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Form Factors for $B(B_c)\to Dl\tilde{\nu}$ in Light-Cone Sum Rules with Chiral Current Correlator*

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Abstract In this paper we calculate the weak form factors of the decays $B(B_c)\to Dl\tilde{\nu}$ by using the chiral current correlator within the framework of the QCD light-cone sum rules (LCSR). The expressions of the form factors only depend on the leading twist distribution amplitude (DA) of the D meson. Three models of the D-meson distribution amplitude are employed and the calculated form factors,$F_{B\to D}(0)$ and $F_{B_c\to D}(0)$ are given. In the velocity transfer region $1.14<y<1.59$, which renders the Operator Product Expansion (OPE) near light-cone $x^2=0$ to go effectively, the yielding behavior of form factor is in agreement with that extracted from the data on $B\to Dl\tilde{\nu}$, within the error. In the large recoil region $1.35<y<1.59$, the form factor $F_{B\to D}(0)$ is observed consistent with that of perturbative QCD (pQCD). The presented calculation can play a bridge role connecting those from the lattice QCD, heavy quark symmetry and pQCD to have an all-around understanding of $B\to Dl\tilde{\nu}$ transitions. Our prediction for $F_{B_c\to D}(0)$, by using the D-meson distribution amplitude with the exponential suppression at the end points, is compatible with other approaches, and favors the three-points sum rules (3PSR) approach with the Coulumb corrections.

Key words form factor, light-cone sum rules, quantum chromodynamics (QCD)

1 Introduction

Calculation of the form factors for semileptonic transitions of B mesons has been being a subject discussed intensely. Recently, it has been shown that the $B\to\pi$ transition form factor can be consistently analyzed by using the different approaches in the different q^2 regions [1—4]. The perturbative QCD (pQCD) can be applied to the $B\to\pi$ form factor in the large recoil (small q^2) region and it is reliable when the involved energy scale is large enough[1]. The QCD light-cone sum rules (LCSR) can involve both the hard and soft contributions to the $B\to\pi$ form factor below $q^2\simeq 18\text{GeV}^2$[2]. The lattice QCD simulations of the $B\to\pi$ transition form factor[3] are available only for the soft region $q^2>15\text{GeV}^2$, because of the restriction to the π energy smaller than the inverse lattice spacing. Thus the results from these three approaches might be complementary to each other. In Ref. [4] we recalculate the $B\to\pi$ form factor in the pQCD approach, with the transverse momentum dependence included for both the hard scattering part and the nonperturbative wave functions(of π and B) to get a more reliable pQCD result. By combining the results from these three methods we obtain a full understanding of the $B\to\pi$ transition form factor in its physical region $0\leqslant q^2\leqslant(M_B-M_\pi)^2\simeq 25\text{GeV}^2$.

It is necessary that there is a reliable estimate of $B\to D$ transition in the whole kinematically accessible range $0\leqslant q^2\leqslant(M_B-M_D)^2\simeq 11.6\text{GeV}^2$, in order to account for the data on $B\to Dl\tilde{\nu}$. For this purpose, it is practical, as shown in $B\to\pi$ case, to combine

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the result of QCD LCSR with those from the lattice QCD, heavy quark symmetry and pQCD. The LCSR approach[2], where the non-perturbative dynamics are effectively parameterized in the so-called light-cone wave functions, is regarded as an effective tool to deal with the heavy-to-light exclusive processes. Although the B→D transition in question is a heavy-to-heavy one, the c-quark is much lighter compared to b-quark and so discussing it with LCSR is plausible for the kinematical range where the OPE near light-cone $x^2=0$ is valid. The other problem with our practical calculation is that the higher twist DA's of D meson, which are important but less studied, would enter into the sum rule result. However, an effective approach[2] has been presented to avoid the pollution by some higher-twist DA's. This improved LSCR method uses a certain chiral current correlator as the starting point so that the relevant twist-3 wave functions make no contributions and the reliability of calculation can be enhanced to a large extent. Its applicability has been examined by a great deal of studies[2]. In this paper we would like to employ the improved LCSR to discuss the form factor for the B→D transition and try to give a full understanding of QCD dynamics involved in the $B\to Dl\tilde{\nu}$.

