Decays of $D_{sj}^*(2317)$ and $D_{sj}(2460)$ Mesons in the Quark Model[*]

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Abstract   We study the decay widths of the narrow resonances $D_{sj}^*(2317)$ and $D_{sj}(2460)$ in the chiral quark model, together with the well-known D^* and D_s^* mesons. All the parameters in our calculation are taken from Godfrey and Isgur's quark model except the π^0-η mixing angle which is fixed by the D_s^* decay widths. The calculated electromagnetic decay widths agree with those from other groups and the experimental data available quite well. However, the pionic decay widths of $D_{sj}(2317)$ and $D_{sj}(2460)$ are too small to fit the experimental data. We suspect that the simple chiral quark pion axial-vector interaction Hamiltonian is not suitable for hadron strong decays of $D_{sj}(2317)$ and $D_{sj}(2460)$.

Key words   quark model, narrow resonance, decay width

1   Introduction

The discovery of narrow resonances $D_{sj}^*(2317)$[1] and $D_{sj}(2460)$[2] raises challenges to the quark model. Masses of these two states are about 100MeV lower than the predictions of the potential quark model[3]. Furthermore, the isospin conserving decay channels $D_{sj}^{(*)} \to D^{(*)}K$ are forbidden by kinematics. The observed pionic decays into $D_s^{(*)}\pi^0$ break the isospin symmetry.

Several non-conventional schemes, such as molecules[4], tetraquark states[5—8], or the chiral partners of D_s and D_s^*[9, 10] etc., have been proposed (for a review, see Refs. [11,12]). But the conventional $c\bar{s}$ interpretation is still attractive if the experimental masses of $D_{sj}^{(*)}$ can be accommodated in the quark model[13—15]. In the heavy quark limit, $D_{sj}(2317)$ and $D_{sj}(2460)$ naturally form a P-wave doublet $J^P = (0^+, 1^+)$ with $j_l = \dfrac{1}{2}$. Couple-channel effects could lead to mass shifts[16, 17]. The observed pionic decays can also be understood through η-π^0 mixing from the chiral perturbation theory[18, 19].

In a previous work[20], we have calculated the pionic decay widths of the $D_{sj}^{(*)}$ mesons using the ${}^3P_0$ strong decay model. Another simple decay model to deal pionic decays is the chiral quark model[21].

In this work, we calculate both the electromagnetic (EM) decays and strong decay widths of $D_{sj}^{(*)}$ mesons in the quark model. The meson wave functions are taken from the well-known Godfrey and Isgur's model[3] which gives an impressively good overall description of meson states. The chiral couplings of light quarks with π, η meson and the isospin violating π^0-η mixing parameter are taken from the chiral perturbation theory[18, 22]. Since the relative momentum is very large in the decays of $D_{sj}^{(*)}$ mesons, the relativistic effect should be important. We do not make the non-relativistic reductions of the transition operators. Instead we keep Dirac spinors in our calculation.

In the next section, we present our formalism. The decays of $D_{sj}^{(*)}$ are evaluated and compared with experimental data in Sec. 3. We give a brief discussion

and summary in Sec. 4.

2　Decays of $\mathrm{D}_{\mathrm{sj}}^{(*)}$ in the quark model

The electromagnetic interaction is standard

$$\mathscr{L}_{\gamma} = e\bar{\Psi}\gamma^{\mu}\Psi A_{\mu} \ . \tag{1}$$

In the chiral quark model, the pions interact with the light up and down quarks through the axial-vector coupling

$$\mathscr{L}_{\pi\mathrm{q}} = \frac{g_{\mathrm{A}}^{q}}{2f_{\pi}}\bar{\Psi}\gamma_{\mu}\gamma_{5}\boldsymbol{\tau}\Psi\cdot\partial^{\mu}\boldsymbol{\phi}_{\pi} \ . \tag{2}$$

As pointed out in Refs [18,22], the reasonable value of g_{A}^{q} ranges from 0.75 to 1.

