# $\Delta$ contribution to the parity－violating nucleon－nucleon force＊ 

LIU Yan－Rui（刘言锐）${ }^{1 ; 11}$ ZHU Shi－Lin（朱世琳 $)^{2 ; 2)}$<br>1 （Institute of High Energy Physics，CAS，P．O．Box 918（4），Beijing 100049，China）<br>2 （Department of Physics，Peking University，Beijing 100871，China）


#### Abstract

Because the nucleon may be excited and transformed into a virtual $\Delta$ resonance easily，we consider the decuplet contribution to the parity－violating（PV）nucleon－nucleon interaction in the chiral effective field theory．The effective PV nucleon－nucleon potential is derived without introducing any unknown coupling constants．


Key words nuclear parity violation，effective field theory，delta
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## 1 Introduction

The strangeness－changing weak processes $(\Delta S=$ 1）can be studied both in the semi－leptonic decays and strangeness－changing hadronic weak interactions．In contrast，the nuclear parity violation is almost the unique way to study the $\Delta S=0$ hadronic weak inter－ action experimentally．Up to now，our knowledge on such processes is still relatively poor．

The search for nuclear parity violation ${ }^{[1]}$ started shortly after the observation of the parity violation in nuclear beta decay ${ }^{[2]}$ ．Thereafter，there had been many experimental investigations of nuclear parity－ violation such as the polarized proton－nucleus scat－ tering，radiative np capture，$\gamma$ decay of nuclei，neu－ tron spin rotation，and atomic parity－violation ${ }^{[3-6]}$ ．

The parity－violating（PV）effect is very small in nuclear processes．Such an effect can be measured through the asymmetry of the cross－sections in the polarized experiments．In some heavy nuclei，there exist two energy levels with different parity which are very close to each other．The PV weak interaction mixes these two levels．Then the asymmetry may be amplified．

However，the few－body nuclear system provides a much cleaner place to study nuclear parity－violation though the asymmetry is only $\sim 10^{-7}$ ．Experimen－
tal progress in this field is very encouraging．Sev－ eral years ago，the longitudinal analyzing power of $\overrightarrow{\mathrm{p}} \mathrm{p}$ scattering was measured at TRIUMF ${ }^{[7,8]}$ ．There are also on－going experiments to measure the photon asymmetry in radiative $\overrightarrow{\mathrm{n}} \mathrm{p}$ capture at LANSCE ${ }^{[9]}$ ，the helicity asymmetry in the photodisintegration of deu－ terium at IASA ${ }^{[10]}$ ，and the spin rotation of polarized neutrons in ${ }^{4} \mathrm{He}$ at $\mathrm{NIST}^{[11,12]}$ ．

Strong interaction dominates the nucleon－nucleon interaction，which is repulsive at the short range． Therefore the weak interactions between nucleons me－ diated directly by W and Z bosons are strongly sup－ pressed since the interaction range is around 0.002 fm ． On the other hand，the meson nucleon interaction ver－ tex can be parity－violating．Thus one can study nu－ clear parity violation after replacing one strong vertex by the weak one in the meson exchange model．

Historically，the study of nuclear parity－violation with the meson－exchange model started in $1964{ }^{[13]}$ ． Later，nuclear parity－violation was extensively stud－ ied in this framework ${ }^{[14-16]}$ ．In 1980，Desplanques， Donoghue，and Holstein（DDH）investigated the PV nuclear force in a general way and considered the ex－ changed mesons up to $\rho$ and $\omega^{[17]}$ ．The DDH method has become a standard way in analyzing experiments since then．In that paper，the PV vertices were pa－ rameterized with seven coupling constants：$h_{\pi}^{1}, h_{\rho}^{0,1,2}$ ，

[^0]$h_{\omega}^{0,1}$ and $h_{\rho}^{1 \prime}$. $h_{\rho}^{1 \prime}$ was found to be small and usually neglected ${ }^{[18]}$. DDH estimated these coupling constants using the quark model and $S U(6)_{\mathrm{w}}$ symmetry. They gave reasonable ranges for the couplings and presented their best guesses. Surprisingly, various experimental constraints are more or less consistent with these DDH estimates, except that the bound on $h_{\pi}^{1}$ from nuclear anapole moment in Cesium ${ }^{[19]}$ does not agree well with those from other experiments ${ }^{[20]}$.

