Possible Heavy Tetraquarks $qQ\bar{q}\bar{Q}$, $qq\bar{Q}\bar{Q}$ and $qQ\bar{Q}\bar{Q}$ *

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Abstract Assuming X(3872) is a $qc\bar{q}\bar{c}$ tetraquark and using its mass as input, we perform a schematic study of the masses of possible heavy tetraquarks using the color-magnetic interaction with the flavor symmetry breaking corrections.

Key words heavy tetraquarks, color-magnetic interaction, quark model

1 Introduction

The past several years have witnessed a renaissance of charmonium spectroscopy. A few charmonium-like new resonances X(3872)[1], X(3940)[2], Y(3940)[3] and Y(4260)[4] were discovered experimentally. Because of its very narrow width, proximity to $D\bar{D}^*$ threshold and special decay patterns, Swanson proposed that X(3872) is mainly a $D^0\bar{D}^{0*}$ molecular state with some admixture of $\rho J/\psi$ and $\omega J/\psi$ components[5, 6]. But the possibility of X(3872) as a conventional 1^{++} charmonium is still not excluded although its mass is smaller than the quark model prediction[7]. Li suggested that X(3872) could be a charmonium hybrid meson[8]. However, both lattice QCD simulation[9] and flux tube mode[10] indicated that the hybrid charmonium lies in the range 4200—4400MeV.

Maiani et al. argued that the X(3872) is a bound state of a pair of diquark and anti-diquark[11]. With the spin-spin interaction and X(3872) mass as input, they predicted a 2^+ $cq\bar{c}\bar{q}$ tetraquark around 3952MeV, which might be identified to the X(3940)[11]. This 2^+ tetraquark state can decay to J/ψ plus a light vector meson, and $D\bar{D}$ via D-wave.

The multi-quark system was studied many years ago using a quark model with color-magnetic (CM) interaction[12—15]. Recently, Richard and Stancu used the model to estimate masses of $qc\bar{q}\bar{c}$ tetraquarks[16—18]. In the complete $qc\bar{q}\bar{c}$ configuration space, they diagonalized the 6×6 Hamiltonian matrix. The CM interaction parameter was determined by the charmed baryon masses. They found the mass of the isoscalar state is around 3910MeV, which lies well above X(3872) and $D\bar{D}^*$ threshold.

In a previous work on light tetraquarks[19], we found that the strength of CM interaction in tetraquarks is quite different from that in conventional $q\bar{q}$ mesons. The strength of CM interaction in charmed baryons is also different from that in the tetraquark system because tetraquarks and charmed baryons have different spatial size. Hence the extraction of the CM interaction parameter in the tetraquark system from the charmed baryon is not justified.

In this work, we identify X(3872) as a tetraquark and use its mass as input to fix the CM strength in the $cq\bar{c}\bar{q}$ system. In Section 2 we present the simple

Received 13 June 2006, Revised 20 July 2006

* Supported by National Natural Science Foundation of China (10375003, 10421503) and Key Grant Project of Chinese Ministry of Education (305001)

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model Hamiltonian with the color-magnetic interaction for the tetraquark system and construct the color-spin-flavor wave functions of 0^+, 1^+ and 2^+ tetraquarks. Section 3 is the numerical analysis. The last section is a short summary.

2 Heavy tetraquark masses

For the tetraquark, the model Hamiltonian reads

$$H=\sum_i m_i + H_{\rm CM} \ , \tag{1}$$

where m_i is the i-th constituent quark mass. $H_{\rm CM}$ is the color-spin interaction Hamiltonian which is derived from one-gluon exchange in MIT bag model[12, 13, 20, 21]:

$$H_{\rm CM}=-\sum_{i>j} v_{ij}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j \ , \tag{2}$$

where $\boldsymbol{\sigma}_i$ is the quark spin operator and $\boldsymbol{\lambda}_i$ the color operator. For the anti-quark, $\boldsymbol{\lambda}_{\bar{q}}=-\boldsymbol{\lambda}^*$ and $\boldsymbol{\sigma}_{\bar{q}}=-\boldsymbol{\sigma}^*$. The values of the coefficients v_{ij} depend on the multiquark system and specific models. In general, v_{ij} takes different values for $\rm q\bar{q}$, qqq and $\rm q\bar{q}q\bar{q}$ systems[19]. Here we discuss the heavy tetraquarks, the v_{ij} in the heavy quark system are also different from the light quark system. We use the following convention

$$v_{ij}=\bar{v}\frac{m_{\rm u}^2}{m_i m_j} \ , \tag{3}$$

where $\bar{v}$ depends on the wave function of the multiquark system. With given flavor context $\rm q_1q_2\bar{q}_3\bar{q}_4$, the expression of the CM matrix element between two states $|\rm k\rangle$ and $|\rm l\rangle$ reads