The CDF Collaboration reported on the observation of the bottom-charm B_c meson at Fermilab[5] in the semileptonic decay mode $B_c \to J/\psi + l + \nu$ with the J/ψ decaying into muon pairs in 1998. Values for the mass and the lifetime of the B_c meson were given as $M(B_c) = 6.40 \pm 0.39 \pm 0.13$GeV and $\tau(B_c) = 0.46^{+0.18}_{-0.16}(\text{stat}) \pm 0.03(\text{syst})$ ps. Recently, CDF reported the first Run II evidence for the B_c meson in the fully reconstructed decay channel $B_c \to J/\psi + \pi$ with $J/\psi \to \mu^+\mu^-$[6]. The mass value quoted for this decay channel is $6.2857 \pm 0.0053(\text{stat}) \pm 0.0012(\text{syst})$GeV with errors significantly smaller than in the first measurement. Also D0 has observed the B_c in the semileptonic mode $B_c \to J/\psi + \mu + X$ and reported preliminary evidence that $M(B_c) = 5.95^{+0.14}_{-0.13} \pm 0.34$GeV and $\tau(B_c) = 0.45^{+0.12}_{-0.10} \pm 0.12$ps[7].

The B_c decays, at first, calculated in the potential models (PM)[8, 9], wherein the variation of techniques results in close estimates after the adjustment on the semileptonic decays of B mesons. The Operator Product Expansion (OPE) evaluation of inclusive decays gave the lifetime and widths[10], which agree well with PM, if one sums up the dominating exclusive modes. That was quite unexpected, when the sum rules (SR) of QCD results in the semileptonic B_c widths[11], which are one order of magnitude less than those of PM and OPE. The reason may be the valuable role of Coulomb corrections, that implies the summation of α_s/v corrections significant in the heavy quarkonia, i.e. in the B_c[12].

In the recent paper[13], we calculate the form factor for $B\to Dl\tilde{\nu}$ transitions within the framework of QCD light-cone sum rules (LCSR). In the velocity transfer region $1.14 < v\cdot v' < 1.59$, which renders the OPE near light-cone $x^2=0$ to go effectively, the yielding behavior of form factor is in agreement with that extracted from the data on $B\to Dl\tilde{\nu}$, within the error. In the larger recoil region $1.35 < v\cdot v' < 1.59$, the results are observed consistent with those of perturbative QCD (pQCD). Now we calculate the form factor of $B_c\to Dl\tilde{\nu}$, which also depends on the D-meson DA. However, due to the different feature of the two processes, the c quark is a spectator in the decay $B_c\to Dl\tilde{\nu}$ and the c quark comes from the b quark decay in the process $B\to Dl\tilde{\nu}$, these two form factors are sensitive to the shape of the DA in two different regions. Combining the information in the two processes, we can find which model is more suitable for describing the D meson. Similar to the case of $B\to\pi l\tilde{\nu}$, the LCSR approach for the $B_c\to Dl\tilde{\nu}$ form factor is reliable only in the region $0 < q^2 < 15\text{GeV}^2$. we extrapolate the result to the whole region and give the decay width and branching ratio for the semileptonic decay[14].

This paper is organized as follows. In the following section we derive the LCSRs for the form factor of $B_c\to Dl\tilde{\nu}$ and $B_c\to Dl\tilde{\nu}$. A discussion of the DA models for the D meson is given in Section 3. Section 4 is devoted to the numerical analysis and comparison with other approaches. The last section is reserved for summary.

2 LCSRs for the $B(B_c)\to D$ form factors

The $B_c \to D$ weak form factors $f(q^2)$ and $\tilde{f}(q^2)$ are usually defined as:

$$\langle D(p)|\bar{u}\gamma_\mu b|B_c(p+q)\rangle = 2f(q^2)p_\mu + \tilde{f}(q^2)q_\mu, \quad (1)$$

with q being the momentum transfer.

To achieve a LCSR estimate of $f(q^2)$, we follow Ref. [2] and use the following chiral current correlator $\Pi_\mu(p,q)$:

$$\begin{aligned}\Pi_\mu(p,q) \;=\; & \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{ipx}\langle D(p)|T\{\bar{u}(x)\gamma_\mu(1+\gamma_5)\times \\ & b(x), \bar{b}(0)\mathrm{i}(1+\gamma_5)c(0)\}|0\rangle = \\ & \Pi(q^2,(p+q)^2)p_\mu + \tilde{\Pi}(q^2,(p+q)^2)q_\mu, \quad (2)\end{aligned}$$

First, we express the hadronic representation for the correlator. This can be done by inserting the complete intermediate states with the same quantum numbers as the current operator $\bar{b}\mathrm{i}(1+\gamma_5)c$. Isolating the pole contribution due to the lowest pseudoscalar B_c meson, we have the hadronic representation in the following:

$$\begin{aligned}\Pi^{\mathrm{H}}_\mu(p,q) = & \Pi^{\mathrm{H}}(q^2,(p+q)^2)p_\mu + \tilde{\Pi}^{\mathrm{H}}(q^2,(p+q)^2)q_\mu = \\ & \frac{\langle D|\bar{u}\gamma_\mu b|B_c\rangle\langle B_c|\bar{b}\mathrm{i}\gamma_5 c|0\rangle}{m^2_{B_c} - (p+q)^2} + \\ & \sum_{\mathrm{H}} \frac{\langle D|\bar{u}\gamma_\mu(1+\gamma_5)b|B^{\mathrm{H}}_c\rangle\langle B^{\mathrm{H}}_c|\bar{b}\mathrm{i}(1+\gamma_5)c|0\rangle}{m^2_{B^{\mathrm{H}}_c} - (p+q)^2}. \quad (3)\end{aligned}$$