In the past decades, there has been important progress in the study of parity violation both in the single-nucleon case ${ }^{[21-25]}$ and NN system ${ }^{[26-30]}$. In order to investigate nuclear parity violation in a model-independent way, Zhu, Maekawa, Holstein, Ramsey-Musolf and van Kolck reformulated the PV nucleon-nucleon interaction in the framework of effective field theory $(\mathrm{EFT})^{[31]}$. At very low energy, the momenta of external fields are very small and the pion can be integrated out. EFT without explicit pions is appropriate. When the external momenta are comparable with the pion mass, EFT with explicit pions is necessary.

For the description of PV NN forces in EFT with explicit pions, the treatment is similar to the study of parity-conserving (PC) NN force in $\mathrm{EFT}^{[32,33]}$. One simply replaces one PC vertex with one PV vertex and imposes chiral symmetry on the PV vertex. In Ref. [31], the PV potential was calculated to $\mathcal{O}(Q)$ in Weinberg's power counting where $Q$ is the typical scale of the processes. The leading order $\left(\mathcal{O}\left(Q^{-1}\right)\right)$ result reproduced the pion exchange part of DDH formalism. At the next leading order $\left(\mathcal{O}\left(Q^{0}\right)\right)$, explicit computation shows there is in fact no contribution. At the third order, the short range potential was described with contact interactions. The medium range potential was deduced from two-pion exchange interactions while the long range potential was obtained by considering corrections to the one-pion exchange interaction. In this framework, Ref. [34] studied a minimal set of parameters to describe low-energy PV observables. In Ref. [35], the authors studied PV asymmetry in $\mathrm{np} \rightarrow \mathrm{d} \gamma$ within EFT.

The decuplet baryon $\Delta$ couples to $N \pi$ strongly. The virtual $\Delta$ may aslo contribute to the PV nucleonnucleon interactions, which was noted long time ago in Refs. [36-38]. The DDH formalism was extended to investigate the effects due to $\Delta^{[39]}$. With development of the modern EFT language, we will extend the former work ${ }^{[31]}$ and calculate the PV potential by considering $\Delta$ as an explicit degree of freedom in EFT in the present work. The present work was part of Y. R. Liu's thesis submitted in April, 2007. It is interesting to note that an independent work dealing with similar topics appeared recently ${ }^{[40]}$. However the way to derive the potential in this work is different from
that in Ref. [40]. In a recent work ${ }^{[41]}$, the calculation of the longitudinal asymmetry in $\mathrm{pp} \rightarrow \mathrm{pp}$ by including $2 \pi$ exchange effects which include NN and $\mathrm{N} \Delta$ intermediate states is presented.

In order to include the $\Delta$ degree of freedom systematically, we employ the heavy baryon chiral perturbation theory with $\Delta$. The expansion scheme was called the small scale expansion (SSE) ${ }^{[42]}$, which was widely used to study the processes involving $\Delta^{[43-51]}$. Both the pion mass and the mass difference between nucleon and $\Delta$ isobar are counted as the order $\mathcal{O}(Q)$. We use this formalism to calculate the $\Delta$ contribution to the PV NN potential. In the following section, we present the relevant Lagrangian. In Section 3, we calculate PV potentials due to the virtual $\Delta$ baryon. The final section is a short summary.

## 2 Lagrangians

In the EFT study of the nucleon-nucleon potential, one performs a systematic expansion of Lagrangians and amplitudes ${ }^{[32,33]}$. We present relevant Lagrangians $\mathcal{L}^{(\nu)}$ in this section. They are grouped with chiral index $\nu=d+f / 2-2$ where $d$ is the number of derivatives and powers of the pion mass and $f$ the number of fermion fields. When we consider the $\Delta$ contribution to the parity-violating potential up to the third order $\mathcal{O}(Q)$, we need only the lowest order chiral Lagrangians.