$$\begin{aligned}V_{\rm CM}({\rm q_1q_2\bar{q}_3\bar{q}_4}) =& V_{12}({\rm q_1q_2})+V_{13}({\rm q_1\bar{q}_3})+V_{14}({\rm q_1\bar{q}_4})+\\ & V_{23}({\rm q_2\bar{q}_3})+V_{24}({\rm q_2\bar{q}_4})+V_{34}({\rm \bar{q}_3\bar{q}_4}) \ ,\end{aligned} \tag{4}$$

where $V_{ij}=\langle {\rm k} \mid \boldsymbol{\lambda}_i\boldsymbol{\lambda}_j\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j \mid {\rm l}\rangle$. The base states $|\rm k\rangle$ and $|\rm l\rangle$ are chosen to be the eigenstates of $SU(6)_{\rm cs}$. Since the $|\rm k\rangle$ and $|\rm l\rangle$ are not eigenstates of the CM interaction, we need calculate every individual term in Eq. (4) and diagonalize the $\langle {\rm k} \mid V_{\rm CM} \mid {\rm l}\rangle$ matrix in the base space in order to obtain the physics mass.

We use the $\rm qq\otimes\bar{q}\bar{q}$ to construct the color-spin wave function of 0^+, 1^+ and 2^+ heavy tetraquarks. A particular multiquark configuration is denoted as $|D_6,D_{3\rm c},S,N\rangle$, where D_6 and $D_{3\rm c}$ are $SU(6)_{\rm cs}$ color-spin and $SU(3)_{\rm c}$ color representations of the multiquark system respectively. S is the spin of the system and N is the total number of quarks and antiquarks. The $SU(6)_{\rm cs}$ eigenstates of the 0^+, 1^+ and 2^+ heavy tetraquark are listed below:

$$\begin{aligned}|1,1_{\rm c},0,4\rangle =& \sqrt{\frac{6}{7}}|21,6_{\rm c},1,2\rangle\otimes|\overline{21},\bar{6}_{\rm c},1,2\rangle+\\ & \sqrt{\frac{1}{7}}|21,\bar{3}_{\rm c},0,2\rangle\otimes|\overline{21},3_{\rm c},0,2\rangle,\end{aligned} \tag{5}$$

$$\begin{aligned}|405,1_{\rm c},0,4\rangle =& \sqrt{\frac{1}{7}}|21,6_{\rm c},1,2\rangle\otimes|\overline{21},\bar{6}_{\rm c},1,2\rangle-\\ & \sqrt{\frac{6}{7}}|21,\bar{3}_{\rm c},0,2\rangle\otimes|\overline{21},3_{\rm c},0,2\rangle,\end{aligned} \tag{6}$$

$$\begin{aligned}|1,1_{\rm c},0,4\rangle =& \sqrt{\frac{3}{5}}|15,\bar{3}_{\rm c},1,2\rangle\otimes|\overline{15},3,1,2\rangle+\\ & \sqrt{\frac{2}{5}}|15,6_{\rm c},0,2\rangle\otimes|\overline{15},\bar{6}_{\rm c},0,2\rangle,\end{aligned} \tag{7}$$

$$\begin{aligned}|189,1_{\rm c},0,4\rangle =& \sqrt{\frac{2}{5}}|15,\bar{3}_{\rm c},1,2\rangle\otimes|\overline{15},3,1,2,\rangle-\\ & \sqrt{\frac{3}{5}}|15,6_{\rm c},0,2\rangle\otimes|\overline{15},\bar{6}_{\rm c},0,2\rangle,\end{aligned} \tag{8}$$

$$|35,1_{\rm c},1,4,\rangle = |21,6_{\rm c},1,2,\rangle\otimes|\overline{21},\bar{6}_{\rm c},1,2,\rangle, \tag{9}$$

$$|35,1_{\rm c},1,4\rangle = |15,\bar{3}_{\rm c},1,2,6_f\rangle\otimes|\overline{15},3_{\rm c},1,2\rangle, \tag{10}$$

$$\begin{aligned}|35,1_{\rm c},1,4\rangle =& \sqrt{\frac{1}{3}}|21,\bar{3}_{\rm c},0,2\rangle\otimes|\overline{15},3_{\rm c},1,2\rangle-\\ & \sqrt{\frac{2}{3}}|21,6_{\rm c},1,2\rangle\otimes|\overline{15},\bar{6}_{\rm c},0,2\rangle,\end{aligned} \tag{11}$$

$$\begin{aligned}|280,1_{\rm c},1,4\rangle =& \sqrt{\frac{2}{3}}|21,\bar{3}_{\rm c},0,2\rangle\otimes|\overline{15},3_{\rm c},1,2,\rangle+\\ & \sqrt{\frac{1}{3}}|21,6_{\rm c},1,2,\rangle\otimes|\overline{15},\bar{6}_{\rm c},0,2,\rangle,\end{aligned} \tag{12}$$