Note that the intermediate states B^{H}_c contain not only the pseudoscalar resonance of masses greater than m_{B_c}, but also the scalar resonances with $J^P = 0^+$, corresponding to the operator $\bar{b}c$. With Eq. (1) and the definition of the decay constant f_{B_c} of the B_c meson

$$\langle B_c|\bar{b}\mathrm{i}\gamma_5 c|0\rangle = m^2_{B_c} f_{B_c}/(m_b + m_c), \quad (4)$$

and expressing the contributions of higher resonances and continuum states in a form of dispersion integration, the invariant amplitudes Π^{H} and $\tilde{\Pi}^{\mathrm{H}}$ read,

$$\begin{aligned}\Pi^{\mathrm{H}}[q^2,(p+q)^2] = & \frac{2f(q^2)m^2_{B_c}f_{B_c}}{(m_b+m_c)(m^2_{B_c} - (p+q)^2)} + \\ & \int_{s_0}^{\infty} \frac{\rho^{\mathrm{H}}(s)}{s-(p+q)^2}\mathrm{d}s + \text{subtractions}, \quad (5)\end{aligned}$$

and

$$\begin{aligned}\tilde{\Pi}^{\mathrm{H}}[q^2,(p+q)^2] = & \frac{\tilde{f}(q^2)m^2_{B_c}f_{B_c}}{(m_b+m_c)(m^2_{B_c} - (p+q)^2)} + \\ & \int_{s_0}^{\infty} \frac{\tilde{\rho}^{\mathrm{H}}(s)}{s-(p+q)^2}\mathrm{d}s + \text{subtractions}, \quad (6)\end{aligned}$$

where the threshold parameter s_0 should be set near the squared mass of the lowest scalar B_c meson, the spectral densities $\rho^{\mathrm{H}}(s)$ and $\tilde{\rho}^{\mathrm{H}}(s)$ can be approximated by invoking the quark-hadron duality ansatz

$$\rho^{\mathrm{H}}(s)(\tilde{\rho}^{\mathrm{H}}(s)) = \rho^{\mathrm{QCD}}(s)(\tilde{\rho}^{\mathrm{QCD}}(s))\theta(s-s_0). \quad (7)$$

On the other hand, we need to calculate the correlator in QCD theory to obtain the desired sum rule result. In fact, there is an effective kinematical region which makes OPE applicable: $(p+q)^2 - m_b^2 \ll 0$ for the $b\bar{d}$ channel and $q^2 \leqslant m_b^2 - 2\Lambda_{\mathrm{QCD}}m_b$ for the momentum transfer. For the present purpose, it is sufficient to consider the invariant amplitude $\Pi(q^2,(p+q)^2)$ which contains the desired form factor. The leading contribution is derived easily by contracting the b-quark operators to a free propagator:

$$\langle 0|Tb(x)\bar{b}(0)|0\rangle = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}kx}\frac{\not{k}+m_b}{k^2-m_b^2}. \quad (8)$$

Substituting Eq. (8) into Eq. (2), we have the two-particle contribution to the correlator,

$$\begin{aligned}\Pi^{(\bar{q}q)}_\mu \;=\; & -2m_b\mathrm{i}\int \frac{\mathrm{d}^4x\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{\mathrm{i}(q-k)x}\frac{1}{k^2-m_b^2}\times \\ & \langle D(p)|T\bar{c}(x)\gamma_\mu\gamma_5 d(0)|0\rangle. \quad (9)\end{aligned}$$

An important observation, as in Ref. [2], is that only the leading non-local matrix element $\langle D(p)|\bar{u}(x)\gamma_\mu\gamma_5 c(0)|0\rangle$ contributions to the correlator, while the nonlocal matrix elements $\langle D(p)|\bar{u}(x)\mathrm{i}\gamma_5 c(0)|0\rangle$ and $\langle D(p)|\bar{u}(x)\sigma_{\mu\nu}\gamma_5 c(0)|0\rangle$ whose leading terms are of twist 3, disappear from the sum rule. Proceeding to Eq. (9), we can expand the nonlocal matrix element $\langle D(p)|T\bar{u}(x)\gamma_\mu\gamma_5 c(0)|0\rangle$ as