For the $\pi \mathrm{NN}$ interaction, the PC part is

$$
\begin{equation*}
\mathcal{L}_{\pi \mathrm{N}, \mathrm{PC}}^{(0)}=\bar{N}\left[\mathrm{i} v \cdot \mathcal{D}+2 g_{\mathrm{A}}^{0} S \cdot A\right] N \tag{1}
\end{equation*}
$$

where

$$
\begin{align*}
\mathcal{D}_{\mu} & =D_{\mu}+V_{\mu}, \quad V_{\mu}=\frac{1}{2}\left(\xi D_{\mu} \xi^{\dagger}+\xi^{\dagger} D_{\mu} \xi\right) \\
A_{\mu} & =-\frac{\mathrm{i}}{2}\left(\xi D_{\mu} \xi^{\dagger}-\xi^{\dagger} D_{\mu} \xi\right)=-\frac{D_{\mu} \pi}{F_{\pi}}+\mathcal{O}\left(\pi^{3}\right) \\
\xi & =\exp \left(\frac{\mathrm{i} \pi^{\mathrm{a}} \tau^{\mathrm{a}}}{2 F_{\pi}}\right)=\exp \left(\frac{\mathrm{i} \pi}{F_{\pi}}\right) \tag{2}
\end{align*}
$$

Here $V_{\mu}$ and $A_{\mu}$ are the chiral connection and the axial field respectively. $v_{\mu}$ is the velocity and $\boldsymbol{S}_{\mu}$ is the Pauli-Lubanski spin vector. $F_{\pi}=92.4 \mathrm{MeV}$ is the pion decay constant and $\boldsymbol{\tau}^{\mathrm{a}}$ is the Pauli matrix. Here $g_{\mathrm{A}} \approx 1.27$ is the nucleon axial vector coupling constant.

The PV part is

$$
\begin{align*}
\mathcal{L}_{\pi \mathrm{N}, \mathrm{PV}}^{(-1)}= & -\frac{h_{\pi}^{1} F_{\pi}}{2 \sqrt{2}} \bar{N} \boldsymbol{X}_{-}^{3} N= \\
& -\mathrm{i} h_{\pi}^{1}\left(\overline{\mathrm{p}} \mathrm{n} \pi^{+}-\overline{\mathrm{n}} \mathrm{p} \pi^{-}\right)+\cdots \tag{3}
\end{align*}
$$

where

$$
\boldsymbol{X}_{-}^{3}=\xi^{+} \boldsymbol{\tau}^{3} \xi-\xi \boldsymbol{\tau}^{3} \xi^{+}
$$

and $h_{\pi}^{1} \sim 10^{-7}$ is the weak coupling constant. The
ellipsis denotes terms involving more pions.
For the part containing $\Delta$, the leading Lagrangian reads ${ }^{[42]}$

$$
\begin{align*}
\mathcal{L}_{\pi \mathrm{N} \Delta, \mathrm{PC}}^{(0)}= & -\mathrm{i} \bar{T}^{\mu \mathrm{a}} v \cdot D^{\mathrm{ab}} T_{\mu}^{\mathrm{b}}+\delta \bar{T}^{\mu \mathrm{a}} T_{\mu}^{\mathrm{a}}+ \\
& 2 g_{\pi \mathrm{N} \Delta}\left(\bar{T}^{\mu \mathrm{a}} A_{\mu}^{\mathrm{a}} N+\bar{N} A_{\mu}^{\mathrm{a}} T^{\mu \mathrm{a}}\right) \tag{4}
\end{align*}
$$

where $\delta=m_{\Delta}-m_{\mathrm{N}}, A_{\mu}^{\mathrm{a}}=\frac{1}{2} \operatorname{Tr}\left(A_{\mu} \boldsymbol{\tau}^{\mathrm{a}}\right)$ and $T^{\mu}$ represents $\Delta$ fields with

$$
\begin{align*}
& T_{\mu}^{1}=\frac{1}{\sqrt{2}}\binom{\Delta^{++}-\Delta^{0} / \sqrt{3}}{\Delta^{+} / \sqrt{3}-\Delta^{-}}_{\mu} \\
& T_{\mu}^{2}=\frac{\mathrm{i}}{\sqrt{2}}\binom{\Delta^{++}+\Delta^{0} / \sqrt{3}}{\Delta^{+} / \sqrt{3}+\Delta^{-}}_{\mu}  \tag{5}\\
& T_{\mu}^{3}=-\sqrt{\frac{2}{3}}\binom{\Delta^{+}}{\Delta^{0}}_{\mu}
\end{align*}
$$

In this Lagrangian, we have used $\mathcal{C}=\sqrt{2} g_{\pi N \Delta}$ with the language in Ref. [52, 53]. The quark model gives the relation $g_{\pi \mathrm{N} \Delta}=\frac{3 \sqrt{2}}{5} g_{\mathrm{A}}$. Since the PV $\pi \mathrm{N} \Delta$ part contributes to the PV potential beyond the order of $\mathcal{O}(Q)^{[22]}$, we do not consider it here.