$$\begin{aligned}|35,1_{\rm c},1,4\rangle =& \sqrt{\frac{1}{3}}|15,\bar{3}_{\rm c},1,2\rangle\otimes|\overline{21},3_{\rm c},0,2\rangle-\\ & \sqrt{\frac{2}{3}}|15,6_{\rm c},0,2\rangle\otimes|\overline{21},\bar{6}_{\rm c},1,2\rangle,\end{aligned} \tag{13}$$

$$\begin{aligned}|280,1_{\rm c},1,4\rangle =& \sqrt{\frac{2}{3}}|15,\bar{3}_{\rm c},1,2\rangle\otimes|\overline{21},3_{\rm c},0,2\rangle+\\ & \sqrt{\frac{1}{3}}|15,6_{\rm c},0,2\rangle\otimes|\overline{21},\bar{6}_{\rm c},1,2\rangle,\end{aligned} \tag{14}$$

$$|405,1_{\rm c},2,4\rangle = |21,6,1,2\rangle\otimes|\overline{21},\bar{6},1,2\rangle, \tag{15}$$

$$|189,1_{\rm c},2,4\rangle = |15,\bar{3},1,2\rangle\otimes|\overline{15},3,1,2\rangle\ . \tag{16}$$

Here we focus on the 0^+, 1^+ and 2^+ states without radial and orbital excitations. Their spatial wave functions are symmetric. Therefore their color-spin wave functions of flavor-symmetric diquark {qq} or

$\{\bar{q}\bar{q}\}$ inside tetraquarks are antisymmetric. Those flavor-antisymmetric diquarks [qq] or $[\bar{q}\bar{q}]$ have symmetric color-spin wave functions. We do not assume that quarks in the tetraquark really form diquarks with strong correlation. Instead we simply use the $qq\otimes\bar{q}\bar{q}$ to construct the convenient base wave functions. On the other hand, one can also start from the $q\bar{q}\otimes q\bar{q}$ basis. In order to calculate the $V_{\rm CM}(q\bar{q})$, we must do some recoupling from the $|qq,D_6,D_{3c},S,2\rangle\otimes|\bar{q}\bar{q},D_6,D_{3c},S,2\rangle$ basis to the $|q\bar{q},D_6,D_{3c},S,2\rangle\otimes|q\bar{q},D_6,D_{3c},S,2\rangle$ basis. Details can be found in Ref. [19]. The recoupling coefficients are listed in the Appendix.

3 Numerical results

In order to estimate the heavy tetraquark mass, we need determine the constituent quark mass and the value of the parameter $\bar{v}$ in Eq. (3) for the heavy tetraquark system. Using the experimental values of $J/\psi,\eta_c,D^*,D,D_s^*,D_s,\Upsilon$ masses, we use Eq. (1) to extract the constituent quark mass:

$$\begin{cases} M(J/\psi)=2m_c+\frac{16}{3}v_{c\bar{c}}\left(\frac{m_u}{m_c}\right)^2 \\ M(\eta_c)=2m_c-16v_{c\bar{c}}\left(\frac{m_u}{m_c}\right)^2 \\ M(D^*)=m_u+m_c+\frac{16}{3}v_{c\bar{u}}\left(\frac{m_u}{m_c}\right) \\ M(D)=m_u+m_c-16v_{c\bar{u}}\left(\frac{m_u}{m_c}\right) \\ M(D_s^*)=m_s+m_c+\frac{16}{3}v_{c\bar{s}}\left(\frac{m_u}{m_c}\right)\left(\frac{m_u}{m_s}\right) \\ M(D_s)=m_s+m_c-16v_{c\bar{s}}\left(\frac{m_u}{m_c}\right)\left(\frac{m_u}{m_s}\right) \\ M(\Upsilon)=2m_b+\frac{16}{3}v_{b\bar{b}}\left(\frac{m_u}{m_b}\right)^2 \end{cases} \Longrightarrow$$

$$\begin{cases} m_c=1534\text{MeV} \\ m_u=437\text{MeV} \\ m_s=542\text{MeV} \\ m_b=4730\text{MeV} \\ v_{c\bar{u}}=23.4\text{MeV} \\ v_{c\bar{s}}=29.3\text{MeV} \\ v_{c\bar{c}}=67.7\text{MeV} \end{cases}, \quad (17)$$

where $v_{c\bar{u}}$, $v_{c\bar{s}}$ and $v_{c\bar{c}}$ are merely the CM interaction parameters in Eq. (2) for the $c\bar{c}$, $c\bar{u}$, $c\bar{s}$ meson system respectively.