$$\begin{aligned}& \langle D(p)|T\bar{u}(x)\gamma_\mu\gamma_5 c(0)|0\rangle = \\ & -\mathrm{i}p_\mu f_D\int_0^1 \mathrm{d}u\mathrm{e}^{\mathrm{i}upx}\varphi_D(\bar{u}) + \text{higher twist terms}, \quad (10)\end{aligned}$$

where $\varphi_D(\bar{u})$ is the twist-2 DA of the D meson with $\bar{u} = 1-u$ being the longitudinal momentum fraction carried by the c quark, those DA's entering the higher-twist terms are of at least twist 4. The use of Eq. (10) yields

$$\Pi^{(\bar{q}q)}[q^2,(p+q)^2] = 2f_{\rm D}m_{\rm b}\int_0^1 {\rm d}u\frac{\varphi_{\rm D}(\bar{u})}{m_{\rm b}^2-(up+q)^2}+ \text{higher twist terms.} \quad (11)$$

Invoking a correction term due to the interaction of the b quark with a background field gluon into Eq. (11), the three-particle contribution $\Pi_\mu^{(\bar{q}qg)}$ is achievable. However, the practical calculation shows that the corresponding matrix element whose leading term is of twist 3 also vanishes. Thus, if we work to the twist-3 accuracy, only the leading twist DA $\varphi_{\rm D}$ is needed to yield a LCSR prediction.

Furthermore, we carry out the subtraction procedure of the continuum spectrum, make the Borel transformations with respect to $(p+q)^2$ in the hadronic and the QCD expressions, and then equate them. Finally, we get the LCSR for $f(q^2)$:

$$f_{{\rm B_c}\to{\rm D}}(q^2)=\frac{m_{\rm b}(m_{\rm b}+m_{\rm c})f_{\rm D}}{m_{\rm B_c}^2 f_{\rm B_c}}{\rm e}^{m_{\rm B_c}^2/M^2}\times \int_{\Delta_{\rm B_c}}^1\frac{{\rm d}u}{u}\exp\left[-\frac{m_{\rm b}^2-(1-u)(q^2-um_{\rm D}^2)}{uM^2}\right]\varphi_{\rm D}(\bar{u}), \quad (12)$$

where

$$\Delta_{\rm B_c}=\frac{\sqrt{(s_0^{\rm B_c}-q^2-m_{\rm D}^2)^2+4m_{\rm D}^2(m_{\rm b}^2-q^2)}-(s_0^{\rm B_c}-q^2-m_{\rm D}^2)}{2m_{\rm D}^2}, \quad (13)$$

and $p^2=m_{\rm D}^2$ has been used.

The LCSR for the form factor of $\rm B\to Dl\tilde{\nu}$ has been derived in Ref. [13], here we just give the result:

$$\mathscr{F}_{\rm B\to D}(v\cdot v')=\frac{2m_{\rm b}^2}{(m_{\rm B}+m_{\rm D})m_{\rm B}}\sqrt{\frac{m_{\rm D}}{m_{\rm B}}}\frac{f_{\rm D}}{f_{\rm B}}{\rm e}^{m_{\rm B}^2/M^2}\times \int_{\Delta_{\rm B}}^1\frac{{\rm d}u}{u}\exp\left[-\frac{m_{\rm b}^2-(1-u)(q^2-um_{\rm D}^2)}{uM^2}\right]\varphi_{\rm D}(u), \quad (14)$$

where

$$\Delta_{\rm B}=\frac{\sqrt{(s_0^{\rm B}-q^2-m_{\rm D}^2)^2+4m_{\rm D}^2(m_{\rm b}^2-q^2)}-(s_0^{\rm B}-q^2-m_{\rm D}^2)}{2m_{\rm D}^2}. \quad (15)$$

3 D-meson distribution amplitude

Now let's discuss an important nonperturbative parameter appearing in the LCSRs, the leading twist DA of D-meson, $\varphi_{\rm D}(x)$. We reexamine the D-meson distribution amplitude since we missed a factor of $\sqrt{2}$ for the decay constant $f_{\rm D}$ in determining the coefficients of the DA model[13].

The D meson is composed of the heavy quark c and the light anti-quark $\bar{q}$. The longitudinal momentum distribution should be asymmetry and the peak of the distribution should be approximately at $x\simeq m_{\rm c}/m_{\rm D}\simeq 0.7$. According to the definition in Eq. (10), $\varphi_{\rm D}(x)$ satisfies the normalization condition

$$\int_0^1 {\rm d}x\varphi_{\rm D}(x)=1. \quad (16)$$

In the pQCD calculations[15], a simple model (we call model I) is adopted as

$$\varphi_{\rm D}^{\rm (I)}(x)=6x(1-x)(1-C_{\rm d}(1-2x)) \quad (17)$$

which is based on the expansion of the Gegenbauer polynomials. Eq. (17) has a free parameter $C_{\rm d}$ which ranges from 0 to 1, and is supposed to approximate 0.7 in order to get consistent results with experiments[15]. Thus we simply take $C_{\rm d}=0.7$.