## $3 \Delta$ contribution to PV NN potential

Because the PV contribution is tiny, one PV vertex is enough for the present study. The intermediate $\Delta$ contribution to parity-violating potential is presented in Fig. 1. We employ the counting scheme


Fig. 1. Diagrams for intermediate $\Delta$ contribution to PV NN potential. The dotted lines are pions. The full lines represent nucleon while the double lines represent $\Delta$ states. Vertices with black dot mean the parity-violating $\pi \mathrm{N}$ interaction.
of SSE and truncate the expansion at the order $\mathcal{O}(Q)$. To this order, the triangle diagrams do not contribute. In the case without the $\Delta$ contribution, the box diagrams are two-particle reducible. Now the diagrams are all two-particle irreducible (2PI). That is, the diagrams in Fig. 1 will not induce double counting problem. In the following, we calculate the effective potentials in detail.

First, we consider the cross diagrams (a)—(d) in Fig. 1. From the vertices, one can construct four cases of transitions which include charge-conserving cases $\mathrm{pp} \rightarrow \mathrm{pp}, \mathrm{nn} \rightarrow \mathrm{nn}$ and $\mathrm{pn} \rightarrow \mathrm{pn}(\mathrm{np} \rightarrow \mathrm{np})$ and charge-changing case $\mathrm{pn} \rightarrow \mathrm{np}(\mathrm{np} \rightarrow \mathrm{pn})$.

For $\mathrm{pp} \rightarrow \mathrm{p}$, the sum of (a) and (b) gives

$$
\begin{equation*}
\mathrm{i} T=-\mathrm{i} \frac{4 \sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} Z(Q) \overline{\mathrm{p}}\left[\boldsymbol{S}_{1} \cdot \mathrm{q}, \boldsymbol{S}_{1 \mu}\right] \mathrm{p} \overline{\mathrm{p}} \boldsymbol{S}_{2}^{\mu} \mathrm{p} \tag{6}
\end{equation*}
$$

where $q=p_{1}-p_{1}^{\prime}=p_{2}^{\prime}-p_{2}, Q^{2}=-q^{2} \approx \boldsymbol{q}^{2}, \Lambda_{\chi}=4 \pi F_{\pi}$ and

$$
\begin{equation*}
Z(Q)=2 L(Q)+\frac{\pi}{2 \delta}\left(4 m_{\pi}^{2}+Q^{2}\right) A(Q)-\frac{2}{\delta} B(Q) \tag{7}
\end{equation*}
$$

with

$$
\begin{align*}
L(Q) & =\frac{\sqrt{4 m_{\pi}^{2}+Q^{2}}}{Q} \ln \frac{Q+\sqrt{4 m_{\pi}^{2}+Q^{2}}}{2 m_{\pi}} \\
A(Q) & =\frac{1}{2 Q} \arctan \frac{Q}{2 m_{\pi}} \\
B(Q) & =\int_{0}^{1} \mathrm{~d} y \int_{\delta}^{\infty} \mathrm{d} \lambda \frac{m_{\pi}^{2}-\delta^{2}+y(1-y) Q^{2}}{\lambda^{2}+m_{\pi}^{2}-\delta^{2}+y(1-y) Q^{2}} \tag{8}
\end{align*}
$$

In calculating the loop integrals, we have used the dimensional regularization. The divergent part could be absorbed by the renormalization of the counterterms at the same chiral order. Here we only retain the non-analytic terms.

For $n n \rightarrow n n$ channel, the sum of (a) and (b) is

$$
\begin{equation*}
\mathrm{i} T=\mathrm{i} \frac{4 \sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} Z(Q) \bar{n}\left[\boldsymbol{S}_{1} \cdot q, \boldsymbol{S}_{1 \mu}\right] n \bar{n} \boldsymbol{S}_{2}^{\mu} n \tag{9}
\end{equation*}
$$

This result is similar to the $\mathrm{pp} \rightarrow \mathrm{pp}$ channel. Similarly, one gets contributions from the mirror diagrams (c) and (d). Since the initial particles are identical, the operator form for these two channels will generate (a), (b) and the mirror diagrams (c), (d) simultaneously.

