Isoscaling Analysis and Symmetry Energy*

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Abstract The status of the isoscaling analysis has been simply reviewed and the corresponding information on the extraction of the symmetry energy is discussed. We discuss the secondary decay effect on isoscaling parameter, the excitation energy and density dependences of the symmetry energy.

Key words isoscaling behavior, symmetry energy, secondary decay, excitation energy, density

1 Introduction

When one makes the isotope yield ratio from two different reactions with the same charge number and the similar temperature, a so-called isoscaling law has been observed experimentally[1—3]. Isoscaling means that the ratio of isotope yields from two different reactions, 1 and 2, $R_{21}(N,Z)=Y_2(N,Z)/Y_1(N,Z)$, is found to exhibit an exponential relationship as a function of the neutron number N and proton number Z

$$R_{21}(N,Z)=\frac{Y_2(N,Z)}{Y_1(N,Z)}=C\exp(\alpha N+\beta Z), \quad (1)$$

where C, α and β are three parameters. In grand-canonical limit, $\alpha=\Delta\mu_{\rm n}/T$ and $\beta=\Delta\mu_{\rm z}/T$ where $\Delta\mu_{\rm n}$ and $\Delta\mu_{\rm z}$ are the differences between the neutron and proton chemical potentials for two reactions, respectively. This behavior is basically attributed to the difference of two reaction systems with different isospin asymmetry. It is potential to probe the isospin dependent nuclear equation of state (EOS) by the studies of isoscaling[4]. So far, the isoscaling behavior has been extensively surveyed experimentally[1, 5—10] and theoretically[2, 3, 9, 11—19] in various reaction mechanisms, ranging from the evaporation, fission and deep inelastic reaction at low energies to the projectile fragmentation and multifragmentation at intermediate energy.

The isospin dependence of the nuclear equation of state is one of the most important properties in nuclear matter. Of which, the asymmetry energy term is of very interesting. Although the nuclear symmetry energy at normal nuclear matter density $\rho_0=0.16{\rm fm}^{-3}$ has been determined from the empirical liquid-drop mass formula[20], however, its values at sub- and super-densities are poorly known. Studies based on various theoretical models also give widely different predictions[21—23]. In Fig. 1 (bottom panel) we show the density dependence of the potential symmetry energy contribution, $E_{\rm sym,pot}$ for three different effective interactions. While all curves cross at normal density ρ_0, large differences are present for values, slopes and curvatures, particularly in high density regions. Even at the relatively well known "crossing point" at normal density the various effective forces are presenting controversial predictions for the momentum dependence of the fields acting on the nucleons and, consequently, for the splitting of the neutron/proton effective masses, which are important in nuclear structure and nuclear reaction dynamics. Isoscaling provides a potential tool to extract the information of potential symmetry energy, as we can see in the following sections.

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In this talk, we will simply review the status of the isoscaling analysis and the corresponding information on the extraction of the symmetry energy. In particular, the excitation energy and density dependences of the symmetry energy will be discussed. Due to the limited page, here we do not review the respective contribution of both surface energy and volume energy to the extracted symmetry energy.

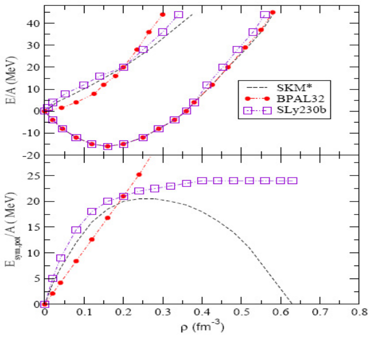

Fig. 1. Equation of State for various effective forces. Top: neutron matter (up), symmetric matter (down). Bottom: potential symmetry energy.