On the other hand, it was suggested in Ref. [16] that the light-cone wave function of the D-meson be taken as:

$$\psi_{\rm D}(x,\boldsymbol{k}_\perp)=A_{\rm D}\exp\left[-b_{\rm D}^2\left(\frac{\boldsymbol{k}_\perp^2+m_{\rm c}^2}{x}+\frac{\boldsymbol{k}_\perp^2+m_{\rm d}^2}{1-x}\right)\right] \quad (18)$$

which is derived from the Brosky-Huang-Lepage (BHL) prescription[17]. One constraint on the wave function is from the leptonic decay process $\rm D\to\mu\nu$:

$$\int_0^1\int\frac{{\rm d}^2\boldsymbol{k}_\perp}{16\pi^3}\psi_{\rm D}(x,\boldsymbol{k}_\perp)=f_{\rm D}/2\sqrt{6}. \quad (19)$$

Here the conventional definition of the decay constant $f_{\rm D}$ has been used, so Eq. (19) differs from that in Ref. [16] by a factor of $\sqrt{2}$. Another constraint comes from an estimation of the probability of finding the $|q\bar{q}\rangle$ Fock state in the D meson:

$$P_{\rm D}=\int_0^1{\rm d}x\int\frac{{\rm d}^2\boldsymbol{k}_\perp}{16\pi^3}|\psi_{\rm D}(x,\boldsymbol{k}_\perp)|^2. \quad (20)$$

As discussed in Ref. [16], $P_{\rm D}\approx 0.8$ is a good approximation for the D meson. Based on these two constraints, the parameters $A_{\rm D}$ and $b_{\rm D}^2$ can be fixed. Taking $P_{\rm D}\approx 0.8$, $f_{\rm D}=222.6$MeV, $m_{\rm c}=1.3$GeV and $m_{\rm d}=0.35$GeV, we have $A_{\rm D}=225{\rm GeV}^{-1}$, $b_{\rm D}^2=0.580{\rm GeV}^{-2}$.

$\psi_{\rm D}(x,\boldsymbol{k}_\perp)$ can be related to the normalized DA $\varphi_{\rm D}(x)$ by the definition:

$$\varphi_{\rm D}(x)=\frac{2\sqrt{6}}{f_{\rm D}}\int\frac{{\rm d}^2\boldsymbol{k}_\perp}{16\pi^3}\psi_{\rm D}(x,\boldsymbol{k}_\perp). \tag{21}$$

Substituting Eq. (18) into Eq. (21), we have a model of the DA(model II)

$$\varphi_{\rm D}^{\rm (II)}(x)=\frac{\sqrt{6}A_{\rm D}}{8\pi^2 f_{\rm D}b_{\rm D}^2}x(1-x)\exp\left[-b_{\rm D}^2\frac{xm_{\rm d}^2+(1-x)m_{\rm c}^2}{x(1-x)}\right], \tag{22}$$

Furthermore, as argued in Ref. [18], a more complete form of the light-cone wave function should include the Melosh rotation effect in spin space:

$$\psi_{\rm D}^f(x,\boldsymbol{k}_\perp)=\chi_{\rm D}(x,\boldsymbol{k}_\perp)A_{\rm D}^f\times \exp\left[-b_{\rm D}^{f\,2}\left(\frac{\boldsymbol{k}_\perp^2+m_{\rm c}^2}{x}+\frac{\boldsymbol{k}_\perp^2+m_{\rm d}^2}{1-x}\right)\right] \tag{23}$$

with the Melosh factor,

$$\chi_{\rm D}(x,\boldsymbol{k}_\perp)=\frac{(1-x)m_{\rm c}+xm_{\rm d}}{\sqrt{\boldsymbol{k}_\perp^2+((1-x)m_{\rm c}+xm_{\rm d})^2}}. \tag{24}$$

It can be seen from Eq. (24) that $\chi_{\rm D}(x,\boldsymbol{k}_\perp)\to 1$ as $m_{\rm c}\to\infty$, since there is no spin interaction between the two quarks in the heavy-flavor meson, i.e., the spin of the heavy constituent decouples from the gluon field, in the heavy quark limit[19]. However the c-quark is not heavy enough to neglect the Melosh factor. After integration over $\boldsymbol{k}_\perp$ the full form of D meson DA can be achieved (Model III):

$$\varphi_{\rm D}^{\rm (III)}(x)=\frac{A_{\rm D}^f\sqrt{6x(1-x)}}{8\pi^{3/2}f_{\rm D}b_{\rm D}^f}y\left[1-{\rm Erf}\left(\frac{b_{\rm D}^f y}{\sqrt{x(1-x)}}\right)\right]\times \exp\left[-b_{\rm D}^{f\,2}\frac{(xm_{\rm d}^2+(1-x)m_{\rm c}^2-y^2)}{x(1-x)}\right], \tag{25}$$