Compared with the case without the $\Delta$ contribution ${ }^{[31]}$, there is an additional channel $\mathrm{pn} \rightarrow$ pn . The sum of (a) - (d) in the operator form reads

$$
\begin{align*}
\mathrm{i} T= & \mathrm{i} \frac{2 \sqrt{2}}{3} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} Z(Q)\left\{\bar{N}\left[\boldsymbol{S} \cdot k, \boldsymbol{S}_{\mu}\right] \boldsymbol{\tau}_{3} N \bar{N} \boldsymbol{S}^{\mu} N-\right. \\
& \left.\bar{N}\left[\boldsymbol{S} \cdot k, \boldsymbol{S}_{\mu}\right] N \bar{N} \boldsymbol{S}^{\mu} \boldsymbol{\tau}_{3} N\right\} \tag{10}
\end{align*}
$$

Here $k$ is the initial momentum minus the final momentum for a nucleon line.

After combining the above three channels, we get

$$
\begin{align*}
& \mathrm{i} \frac{4 \sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} Z(Q) \bar{N}\left[\boldsymbol{S} \cdot k, \boldsymbol{S}_{\mu}\right] \boldsymbol{\tau}_{3} N \bar{N} \boldsymbol{S}^{\mu} N- \\
& \mathrm{i} \frac{8 \sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} Z(Q) \bar{N}\left[\boldsymbol{S} \cdot k, \boldsymbol{S}_{\mu}\right] N \bar{N} \boldsymbol{S}^{\mu} \boldsymbol{\tau}_{3} N . \tag{11}
\end{align*}
$$

For the charge-changing case $\mathrm{pn} \rightarrow \mathrm{np}$, the sum of diagrams (a)-(d) gives

$$
\begin{equation*}
-\frac{\sqrt{2}}{6} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} Y(Q) \epsilon^{\mathrm{ij} 3} \bar{N} \boldsymbol{\tau}^{\mathrm{i}} N \bar{N} \boldsymbol{\tau}^{\mathrm{j}} \boldsymbol{\sigma} \cdot \boldsymbol{k} N \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
Y(Q)=2 L(Q)+\frac{2 \pi}{3 \delta}\left(2 m_{\pi}^{2}+Q^{2}\right) A(Q)-\frac{2}{\delta} C(Q) \tag{13}
\end{equation*}
$$

with

$$
\begin{equation*}
C(Q)=\int_{0}^{1} \mathrm{~d} y \int_{\delta}^{\infty} \mathrm{d} \lambda \frac{m_{\pi}^{2}-\delta^{2}+\frac{4}{3} y(1-y) Q^{2}}{\lambda^{2}+m_{\pi}^{2}-\delta^{2}+y(1-y) Q^{2}} \tag{14}
\end{equation*}
$$

Next, we consider the box diagrams (e)-(h) in Fig. 1. There are also four cases: charge-conserving processes $\mathrm{np} \rightarrow \mathrm{np}(\mathrm{pn} \rightarrow \mathrm{pn})$, $\mathrm{pp} \rightarrow \mathrm{pp}$ and $\mathrm{nn} \rightarrow \mathrm{nn}$ and charge-changing process $n p \rightarrow p n(p n \rightarrow n p)$.

For the channel $n p \rightarrow n p$, the sum of the diagrams (e)—(h) gives

$$
\begin{align*}
\mathrm{i} T= & \mathrm{i} \frac{2 \sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} W(Q)\left\{\bar{N}\left[\boldsymbol{S} \cdot k, \boldsymbol{S}_{\mu}\right] \boldsymbol{\tau}_{3} N \bar{N} \boldsymbol{S}^{\mu} N-\right. \\
& \left.\bar{N}\left[\boldsymbol{S} \cdot k, \boldsymbol{S}_{\mu}\right] N \bar{N} \boldsymbol{S}^{\mu} \boldsymbol{\tau}_{3} N\right\} \tag{15}
\end{align*}
$$

where

$$
\begin{equation*}
W(Q)=2 L(Q)-\frac{\pi}{2 \delta}\left(4 m_{\pi}^{2}+Q^{2}\right) A(Q)-\frac{2}{\delta} B(Q) \tag{16}
\end{equation*}
$$

In calculating the amplitudes, we use the following formula

$$
\begin{equation*}
\frac{1}{v \cdot k+\mathrm{i} \epsilon}=-\frac{1}{-v \cdot k+\mathrm{i} \epsilon}-2 \pi \mathrm{i} \delta(v \cdot k) . \tag{17}
\end{equation*}
$$

In the case without the $\Delta$ contribution, the part from the $\delta$ function was subtracted to separate the contributions from the iterated one-pion exchange and those from the irreducible two-pion exchange. Now this part is included because the diagram is 2 PI .