2 Isoscaling and symmetry energy

Fig. 2 shows the isotope ratios, R_{21}, plotted as a function of N (upper panel) and Z (lower panel), for the central collisions of $^{124}Sn+^{124}Sn$ and $^{112}Sn+^{112}Sn$, at 50MeV/nucleon[2]. R_{21} clearly exhibits an exponential dependence on N and Z, i.e. isoscaling behavior. Alternatively, the data in the left panels can be displayed compactly as a function of one variable (right panels), either N or Z, by removing the dependence of the other variable using the scaled isotope or isotone functions:

$$S(N)=R_{21}\exp(-\beta Z) \quad \text{or} \quad S(Z)=R_{21}\exp(-\alpha N).$$

It has been shown[3, 12] that the isoscaling parameter α is directly related to the coefficient C_{sym} of the symmetry energy term of the nuclear binding energy, the following relation has been obtained both in the framework of the grand-canonical limit of the statistical multifragmentation model and in the expanding-emitting source model:

$$\alpha=4\frac{C_{sym}}{T}\left[\left(\frac{Z_1}{A_1}\right)^2-\left(\frac{Z_2}{A_2}\right)^2\right], \tag{2}$$

where Z_1, A_1, and Z_2, A_2 refer to the charge number and mass number of the fragments from reactions 1 and 2 respectively. Using this relation, from the extracted values of α and T of the fragments in the reaction, symmetry energy coefficient C_{sym} in the EOS could be derived.

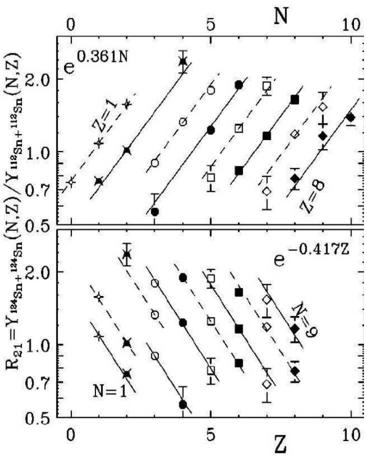

Fig. 2. The yield ratio R_{21} is plotted as a function of N (upper panel) or Z (lower panel). The central reactions considered are $^{124}Sn+^{124}Sn$ and $^{112}Sn+^{112}Sn$, at 50MeV/nucleon.

3 Sequential decay effect on isoscaling

Sequential decay might have an important effect on the values of isoscaling parameters due to the deexcitation of hot fragments themselves as well as the side feeding effect. To quantitatively investigate this effect, some statistical models have been performed for checking this influence.

For instance, the sequential decay effect on isoscaling was investigated by the isospin-dependent quantum molecular dynamics (IQMD) model followed by the afterburner: GEMINI code calculation. α parameters were extracted as a function of the fragment atomic number Z, one found that the dynamical effect due to different reaction times is presented for the light fragments[17], here the different reaction time refers to the switching time from hot fragments which are recognized in IQMD calculation to the GEMINI calculations. The secondary decay effect, however, does not depend on the switching time of IQMD to GEMINI code for the final products after secondary decay. The ratio between α_{GEMINI} and α_{IQMD} was plotted in Fig. 3 to show the secondary sequential decay effect: no big effect for lighter fragments with $Z \leqslant 8$ but it will increase the isoscaling parameter α slightly for larger fragments.

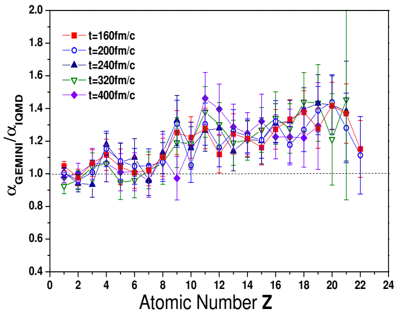

Fig. 3. The ratio of isoscaling parameters α between final and primary products by IQMD followed by GEMINI.

However, there is no consensus about how the decay effects on isoscaling parameter so far. For examples, the calculation by the AMD model followed by the SMM-MSU/GEMINI code shows the secondary effect will make the isoscaling parameter of the final fragments much smaller than that of primary fragments, roughly reduced by 50%—60% for the system ^{48}Ca+^{48}Ca and ^{40}Ca+^{40}Ca[24]. Somewhat large reductions of $\alpha_{\rm pri}$ due to secondary decays have also been found in the dynamical statistical mean field simulations of Sn+Sn collisions[9]. Moreover, it was noted that the secondary decay corrections to dynamical simulations are larger than the corresponding corrections to equilibrium statistical models[3], which shows no big effect on isoscaling parameters on light nuclear clusters was found for Sn+Sn collisions. The difference may arise from the different condition of different ensembles of excited fragments.