where $y=xm_{\rm d}+(1-x)m_{\rm c}$ and the error function ${\rm Erf}(x)$ is defined as ${\rm Erf}(x)=\dfrac{2}{\pi}\displaystyle\int_0^x\exp(-t^2){\rm d}t$. Using the same constraints as in Eqs. (19) and (20), the parameters $A_{\rm D}^f$ and $b_{\rm D}^f$ are fixed as $A_{\rm D}^f=209{\rm GeV}^{-1}$ and $b_{\rm D}^{f\,2}=0.540{\rm GeV}^{-2}$.

In this paper we will employ the above three kinds of models to do numerical calculation. All these DA's of the D-meson are plotted in Fig. 1 for comparison. It can be seen that although they all have a maximum at $x\simeq 0.65$, the shapes of them are rather different, especial in the region $0<x<0.3$ and $0.5<x<0.8$.

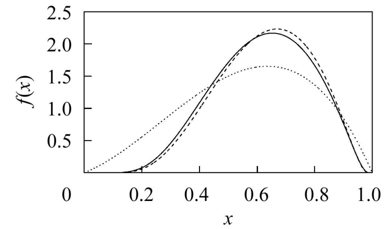

Fig. 1. Different kinds of D-meson DA's,solid and dashed curves correspond to Model III and II, while the dotted line expresses Model I.

4 Numerical results and discussion

Apart from the DA of the D meson, the decay constant of $\rm B_c$-meson $f_{\rm B_c}$ is among the important nonperturbative inputs. For consistency, we use the following corrector

$$K(q^2)={\rm i}\int{\rm d}^4x{\rm e}^{{\rm i}qx}\langle 0|\bar{c}(x)(1+\gamma_5)b(x), \bar{b}(0)(1-\gamma_5)c(0)|0\rangle, \tag{26}$$

to recalculate it in the two-point sum rules. The calculation should be limited to leading order in QCD, since the QCD radiative corrections to the sum rule for $f_{\rm B_c\to D}(q^2)$ are not taken into account. The value of the threshold parameter $s_0^{\rm B_c}$ is determined by a best fit requirement in the region $8{\rm GeV}^2\leqslant M^2\leqslant 12{\rm GeV}^2$, where M^2 is the corresponding Borel parameter. The same procedure is performed for $f_{\rm B}$, in almost the same Borel "window". The results are listed in Table 1. As we have ignored all the radiation corrections, we don't expect our values of $f_{\rm B_c}$ and $f_{\rm B}$ to be good predictions of that quantity. We use the same threshold parameters for the corresponding form factors in the LCSRs, except for the Borel parameter $M_{\rm LC}^2$, which is taken as $M_{\rm LC}^2\simeq M^2/\langle u\rangle$, with $\langle u\rangle$ being the average momentum faraction involved. It turns out that the form factors depend little on $M_{\rm LC}^2$ in the region $15<M_{\rm LC}^2<20$. The other input parameters are taken as $m_{\rm B}=5.279{\rm GeV}$, $m_{\rm D}=1.869{\rm GeV}$, $m_{\rm B_c}=6.286{\rm GeV}$.

Table 1. Parameter sets for $f_{\rm B_c}$ and $f_{\rm B}$, $s_0^{\rm B_c}$ and $s_0^{\rm B}$ for $f_{\rm B_c}$ and $f_{\rm B}$ respectively; $m_{\rm b}$, $f_{\rm B_c}$ and $f_{\rm B}$ are given in GeV, $s_0^{\rm B_c}$ and $s_0^{\rm B}$ in ${\rm GeV}^2$.

	$m_{\rm b}$	$s_0^{\rm B_c}$	$f_{\rm B_c}$	$s_0^{\rm B}$	$f_{\rm B}$
set 1	4.6	43.0	0.243	30.7	0.145
set 2	4.7	42.0	0.189	30.2	0.117
set 3	4.8	41.2	0.137	29.8	0.090