With the assumption that X(3872) is a $qc\bar{q}\bar{c}$ tetraquark, the $V_{\rm CM}(qc\bar{q}\bar{c})$ reads

$$V_{\rm CM}(qc\bar{q}\bar{c})=\zeta_cV_{12}+V_{13}+\zeta_cV_{14}+\zeta_cV_{23}+\zeta_c^2V_{24}+\zeta_cV_{34}, \quad (18)$$

where $\zeta_c=\frac{m_u}{m_c}$. After diagonalizing the full matrix $\langle k\mid V_{\rm CM}(qc\bar{q}\bar{c})\mid l\rangle$, we obtain CM interaction eigenvalue and mass. The lowest eigenvalue is

$$V_{\rm CM}=-15.9v_Q\,,$$

where $v_Q=\bar{v}_{qc\bar{q}\bar{c}}$. In the heavy quark limit $\zeta_c=\frac{m_u}{m_c}\to 0$, the CM energy of $qc\bar{q}\bar{c}$ system is dominated by the $q\bar{q}$ subsystem $V_{\rm CM}(qc\bar{q}\bar{c})\to V_{13}=-16v_Q$. The wave function of this lowest-lying tetraquark state will be $|c\bar{c}35,1_c,1,2\rangle\otimes|q\bar{q}1,1_c,0,2\rangle$ with charge conjugate parity $C=-$. This state is strongly coupled to J/ψ and a light pesudoscalar. Therefore it fall parts very easily and has a very broad decay width. Thus it is unlikely to observe it experimentally. The second lowest-lying tetraquark state will be $|c\bar{c}35,8_c,1,2\rangle\otimes|q\bar{q}35,8_c,1,2\rangle$ with $C=+$. We identify it to the observed X(3872) state:

$$V_{\rm CM}(X(3872))\;=\;-4.1v_Q\,,$$

$$M(X(3872))\;=\;2m_c+2m_u-4.1v_Q\,.$$

In the heavy quark limit $\zeta_c=\frac{m_u}{m_c}\to 0$, $V_{\rm CM}(qc\bar{q}\bar{c})\to V_{13}=-4v_Q$. Using the experimental value of X(3872) mass as input, we get

$$v_Q\approx 17.1\text{MeV}\,.$$

We will use this v_Q to estimate the mass of other $qq\bar{Q}\bar{Q}$ and $qQ\bar{q}\bar{Q}$ tetraquarks.

The CM strength parameter (v) depends on the constituent quark mass and spatial wave function of quark model. Since we consider only the 0^+, 1^+ and 2^+ heavy tetraquarks without radial and orbital excitations, their spatial wave functions are expected to be roughly the same for $qc\bar{q}\bar{c}$ and $qq\bar{c}\bar{c}$ systems. Hence, we use the same parameter v_Q to estimate the CM interaction of 0^+, 1^+ and 2^+ states of all $qQ\bar{q}\bar{Q}$ and $qq\bar{Q}\bar{Q}$ tetraquark systems with

Q = c or b. We also use the same $v_{\rm Q}$ to estimate the $\rm qQ\bar{Q}\bar{Q}$ heavy tetraquarks. The masses are collected in Tables 1—3. As pointed above, the lowest-lying tetraquark state of each $\rm qQ\bar{q}\bar{Q}$ system will be $|{\rm Q\bar{Q}}35,1_{\rm c},1,2\rangle\otimes|{\rm q\bar{q}}1,1_{\rm c},0,2\rangle$ in the heavy quark limit, which will be too broad to be observed experimentally. Thus in Table 1 we only list the masses of the second lowest-lying tetraquark states. The neutral ones of these states have positive C-parity.

Table 1. The CM energy and masses of 0^+, 1^+ and 2^+ of $\rm qQ\bar{q}\bar{Q}$. The CM energy is in unit $v_{\rm Q}$.

flavor	(0^+) $V_{\rm CM}(v_{\rm Q})$	$(0^+)M/$ MeV	(1^+) $V_{\rm CM}(v_{\rm Q})$	$(1^+)M/$ MeV	(2^+) $V_{\rm CM}(v_{\rm Q})$	$(2^+)M/$ MeV
$\rm qc\bar{q}\bar{c}$	−7.4	3816	−4.1	3872	2.7	3988
$\rm qc\bar{s}\bar{c}$	−6.5	3936	−3.8	3982	2.5	4090
$\rm sc\bar{s}\bar{c}$	−5.6	4056	−3.2	4097	2.3	4191
$\rm qb\bar{q}\bar{b}$	−3.0	10283	−1.8	10303	0.4	10341
$\rm qb\bar{s}\bar{b}$	−2.6	10395	−1.6	10412	0.5	10448
$\rm sb\bar{s}\bar{b}$	−2.3	10505	−1.3	10522	0.5	10553
$\rm qc\bar{q}\bar{b}$	−5.4	7046	−3.9	7071	1.6	7165
$\rm qc\bar{s}\bar{b}$	−4.5	7166	−3.1	7190	1.4	7267
$\rm sc\bar{s}\bar{b}$	−4.1	7278	−3.0	7297	1.4	7371

Table 2. The CM energy and masses of 0^+, 1^+ and 2^+ of $\rm qq\bar{Q}\bar{Q}$. The CM energy is in unit $v_{\rm Q}$. [qq] is the antisymmetrical representation in $SU(3)_{\rm f}$, {qq} is the symmetrical representation.