4 Excitation energy dependence of symmetry energy

Heavy residue isoscaling has been investigated in Texas A&M University (TAMU) from the peripheral collisions of ^{86}Kr (25MeV/nucleon), ^{64}Ni (25MeV/nucleon) and ^{136}Xe (20MeV/nucleon) beams on various target pairs[25]. The excitation energies are extracted by the measured average velocities, typically E^*/A=2—3MeV. The symmetry energy has been extracted from the isoscaling parameter α and the excitation energy dependence of the symmetry energy has been constructed. The stars in Fig. 4 are the experimental results. They show a decreasing function with the increasing of excitation energy.

Heavy fragment isoscaling was also studied in the statistical ablation-abration (SAA) model . The projectile fragmentation, reactions of $^{40/36}$Ar, $^{48/40}$Ca, $^{64/58}$Ni, $^{86/78}$Kr, $^{124/112}$Sn and $^{129/136}$Xe on ^{112}Sn at 60A MeV have been simulated by the SAA model[18]. Good isoscaling behavior has been obtained for different projectile-like fragments (PLFs). Different from the light fragments, the value of α of each PLF has a strong dependence on the fragment charge, as shown in Fig. 4, which may indicate that the surface symmetry energy is playing an important role in PLF isoscaling. Since there is a relationship between each PLF's charge number and the average excitation energy, we can deduce the excitation energy dependence of the symmetry energy. The points shown in Fig. 4 are our calculated results which show similar decreasing behavior with excitation energy. Even though different systems and reaction mechanism from TAMU data (stars), it looks the behavior for our PLF results and TAMU heavy residue data are qualitatively similar.

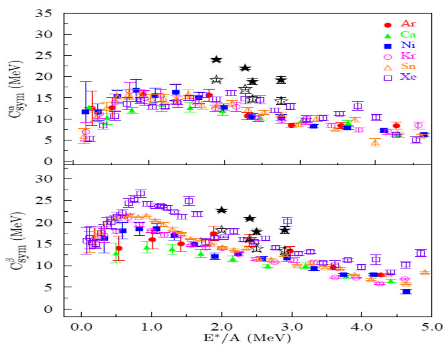

Fig. 4. Symmetry energies extracted from α ($C^{\alpha}_{\rm sym}$, upper panel) and β ($C^{\beta}_{\rm sym}$, lower panel) as a function of the excitation energy per nucleon. The solid and open stars are the experimental data taken from Ref. [25] with temperature calculated by Fermi gas and expanding mono-nucleus model respectively.

INDRA@GSI data for ^{12}C+^{124,112}Sn shows the similar decreasing trend for symmetry energy parameter from the peripheral to central collision, i.e. from lower temperature to higher temperature[22].

5 Density dependence of symmetry energy

Since the phase diagram of multifragmentation system is two dimensional and hence the excitation energy dependence and the density dependence is usually related each other. Usually the density of the system decreases with the increasing of the excitation energy, while the temperature may show a plateau as a function of the excitation energy. The density dependence of symmetry energy has been attempted to extract from the isoscaling data analysis for ^{58}Ni+^{58}Ni, ^{58}Fe+^{58}Ni, ^{58}Fe+^{58}Fe reactions at beam energies of 30, 40 and 47MeV/nucleon at TAMU[26] (see Fig. 5). The solid curve corresponds to the dependence, $E_{\rm sym}(\rho)=31.6(\rho/\rho_0)^{0.69}$. Recent IBUU simulation for the isospin diffusion gives the similar relation for $E_{\rm sym}$ versus ρ but with a stiffer dependence on density[23]. In our IQMD simulation, the density dependence is also extracted and the similar $E_{\rm sym}(\rho)$ exhibits in lower density region[17].