With these inputs, we can carry out the numerical analysis. In particular, we redo the previous calculation for $B \to Dl\tilde{\nu}$ in Ref. [13] and show the corresponding form factor $\mathscr{F}_{B\to D}(v\cdot v')$ in Fig. 2. The result for the form factor of $B_c \to Dl\tilde{\nu}$ is given in Fig. 3. For $\mathscr{F}_{B\to D}(v\cdot v')$, similar results can be obtained by applying the various model DA's at large recoil region $v\cdot v' \simeq 1.59$, i.e., $q^2 \simeq 0$, but rather different values at the zero recoil point $q^2 = q^2_{\max}$. It can be understood easily from the involved region of the DA. While $q^2 = 0$ corresponds to $\varDelta_B \simeq 0.75$ according to Eq. (15), $q^2 = q^2_{\max}$ corresponds to $\varDelta_B \simeq 0.6$, and the models of the D-meson DA in the region $0.5 < x < 0.8$ are rather different. However, the LCSR result at the zero recoil point ($q^2 = q^2_{\max}$) is less reliable, we can not get a final conclusion from the difference of the form factor at this point. Fortunately, the case for $B_c \to Dl\tilde{\nu}$ is just opposite, which can be seen from Fig. 3. There is a big difference of the form factor at the point $q^2 = 0$. A detailed comparison for the form factor at this point with other approaches is shown in Table 2. The big difference between model I and others comes from the different contributions of the DA's in the involved region $0 < x < 0.45$. Due to the exponential suppression at the end points, the results from Model II and III are much smaller than that from Model I, and are consistent with the 3PSR results with the Coulumb corrections included, and the PM result. It can also be seen from Fig. 2 and Fig. 3 that, in both cases, Model II and III actually differ little, which means that the influence of the Melosh factor is not so important due to the heavy c quark.

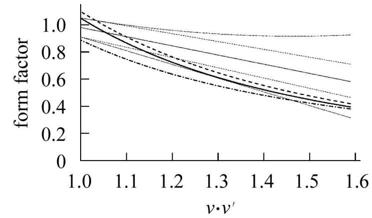

Fig. 2. $\mathscr{F}_{B\to D}$ as a function of the velocity transfer (with the parameters in the set 2). The thin lines express the experiment fits results, the solid line represents the central values, the dashed(dash-dotted) lines give the bounds from the linear(quadratic) fits. The thick lines correspond to our results, with the solid, dashed and dash-dotted lines for Model III, II and I respectively.

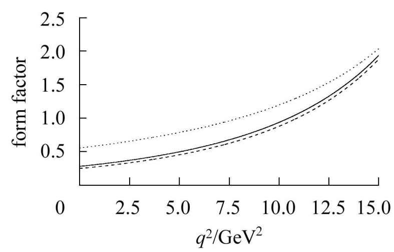

Fig. 3. $f_{B_c\to D}(q^2)$ calculated by using different kinds of DA models. The solid, dashed and dash-dotted lines correspond to Model III, II and I respectively. Here the threshold parameter set 2 has been used.

Table 2. Form factor $f_{B_c\to D}(0)$ of $B_c \to Dl\tilde{\nu}$ calculated with different kinds of D-meson DA's, in comparison with that of the 3-Points Sum Rule (3PSR) without[11] and with[20] the Coulumb corrections and Potential Model (PM)[20].

	model I	model II	model III	3PSR[11]	3PSR[20]	PM[20]
$f_{B_c\to D}(0)$	0.55	0.25	0.28	0.13±0.05	0.32	0.29

The calculated form factor for $B_c \to Dl\tilde{\nu}$ can be fitted excellently in the calculated region $0 < q^2 < 15\text{GeV}^2$ by the parametrization:

$$f_{B_c\to D}(q^2) = \frac{f(0)}{1 - a_f q^2/m^2_{B_c} + b_f (q^2/m^2_{B_c})^2}. \qquad (27)$$

The values of $f_{B_c\to D}(0)$, a_f and b_f are listed in Table 3.

Extrapolating the form factor to the whole kinetic region $0 < q^2 < (m_{B_c} - m_D)^2 \approx 19.5\text{GeV}^2$ using this parametrization, we get:

$$\varGamma(B_c \to Dl\tilde{\nu}) = (0.197 \pm 0.013) \times 10^{-15}\text{GeV}, \qquad (28)$$

and

$$\text{BR}(B_c \to Dl\tilde{\nu}) = (1.35 \pm 0.05) \times 10^{-4}. \qquad (29)$$

where $\tau(B_c) = 0.45\text{ps}$ and $V_{ub} = 0.0037$ have been used. The central values are calculated by using the parameters set 2, while the upper and lower bounds are given by using set 3 and set 1 respectively. Our result for the branching ratio is much larger than

$BR(B_c\to Dl\tilde{\nu})=0.4\times10^{-4}$ from Ref. [20], they employed a simple pole approximation to extrapolate the form factor to the whole region. It is also much larger that the PM result $BR(B_c\to Dl\tilde{\nu})=0.35\times10^{-4}$[21], and the result $BR(B_c\to Dl\tilde{\nu})=0.6\times10^{-4}$ from Ref. [22].

Table 3. Form factor $f_{B_c\to D}(q^2)$ in a three-parameter fit (27). The three rows correspond to the calculated form factors using different sets of parameters, respectively.