flavor	(0^+) $V_{\rm CM}(v_{\rm Q})$	$(0^+)M/$ MeV	(1^+) $V_{\rm CM}(v_{\rm Q})$	$(1^+)M/$ MeV	(2^+) $V_{\rm CM}(v_{\rm Q})$	$(2^+)M/$ MeV
$\rm \{qq\}\bar{c}\bar{c}$	−3.9	3875	1.4	3966	4.4	4017
$\rm \{ss\}\bar{c}\bar{c}$	−3.6	4090	0.7	4164	3.2	4207
$\rm [qq]\bar{c}\bar{c}$			−9.1	3786		
$\rm qs\bar{c}\bar{c}$	−3.8	3982	−7.5	3919	3.7	4110
$\rm \{qq\}\bar{b}\bar{b}$	0.7	10346	2.2	10372	3.2	10389
$\rm \{ss\}\bar{b}\bar{b}$	0.1	10546	1.4	10568	2.2	10582
$\rm [qq]\bar{b}\bar{b}$			−8.1	10195		
$\rm us\bar{b}\bar{b}$	0.4	10446	−6.6	10326	2.6	10483
$\rm \{qq\}\bar{c}\bar{b}$	−1.6	7111	−0.2	7135	3.7	7201
$\rm \{ss\}\bar{c}\bar{b}$	−1.8	7317	−0.7	7336	2.6	7392
$\rm [qq]\bar{c}\bar{b}$	−11.1	6948	−9.3	6979	1.1	7157
$\rm us\bar{c}\bar{b}$	−9.5	7081	−7.8	7110	1.2	7264

Table 3. The CM energy and masses of 0^+, 1^+ and 2^+ of $\rm qQ\bar{Q}\bar{Q}$. The CM energy is in unit $v_{\rm Q}$. [qq] is antisymmetrical representation in $SU(3)_{\rm f}$, {qq} is symmetrical representation.

flavor	(0^+) $V_{\rm CM}(v_{\rm Q})$	$(0^+)M/$ MeV	(1^+) $V_{\rm CM}(v_{\rm Q})$	$(1^+)M/$ MeV	(2^+) $V_{\rm CM}(v_{\rm Q})$	$(2^+)M/$ MeV
$\rm qc\bar{c}\bar{c}$	−3.5	4979	−4.0	4971	2.0	5073
$\rm sc\bar{c}\bar{c}$	−3.0	5093	−3.1	5091	1.7	5173
$\rm qb\bar{c}\bar{c}$	−3.4	8177	−3.4	8177	1.3	8257
$\rm sb\bar{c}\bar{c}$	−2.8	8292	−2.7	8294	1.1	8359
$\rm qb\bar{b}\bar{b}$	−1.0	14610	−1.4	14603	0.5	14636
$\rm sb\bar{b}\bar{b}$	−0.8	14718	−1.1	14713	0.4	14739
$\rm qc\bar{b}\bar{b}$	−0.6	11421	−2.5	11388	1.1	11450
$\rm sc\bar{b}\bar{b}$	−0.6	11526	−2.0	11502	0.9	11551
$\rm qc\bar{c}\bar{b}$	−5.8	8136	−4.8	8153	0.9	8250
$\rm sc\bar{c}\bar{b}$	−4.8	8258	−3.9	8273	0.8	8354
$\rm qb\bar{c}\bar{b}$	−4.9	11347	−4.6	11352	0.5	11440
$\rm sb\bar{c}\bar{b}$	−4.0	11468	−3.7	11473	0.4	11543

4 Discussions

In short summary, we have performed a schematic study of the masses of possible heavy tetraquarks using the color-magnetic interaction with the flavor symmetry breaking corrections. Treating X(3872) as a $\rm qc\bar{q}\bar{c}$ tetraquark and using its mass as input, we extract the CM interaction parameter $v_{\rm Q}$ for the heavy tetraquark system. With the same $v_{\rm Q}$, we have estimated other heavy tetraquark masses. It's interesting to note from the above tables that the CM interaction is repulsive for 2^+ heavy tetraquarks. From Table 1, it's clear that 0^+ $\rm qc\bar{q}\bar{c}$ states will also exist if X(3872) is really a 1^+ tetraquark.

C.Y. thanks Y. R. Liu and W. Wei for helpful discussions.