The diagrams (e) and (f) result in

$$
\begin{equation*}
\mathrm{i} T=-\mathrm{i} \frac{4 \sqrt{2}}{3} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} W(Q) \bar{p}\left[\boldsymbol{S}_{1} \cdot q, \boldsymbol{S}_{1 \mu}\right] p \bar{p} \boldsymbol{S}_{2}^{\mu} p \tag{18}
\end{equation*}
$$

for the channel $\mathrm{pp} \rightarrow \mathrm{pp}$ and

$$
\begin{equation*}
\mathrm{i} T=\mathrm{i} \frac{4 \sqrt{2}}{3} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} W(Q) \bar{n}\left[\boldsymbol{S}_{1} \cdot q, \boldsymbol{S}_{1 \mu}\right] n \bar{n} \boldsymbol{S}_{2}^{\mu} n \tag{19}
\end{equation*}
$$

for $\mathrm{nn} \rightarrow \mathrm{nn}$.

After combining these results, we get the chargeconserving amplitude from box diagrams

$$
\begin{align*}
& -\mathrm{i} \frac{4 \sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} W(Q) \bar{N}\left[\boldsymbol{S} \cdot k, \boldsymbol{S}_{\mu}\right] \boldsymbol{\tau}_{3} N \bar{N} \boldsymbol{S}^{\mu} N- \\
& \mathrm{i} \frac{8 \sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} W(Q) \bar{N}\left[\boldsymbol{S} \cdot k, \boldsymbol{S}_{\mu}\right] N \bar{N} \boldsymbol{S}^{\mu} \boldsymbol{\tau}_{3} N .(20 \tag{20}
\end{align*}
$$

For the charge-changing case $\mathrm{np} \rightarrow \mathrm{pn}$, one sums the amplitudes from (e)-(h) and gets

$$
\begin{equation*}
-\frac{\sqrt{2}}{6} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}} X(Q) \epsilon^{\mathrm{i} 3} \bar{N} \boldsymbol{\tau}^{\mathrm{i}} N \bar{N} \boldsymbol{\tau}^{\mathrm{j}} \boldsymbol{\sigma} \cdot \boldsymbol{k} N \tag{21}
\end{equation*}
$$

where

$$
\begin{equation*}
X(Q)=2 L(Q)-\frac{2 \pi}{3 \delta}\left(2 m_{\pi}^{2}+Q^{2}\right) A(Q)-\frac{2}{\delta} C(Q) \tag{22}
\end{equation*}
$$

After combing Eqs. (11), (12), (20) and (21), we finally get the $\Delta$ contribution to nuclear parity violation

$$
\begin{align*}
& -\frac{\sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}}[W(Q)-Z(Q)] \epsilon^{\mathrm{ijk}} N^{\dagger} k^{\mathrm{i}} \boldsymbol{\sigma}^{\mathrm{j}} \boldsymbol{\tau}_{3} N N^{\dagger} \boldsymbol{\sigma}^{k} N \\
& -\frac{2 \sqrt{2}}{9} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}}[W(Q)+Z(Q)] \epsilon^{\mathrm{ijk}} N^{\dagger} k^{\mathrm{i}} \boldsymbol{\sigma}^{\mathrm{j}} N N^{\dagger} \boldsymbol{\sigma}^{k} \boldsymbol{\tau}_{3} N \\
& -\frac{\sqrt{2}}{6} \frac{g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}}{\Lambda_{\chi}^{2} F_{\pi}}[X(Q)+Y(Q)] \epsilon^{\mathrm{i} 3} N^{\dagger} \boldsymbol{\tau}^{\mathrm{i}} N N^{\dagger} \boldsymbol{\tau}^{\mathrm{j}} \boldsymbol{\sigma} \cdot \boldsymbol{k} N . \tag{23}
\end{align*}
$$

