the enhancements of the sensitivity to nucleon mass and meson masses variations are about 5—6 orders of magnitude (see Ref. [36] for details).

3 Summary

Using formulae obtained by the Nilsson model and Feynman-Hellmann theorem, we investigate sensitivities of the variation of the transition frequency to variations of the fine structure constant, nucleon mass, σ-, ω- and ρ-meson masses in ^{229}Th by RMF theory. Consistent results are obtained by these two method. The enhancement factor κ_α is about 3—4 orders of magnitude. The enhancement factors $\kappa_\text{n}, \kappa_\sigma, \kappa_\omega$ and κ_ρ are about 4—6 orders of magnitude. We conclude that the 7.6 eV isomer state in ^{229}Th is a very narrow state with large enhancement factors for variations of α, nucleon mass and meson masses. The transition between this isomer and the ground state would be a important candidate in researching the temporal variation of some physical fundamental constants.

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