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Appendix A

The recoupling coefficients

In order to calculate the terms of $V(\mathrm{q\bar{q}})$, we need do the following recouplings[22, 23]

$$\begin{aligned}&\{|\mathrm{q}_1\mathrm{q}_2 D_6(\mathrm{Q})D_3(\mathrm{Q})S(\mathrm{Q})\rangle\otimes|\bar{\mathrm{q}}_3\bar{\mathrm{q}}_4 D_6(\bar{\mathrm{Q}})D_3(\bar{\mathrm{Q}})S(\bar{\mathrm{Q}})\rangle\}_{(D_3,S)}=\sum R(D_3(\mathrm{Q})D_3(\bar{\mathrm{Q}});D_3(13)D_3(24);D)\times\\&R(S(\mathrm{Q})S(\bar{\mathrm{Q}});S(13)S(24);S)\{|\mathrm{q}_1\bar{\mathrm{q}}_3 D_6(13)D_3(13)S(13)\rangle\otimes|\mathrm{q}_2\bar{\mathrm{q}}_4 D_6(24)D_3(24)S(24)\rangle\}_{(D_3,S)}\,,\end{aligned}\tag{A1}$$

where the recoupling coeffiecients are:

$$R(S(\mathrm{Q})S(\bar{\mathrm{Q}});S(13)S(24);S)=\sqrt{(2S(\mathrm{Q})+1)(2S(\bar{\mathrm{Q}})+1)(2S(13)+1)(2S(24)+1)}\begin{pmatrix}\frac{1}{2}&\frac{1}{2}&S(\mathrm{Q})\\\frac{1}{2}&\frac{1}{2}&S(\bar{\mathrm{Q}})\\S(13)&S(24)&S\end{pmatrix},\tag{A2}$$

$$R((\lambda_{\mathrm{Q}}\mu_{\mathrm{Q}})(\lambda_{\bar{\mathrm{Q}}}\mu_{\bar{\mathrm{Q}}});(\lambda_{13}\mu_{13})(\lambda_{24}\mu_{24}))=(-1)^{\lambda_{\mathrm{Q}}+\mu_{\mathrm{Q}}+\lambda_{13}+\mu_{13}}U((10)(10)(10)(01);(\lambda_{\mathrm{Q}}\mu_{\mathrm{Q}})(\lambda_{13}\mu_{13}))\,.\tag{A3}$$

We will list the recoupling coefficients of 0^+, 1^+ and 2^+ heavy tetraquarks below. With $q_1q_2\otimes\bar{q}_3\bar{q}_4$ construction, the 0^+ heavy tetraquarks are:

$$\begin{aligned}|1,1_{\mathrm{c}},S=0,4\rangle&=\sqrt{\frac{6}{7}}|21,6_{\mathrm{c}},1,2;\overline{21},\bar{6}_{\mathrm{c}},1,2\rangle+\sqrt{\frac{1}{7}}|21,\bar{3}_{\mathrm{c}},0,2;\overline{21},3_{\mathrm{c}},0,2\rangle,\\|405,1_{\mathrm{c}},0,4\rangle&=\sqrt{\frac{1}{7}}|21,6_{\mathrm{c}},1,2;\overline{21},\bar{6}_{\mathrm{c}},1,2\rangle-\sqrt{\frac{6}{7}}|21,\bar{3}_{\mathrm{c}},0,2;\overline{21},3_{\mathrm{c}},0,2\rangle,\\|1,1_{\mathrm{c}},0,4\rangle&=\sqrt{\frac{3}{5}}|15,\bar{3}_{\mathrm{c}},1,2;\overline{15},3,1,2\rangle+\sqrt{\frac{2}{5}}|15,6_{\mathrm{c}},0,2;\overline{15},\bar{6}_{\mathrm{c}},0,2\rangle,\\|189,1_{\mathrm{c}},0,4\rangle&=\sqrt{\frac{2}{5}}|15,\bar{3}_{\mathrm{c}},1,2;\overline{15},3,1,2,\rangle-\sqrt{\frac{3}{5}}|15,6_{\mathrm{c}},0,2;\overline{15},\bar{6}_{\mathrm{c}},0,2\rangle.\end{aligned}\tag{A4}$$

For 0^+ tetraquarks, the basis states of the $\mathrm{q_1q_2}\otimes\mathrm{\bar{q}_3\bar{q}_4}$ construction are:

$$\begin{aligned}a_1^0&=|\mathrm{q_1q_2}21,6_{\mathrm{c}},S=1;\mathrm{\bar{q}_3\bar{q}_4}\overline{21},\bar{6}_{\mathrm{c}},1\rangle; & a_2^0&=|\mathrm{q_1q_2}21,\bar{3}_{\mathrm{c}},0;\mathrm{\bar{q}_3\bar{q}_4}\overline{21},3_{\mathrm{c}},0\rangle;\\a_3^0&=|\mathrm{q_1q_2}15,\bar{3}_{\mathrm{c}},1;\mathrm{\bar{q}_3\bar{q}_4}\overline{15},3,1\rangle; & a_4^0&=|\mathrm{q_1q_2}15,6_{\mathrm{c}},0;\mathrm{\bar{q}_3\bar{q}_4}\overline{15},\bar{6}_{\mathrm{c}},0\rangle.\end{aligned}\tag{A5}$$