Acordingly, one gets the PV potential

$$
\begin{align*}
V= & -\frac{\mathrm{i}}{\Lambda_{\chi}^{3}}\left\{\tilde{C}_{2}^{\Delta}(Q) \frac{\tau_{1}^{z}+\tau_{2}^{z}}{2}\left(\boldsymbol{\sigma}_{1} \times \boldsymbol{\sigma}_{2}\right) \cdot \boldsymbol{q}+\right. \\
& \left.C_{6}^{\Delta}(Q)\left(\boldsymbol{\tau}_{1} \times \boldsymbol{\tau}_{2}\right)^{z}\left(\boldsymbol{\sigma}_{1}+\boldsymbol{\sigma}_{2}\right) \cdot \boldsymbol{q}\right\} \tag{24}
\end{align*}
$$

where

$$
\begin{align*}
\tilde{C}_{2}^{\Delta}(Q)= & \frac{8 \sqrt{2}}{9} \pi g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}\left[4 L(Q)-\frac{4}{\delta} C(Q)\right] \\
C_{6}^{\Delta}(Q)= & -\frac{2 \sqrt{2}}{3} \pi g_{\pi \mathrm{N} \Delta}^{2} g_{\mathrm{A}} h_{\pi}^{1}[8 L(Q)- \\
& \left.\frac{\pi}{\delta}\left(4 m_{\pi}^{2}+Q^{2}\right) A(Q)-\frac{8}{\delta} B(Q)\right] . \tag{25}
\end{align*}
$$

## 4 Discussions

In short summary, we have calculated the $\Delta$ contribution to the parity-violating nucleon-nucleon potential. The $\Delta$ resonance couples to the nucleon and pion strongly. A nucleon may be excited and transformed into a virtual $\Delta$ quite easily. The $\Delta$ will certainly contribute to the hadronic parity violation in nuclear processes. In this work, we employ the small scale expansion formalism and extend the former investigation of PV NN interaction in EFT in Ref. [31] through the inclusion of the $\Delta$ contribution. To the next-next-leading order, the new potential contains
no more unknown PV coupling constants. The only new parameter is the strong coupling constant $g_{\pi \mathrm{N} \Delta}$, which is known from the decay width of the $\Delta$ baryon.


Fig. 2. The momentum dependence of coefficients in the PV two-pion exchange potentials: $\tilde{C}_{2}^{2 \pi}(Q)$ (thick solid line), $C_{6}^{2 \pi}(Q)$ (thin solid line), $\tilde{C}_{2}^{\Delta}(Q)$ (dash line) and $C_{6}^{\Delta}(Q)$ (dotted line).

The structure of the obtained potential is similar to the medium-range potential derived in Refs. $[6,31]$.

For comparison, we plot the momentum dependence of coefficients $\tilde{C}_{2}^{\Delta}(Q), \quad C_{6}^{\Delta}(Q), \quad \tilde{C}_{2}^{2 \pi}(Q)=$ $-8 \sqrt{2} \pi g_{\mathrm{A}}^{3} h_{\pi}^{1} L(Q)$ and $C_{6}^{2 \pi}(Q)=-\sqrt{2} \pi g_{\mathrm{A}} h_{\pi}^{1} L(Q)+$ $\sqrt{2} \pi[3 L(Q)-H(Q)] g_{\mathrm{A}}^{3} h_{\pi}^{1}$ with $H(Q)=\frac{4 m_{\pi}^{2}}{4 m_{\pi}^{2}+Q^{2}} L(Q)$ in Fig. 2. We take $m_{\pi}=135 \mathrm{MeV}, \delta=294 \mathrm{MeV}$, $g_{\mathrm{A}}=1.27, g_{\pi \mathrm{N} \Delta}=\frac{3 \sqrt{2}}{5} g_{\mathrm{A}}$. From the figure, one notes $C_{6}^{\Delta}(Q)$ is bigger than $C_{6}^{2 \pi}(Q)$ at small momentum. It is also important to note that $\tilde{C}_{2}^{\Delta}(Q)$ and $\tilde{C}_{2}^{2 \pi}(Q)$ are comparable in magnitude but they have opposite signs! Therefore it is highly desirable to include the new parity violating nucleon-nucleon arising from the $\Delta$ correction in the future theoretical calculation of PV observables in the hadronic weak interaction processes.

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    1）E－mail：yrliu＠ihep．ac．cn
    2）E－mail：zhusl＠phy．pku．edu．cn