Eq. (2) is reasonable for systems near normal density, it becomes increasingly less accurate at low densities where entropic effects become increasingly more important[28]. Indeed a recent paper has demonstrated the existence of isoscaling effects in a non-interacting system with no symmetry energy, resulting entirely from maximal entropy[28]. Actually in Eq. (2) $C_{\rm sym}$ should be $F_{\rm sym}$, which includes all contributions not only from the symmetry energy, $C_{\rm sym}$ but also arising from, e.g. bulk, Coulomb and surface. Recently TAMU experimental analyses of moderate temperature nuclear gases produced in the violent collisions of 35 MeV/nucleon ^{64}Zn projectiles with ^{92}Mo and ^{197}Au target nuclei reveal a large degree of alpha particle clustering at low densities[29]. For these gases, temperature and density dependent symmetry energy coefficients have been derived from isoscaling analyses of the yields of nuclei with $A\leqslant 4$. At densities of 0.01 to 0.05 times the ground state density of symmetric nuclear matter, the temperature and density dependent symmetry energies are 10.7 to 13.5MeV. These values are much larger than those obtained in mean field calculations. They are in quite good agreement with results of a recently proposed Virial Equation of State calculation. The symmetry energy coefficients are also plotted against density in Fig. 6 where they are compared to those which are predicted by the Gogny effective interaction[30] and to the $31.6(\rho/\rho_0)^{1.05}$ dependence suggested by a recent analysis of isospin diffusion data[23]. The derived values of E_{sym} are much higher than those predicted by mean field calculations. This work shows that the entropic contributions to the symmetry free energies derived from isoscaling analyses are very important at low densities and are strongly affected by the cluster formation.

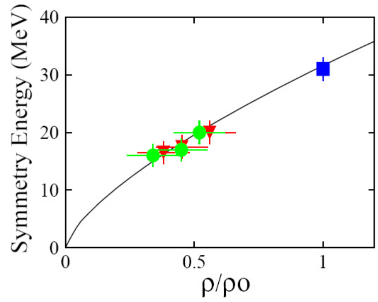

Fig. 5. Symmetry energy as a function of density for the Fe+Fe and Ni+Ni pair of reactions (inverted triangles) and Fe+Ni and Ni+Ni pair of reactions (solid circles) for the 30, 40 and 47MeV/nucleon. The solid square corresponds to those from Ref. [27].

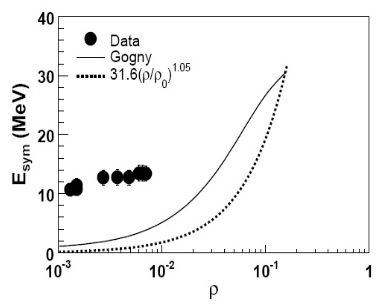

Fig. 6. Derived symmetry energy coefficients as a function of baryon density. Solid line indicates the variation predicted by the Gogny interaction. The dotted line represents the function $31.6(\rho/\rho_0)^{1.05}$[23].

6 Conclusions

Isoscaling behavior has been simply reviewed and the extracted information of symmetry energy is focused. Since fragmenting nuclei is dynamical system, the secondary decay effect is discussed on the isoscaling effect. Some models predict that the decay will increase the apparent isoscaling parameter, but some models predict no big effect even it will decrease the parameter. It looks that there exists model dependence for this decay effect, also different ensemble for excited fragments play an important role. The excitation energy dependence of the symmetry energy has been deduced in some data and model, generally $E_{\rm sym}$ decreases with the excitation energy. The density dependence of $E_{\rm sym}$ indicates that $31.6\rho/\rho_0^{\gamma}$ with γ=0.6—1 might be reasonable in the density not far from the normal density. But at densities of 0.01 to 0.05 ρ_0, the temperature and density dependent symmetry energies are 10.7 to 13.5MeV. The derived values of $E_{\rm sym}$ are much higher than those predicted by mean field calculations. However, the definite conclusion for the density and excitation energy (or temperature) dependences of the symmetry energy is still open, many experimental and theoretical researches have to be performed.

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同位旋标度与对称能*

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摘要 简要地评论了同位旋标度率分析的现状以及从中提取的对称能的信息. 首先提出了激发核的次级衰变对同位旋标度系数的影响, 然后提出了对称能对激发能的依赖性, 最后讨论了对称能对密度的依赖性.

关键词 同位旋标度 对称能系数 次级衰变 激发能 密度

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