The corresponding basis states of the $\mathrm{q_1\bar{q}_3}\otimes\mathrm{q_2\bar{q}_4}$ construction are:

$$\begin{aligned}b_1^0&=|\mathrm{q_1\bar{q}_3}1,1,0;\mathrm{q_2\bar{q}_4}1,1,0\rangle; & b_2^0&=|\mathrm{q_1\bar{q}_3}35,1,1;\mathrm{q_2\bar{q}_4}35,1,1\rangle;\\b_3^0&=|\mathrm{q_1\bar{q}_3}35,8,0;\mathrm{q_2\bar{q}_4}35,8,0\rangle; & b_4^0&=||\mathrm{q_1\bar{q}_3}35,8,1;\mathrm{q_2\bar{q}_4}35,8,1\rangle.\end{aligned}\tag{A6}$$

The transform matrix from Eq. (A5) to Eq. (A6) is:

$$\begin{pmatrix}\frac{\sqrt{2}}{2}&-\frac{\sqrt{6}}{6}&\frac{1}{2}&-\frac{\sqrt{3}}{6}\\\frac{\sqrt{3}}{6}&\frac{1}{2}&-\frac{\sqrt{6}}{6}&-\frac{\sqrt{2}}{2}\\\frac{1}{2}&-\frac{\sqrt{3}}{6}&-\frac{\sqrt{2}}{2}&\frac{\sqrt{6}}{6}\\\frac{\sqrt{6}}{6}&\frac{\sqrt{2}}{2}&\frac{\sqrt{3}}{6}&\frac{1}{2}\end{pmatrix}.\tag{A7}$$

Then we can write the 0^+ $SU(6)$ eigenstates in terms of two pairs of $\mathrm{q_1\bar{q}_3 \otimes q_2\bar{q}_4}$:

$$|1,1_c,0\rangle = \frac{\sqrt{21}}{6}|q_1\bar{q}_3 1,1,0;q_2\bar{q}_4 1,1,0\rangle - \frac{\sqrt{7}}{14}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle + \frac{\sqrt{42}}{21}|q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,0\rangle - \frac{\sqrt{14}}{7}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle, \tag{A8}$$

$$|405,1_c,0\rangle = -\frac{2\sqrt{42}}{21}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle + \frac{3\sqrt{7}}{14}|q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,0\rangle + \frac{5\sqrt{21}}{42}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle, \tag{A9}$$

$$|1,1_c,0\rangle = \frac{\sqrt{15}}{6}|q_1\bar{q}_3 1,1,0;q_2\bar{q}_4 1,1,0\rangle + \frac{\sqrt{5}}{10}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle - \frac{\sqrt{30}}{15}|q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,0\rangle + \frac{\sqrt{10}}{5}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle, \tag{A10}$$

$$|189,1_c,0\rangle = -\frac{2\sqrt{30}}{15}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle - \frac{3\sqrt{5}}{10}|q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,0\rangle - \frac{\sqrt{15}}{30}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle. \tag{A11}$$

For the 1^+ heavy tetraquarks, the states with the $\mathrm{q_1q_2 \otimes \bar{q}_3\bar{q}_4}$ construction are:

$$\begin{aligned}
|35,1_c,1,4,\rangle &= |21,6_c,1,2,\rangle \otimes |\overline{21},\bar{6}_c,1,2,\rangle, \\
|35,1_c,1,4\rangle &= |15,\bar{3}_c,1,2,6_f\rangle \otimes |\overline{15},3_c,1,2\rangle, \\
|35,1_c,1,4\rangle &= \sqrt{\frac{1}{3}}|21,\bar{3}_c,0,2\rangle \otimes |\overline{15},3_c,1,2\rangle - \sqrt{\frac{2}{3}}|21,6_c,1,2\rangle \otimes |\overline{15},\bar{6}_c,0,2\rangle, \\
|280,1_c,1,4\rangle &= \sqrt{\frac{2}{3}}|21,\bar{3}_c,0,2\rangle \otimes |\overline{15},3_c,1,2,\rangle + \sqrt{\frac{1}{3}}|21,6_c,1,2,\rangle \otimes |\overline{15},\bar{6}_c,0,2,\rangle, \\
|35,1_c,1,4\rangle &= \sqrt{\frac{1}{3}}|15,\bar{3}_c,1,2\rangle \otimes |\overline{21},3_c,0,2\rangle - \sqrt{\frac{2}{3}}|15,6_c,0,2\rangle \otimes |\overline{21},\bar{6}_c,1,2\rangle, \\
|280,1_c,1,4\rangle &= \sqrt{\frac{2}{3}}|15,\bar{3}_c,1,2\rangle \otimes |\overline{21},3_c,0,2\rangle + \sqrt{\frac{1}{3}}|15,6_c,0,2\rangle \otimes |\overline{21},\bar{6}_c,1,2\rangle.
\end{aligned} \tag{A12}$$