	$f(0)$	a_f	b_f
set 1	0.288	3.79	4.23
set 2	0.283	3.92	4.47
set 3	0.288	4.03	4.77

5 Summary

We have discussed the form factor for $B\to D$ transitions $\mathscr{F}_{B\to D}(y)$, using the improved QCD LCSR approach where with the chiral current correlator chosen only the leading twist DA of the D-meson is relevant at twist-3 accuracy. The resulting LCSR's for $\mathscr{F}_{B\to D}(y)$ are available in the velocity transfer region $1.14<y<1.59$. Calculation is done using three different twist-2 DA models for D meson. It has been shown the numerical results are less sensitive to the choice of DA, and are of a central value slight smaller than but within the error in a agreement with those obtained by fitting the data on $B\to Dl\tilde{\nu}$. In the larger recoil region $1.35<y<1.59$ where pQCD is applicable, the results presented here are consistent with the ones of pQCD. From the practical calculations, we find that the present results might be extrapolated to the smaller recoil region so that the $B\to D$ transitions are calculable in the whole kinematically accessible range, using the improved LCSR approach.

Also, we argue that for understanding the form factor for $B\to Dl\tilde{\nu}$ in the whole kinematical range a combined use is necessary of three different methods: the lattice QCD (with the heavy quark symmetry considered), the improved LCSR and the pQCD approaches, which are adequate to do calculation in different kinematical regions and so could be complementary to each other. The LCSR approach plays a bridge role in doing such calculation.

The present findings can be improved once the QCD radiative correction to the LCSR is taken into account and a more reliable twist-2 DA of D meson becomes available. From the previous discussion in Ref. [2], however, it is expected that the QCD radiative correction cannot change the present results too much.

The B_c meson has been observed by the CDF and D0 groups in the different channels. In this paper we study the weak form factor of the decay process $B_c(B)\to Dl\tilde{\nu}$ by using the chiral current correlator within the framework of the QCD light-cone sum rules, which is similar to the approach for the weak form factor $f_{B\pi}(q^2)$ in Ref. [2]. The calculated form factors depend on the distribution amplitude of the D meson, and we employ the three different models for the D meson. It has been shown that the results using the model with a exponential suppression at the end points are consistent with other approaches. Our results can also confirm the including of the Coulumb corrections in the 3PSR calculations for the semileptonic decay $B_c\to Dl\tilde{\nu}$. In the LCSRs for the form factors of $B_c(B)\to Dl\tilde{\nu}$, the involved region of the D meson distribution amplitude is rather different. Combining the information in the two processes, we can find which model is more suitable for describing the D meson.

We have made a parametrization (27) to the form factor by fitting our calculation in the region $0<q^2<15\text{GeV}^2$, and the decay width and the branching ratio of the process $B_c\to Dl\tilde{\nu}$ have been calculated. It has been shown that $\Gamma(B_c\to Dl\tilde{\nu})=(0.197\pm0.013)\times10^{-15}\text{GeV}$ and $BR(B_c\to Dl\tilde{\nu})=(1.35\pm0.05)\times10^{-4}$. The results are different from other approaches. It will be expected to test the different predictions in the coming LHC experiments.

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应用具有手征流关联函数的光锥求和规则计算 $\mathrm{B(B_c)\to Dl\tilde{\nu}}$ 过程的形状因子

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摘要 在QCD光锥求和规则(LCSR)框架内应用具有手征流关联函数计算$\mathrm{B(B_C)\to Dl\tilde{\nu}}$衰变过程的弱形状因子. 所获得的形状因子的表达式仅依赖于D介子的主导级分布振幅(DA). 应用了三类D介子的分布振幅计算了形状因子$F_{\mathrm{B\to D}}(0)$ 和 $F_{\mathrm{B_c\to D}}(0)$. 在速度迁移$1.14< y <1.59$的区域内使在光锥$x^2=0$附近算符乘积展开(OPE)得以有效的情况下所计算的形状因子行为在误差范围内与$\mathrm{B\to Dl\tilde{\nu}}$过程实验数据相一致. 在大反冲区域$1.35< y <1.59$获得的形状因子$F_{\mathrm{B\to D}}(0)$是与微扰QCD(pQCD)结果相一致的. 所以本文的计算在联接格点QCD, 重夸克对称性和pQCD之间起桥梁作用, 有助于进一步对$\mathrm{B\to Dl\tilde{\nu}}$跃迁过程的理解. 计算使用了在端点具有指数压低的分布振幅行为, 对$F_{\mathrm{B_c\to D}}(0)$的预言与其他方法获得的结果是可比的, 有利于具有库仑修正的三点求和规则(3PSR)方法所得的结果.

关键词 形状因子 光锥求和规则 量子色动力学(QCD)

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