The pionic decay of $\mathrm{D}_{\mathrm{sj}}^{(*)}$ cannot occur from the above isospin conserving interaction since $\mathrm{D}_{\mathrm{sj}}^{(*)}$ has no u- or d- light flavor quarks in the conventional $c\bar{s}$ configuration. However, this isospin breaking decay can occur through π^{0}-η mixing due to the up and down quark mass difference. In the chiral perturbation theory, the η-π^{0} mixing amplitude reads[18, 19]

$$\theta_{\mathrm{m}} = \frac{\sqrt{3}}{4}\frac{m_{\mathrm{d}}-m_{\mathrm{u}}}{m_{\mathrm{s}}-\dfrac{m_{\mathrm{d}}+m_{\mathrm{u}}}{2}} \simeq 0.010 \ . \tag{3}$$

From $SU_{\mathrm{F}}(3)$ symmetry, the η coupling should be

$$\mathscr{L}_{\eta\mathrm{q}} = \frac{g_{\mathrm{A}}^{q}}{2f_{\eta}}\bar{\Psi}\gamma_{\mu}\lambda_{8}\Psi\cdot\partial^{\mu}\phi_{\eta} \ . \tag{4}$$

Thus, we have[1]

$$\mathscr{L}_{\pi\mathrm{s}} = -\frac{g_{\mathrm{A}}^{q}}{\sqrt{3}f_{\eta}}\theta_{\mathrm{m}}\bar{s}\gamma^{\mu}\gamma_{5}s\,\partial_{\mu}\pi^{0} \ . \tag{5}$$

In our calculation, we do not make the non-relativistic reductions of the transition operators. The quark fields are kept in the form with Dirac spinors

$$\Psi(x) = \sum_{s}\int\frac{\mathrm{d}^{3}p}{(2\pi)^{3}}\frac{1}{2p^{0}}\Big[d^{\dagger}(\boldsymbol{p},s)v(\boldsymbol{p},s)\mathrm{e}^{\mathrm{i}p\cdot x}+$$
$$b(\boldsymbol{p},s)u(\boldsymbol{p},s)\mathrm{e}^{-\mathrm{i}p\cdot x}\Big], \tag{6}$$

where the anti-commutation relation of quarks reads

$$\{d(\boldsymbol{p},s),d^{\dagger}(\boldsymbol{p}',s')\} = \{b(\boldsymbol{p},s),b^{\dagger}(\boldsymbol{p}',s')\} =$$
$$(2\pi)^{3}(2p^{0})\delta_{ss'}\delta(\boldsymbol{p}-\boldsymbol{p}') \ . \tag{7}$$

Accordingly, the meson wave functions are expressed

with the quark operators

$$|M,\boldsymbol{P}\rangle = \frac{1}{\sqrt{N_{\mathrm{c}}}}\int\frac{\mathrm{d}^{3}k}{(2\pi)^{3}}\frac{1}{2\sqrt{p_{\mathrm{Q}}^{0}p_{\bar{\mathrm{q}}}^{0}}}\phi(\boldsymbol{k},\boldsymbol{P},\mathrm{D}_{s})b_{\mathrm{Q}}^{\dagger}T_{\mathrm{M}}d_{\bar{\mathrm{q}}}^{\dagger}|0\rangle \ . \tag{8}$$

Here we treat the quark operators in a matrix form

$$b_{\mathrm{Q}}^{\dagger} = \begin{pmatrix} b_{\mathrm{Q},\uparrow}^{\dagger} \\ b_{\mathrm{Q},\downarrow}^{\dagger} \end{pmatrix}^{\mathrm{T}} \qquad d_{\bar{\mathrm{q}}}^{\dagger} = \begin{pmatrix} b_{\bar{\mathrm{q}},\uparrow}^{\dagger} \\ b_{\bar{\mathrm{q}},\downarrow}^{\dagger} \end{pmatrix} \ . \tag{9}$$

The inner $\mathrm{Q}\bar{\mathrm{q}}$ structure matrix T_{M} of D_{s}, D_{s}^{*}, $\mathrm{D}_{\mathrm{sj}}^{*}(2317)$ and $\mathrm{D}_{\mathrm{sj}}(2460)$ mesons are taken from the quark model[3, 23]. They are listed in Table 1. The spatial wave functions $\phi(\boldsymbol{k},\boldsymbol{P})$ are normalized as

Table 1.　The inner tensor structure matrix of $\mathrm{D}_{s}^{(*)}$ mesons.