The basis states are:

$$\begin{aligned}
a_1^1 &= |q_1q_2 21,\bar{3}_c,0;\bar{q}_3\bar{q}_4\overline{15},3_c,1\rangle; & a_2^1 &= |q_1q_2 21,6,1;\bar{q}_3\bar{q}_4\overline{15},\bar{6}_c,0\rangle; \\
a_3^1 &= |q_1q_2 21,6,1;\bar{q}_3\bar{q}_4\overline{21},\bar{6}_c,1\rangle; & a_4^1 &= |q_1q_2 15,\bar{3},1;\bar{q}_3\bar{q}_4\overline{15},3,1\rangle; \\
a_5^1 &= |q_1q_2 15,\bar{3},1;\bar{q}_3\bar{q}_4\overline{21},3,0\rangle; & a_6^1 &= |q_1q_2 15,6,0;\bar{q}_3\bar{q}_4\overline{21},\bar{6},1\rangle.
\end{aligned} \tag{A13}$$

With $\mathrm{q_1\bar{q}_3 \otimes q_2\bar{q}_4}$ construction, the basis states are:

$$\begin{aligned}
b_1^1 &= |q_1\bar{q}_3 1,1,0;q_2\bar{q}_4 35,1,1\rangle; & b_2^1 &= |q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,1\rangle; \\
b_3^1 &= |q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 1,1,0\rangle; & b_4^1 &= ||q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,0\rangle; \\
b_5^1 &= |q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle; & b_6^1 &= |q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle.
\end{aligned} \tag{A14}$$

The transform matrix from Eq. (A13) to Eq. (A14) is:

$$\begin{pmatrix}
\frac{\sqrt{3}}{6} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} \\
-\frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} \\
\frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} & 0 & 0 \\
\frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & 0 & 0 \\
\frac{\sqrt{3}}{6} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} \\
-\frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3}
\end{pmatrix}. \tag{A15}$$

Then the 1^+ $SU(6)$ flavor eigenstates in terms of two pairs of $q_1\bar{q}_3\otimes q_2\bar{q}_4$ read

$$|35,1_c,1\rangle = \frac{1}{2}|q_1\bar{q}_3 1,1,0;q_2\bar{q}_4 35,1,1\rangle - \frac{1}{2}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 1,1,0\rangle - \frac{2}{3}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle, \tag{A16}$$

$$|280,1_c,1\rangle = -\frac{1}{2}|q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,1\rangle + \frac{1}{2}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,0\rangle - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle + \frac{2}{3}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle, \tag{A17}$$

$$|35,1_c,1\rangle = \frac{\sqrt{3}}{3}|q_1\bar{q}_3 1,1,0;q_2\bar{q}_4 35,1,1\rangle + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,1\rangle + \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 1,1,0\rangle + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,0\rangle, \tag{A18}$$

$$|35,1_c,1\rangle = \frac{\sqrt{6}}{6}|q_1\bar{q}_3 1,1,0;q_2\bar{q}_4 35,1,1\rangle - \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,1\rangle + \frac{\sqrt{6}}{6}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 1,1,0\rangle - \frac{\sqrt{3}}{3}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,0\rangle, \tag{A19}$$

$$|35,1_c,1\rangle = \frac{1}{2}|q_1\bar{q}_3 1,1,0;q_2\bar{q}_4 35,1,1\rangle - \frac{1}{2}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 1,1,0\rangle - \frac{2}{3}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle, \tag{A20}$$

$$|280,1_c,1\rangle = -\frac{1}{2}|q_1\bar{q}_3 35,8,0;q_2\bar{q}_4 35,8,1\rangle + \frac{1}{2}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,0\rangle - \frac{\sqrt{2}}{6}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle + \frac{2}{3}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle. \tag{A21}$$

For the 2^+ heavy tetraquarks,

$$|405,1_c,2\rangle = \{|q_1q_2 21,6,1;\bar{q}_3\bar{q}_4\overline{21},\bar{6},S=1\rangle\} = \sqrt{\frac{1}{3}}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle + \sqrt{\frac{2}{3}}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle, \tag{A22}$$

$$|189,1_c,2\rangle = \{|q_1q_2 15,\bar{3},1;\bar{q}_3\bar{q}_4\overline{15},3,1\rangle\} = \sqrt{\frac{2}{3}}|q_1\bar{q}_3 35,8,1;q_2\bar{q}_4 35,8,1\rangle - \sqrt{\frac{1}{3}}|q_1\bar{q}_3 35,1,1;q_2\bar{q}_4 35,1,1\rangle. \tag{A23}$$

With $SU_c(3)$ and $SU_s(2)$ symmetry, similar recoupling coefficients can also be obtained.

可能的 $qQ\bar{q}\bar{Q}$, $qq\bar{Q}\bar{Q}$ 和 $qQ\bar{Q}\bar{Q}$ 重四夸克态*

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摘要　假设 X(3872) 是一个 $qc\bar{q}\bar{c}$ 四夸克态, 并用它的质量作为输入, 用具有味对称性破坏的色磁相互作用系统研究了可能的重四夸克态的质量谱.

关键词　重四夸克态　色磁相互作用　夸克模型

2006 – 06 – 13 收稿,　2006 – 07 – 20 收修改稿

∗ 国家自然科学基金(10375003, 10421503)和教育部重大项目(305001)资助

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