meson	J^{P}	T_{M}
D_{s}	0^{-}	$\dfrac{\mathbb{1}}{\sqrt{2}}$
D_{s}^{*}	1^{-}	$\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}}{\sqrt{2}}$
$\mathrm{D}_{\mathrm{sj}}^{*}(2317)$	0^{+}	$\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{\sqrt{6}}$
$\mathrm{D}_{\mathrm{sj}}(2460)$	1^{+}	$\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\boldsymbol{\sigma}\cdot\boldsymbol{k}}{\sqrt{6}}$
$^{1}\mathrm{P}_{1}$	1^{+}	$\dfrac{\boldsymbol{k}\cdot\boldsymbol{\epsilon}}{\sqrt{2}}$
$^{3}\mathrm{P}_{1}$	1^{+}	$\dfrac{\mathrm{i}\boldsymbol{k}\times\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}}{2}$

$$2E = \int\frac{\mathrm{d}^{3}k}{(2\pi)^{3}}|\phi(\boldsymbol{k},\boldsymbol{p})|^{2} \ . \tag{10}$$

In the heavy quark limit, $j_{\mathrm{l}} = L+s_{\mathrm{q}}$ is a good quantum number when $m_{\mathrm{Q}}\to\infty$. The lowest 0^{+} and 1^{+} excitation states $\mathrm{D}_{\mathrm{sj}}^{*}(2317)$ and $\mathrm{D}_{\mathrm{sj}}(2460)$ have $j_{\mathrm{l}} = 1/2$. They form the $(0^{+},1^{+})$ doublet of the P-wave orbital excitation. $\mathrm{D}_{\mathrm{sj}}(2460)$ is an ideal mixture of the $^{1}\mathrm{P}_{1}$ and $^{3}\mathrm{P}_{1}$ states in this limit

$$|\mathrm{D}_{\mathrm{sj}}(2460),\boldsymbol{P}\rangle = \sqrt{\frac{1}{3}}|\mathrm{D}_{\mathrm{sj}}(^{1}\mathrm{P}_{1}),\boldsymbol{P}\rangle + \sqrt{\frac{2}{3}}|\mathrm{D}_{\mathrm{sj}}(^{3}\mathrm{P}_{1}),\boldsymbol{P}\rangle \ . \tag{11}$$

The spatial wave functions are related to the simple harmonic oscillator (SHO) wave functions in Ref. [3]

$$\frac{1}{\sqrt{2E}}\phi(\boldsymbol{k},\boldsymbol{P}) = \phi(\boldsymbol{k}^{2}) \ . \tag{12}$$

For the ground states,

$$\phi(\boldsymbol{k}^{2}) = \left(\frac{2\sqrt{\pi}}{\beta}\right)^{3/2}\mathrm{e}^{-\boldsymbol{k}^{2}/2\beta^{2}} \ . \tag{13}$$

For the P-wave states,

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1) Here the mixing angle between λ_{8} and λ_{0} can be neglected since it is too small in our discussion of isospin violating decays.

$$\phi(\boldsymbol{k}^2) = \frac{\sqrt{2}}{\beta}\left(\frac{2\sqrt{\pi}}{\beta}\right)^{3/2} e^{-\boldsymbol{k}^2/2\beta^2}. \quad (14)$$

In the rest frame of A, the decay width of a process $A \to B + C$ is

$$\Gamma(A \to B+C) = \frac{|\boldsymbol{p}_C|}{32\pi^2 M_A^2}\int d\Omega \, |\mathscr{M}(A \to B+C)|^2, \quad (15)$$

$$\mathscr{M}(A \to B+C) = \langle BC|\mathscr{L}(0)|A\rangle, \quad (16)$$

with

$$|\boldsymbol{p}_C| = \frac{[(M_A^2 - (M_B + M_C)^2)(M_A^2 - (M_B - M_C)^2)]^{1/2}}{2M_A}. \quad (17)$$

The wave function in Eq. (12) is calculated in the rest frame, which is an approximation valid only for small $|\boldsymbol{P}|$ for mesons in motion. The calculation of the Lorentz invariant $\mathscr{M}$ matrix element should be done in a suitable frame in which the relativistic effect due to $|\boldsymbol{P}|$ is small. In our calculation, the C particle is π or γ, which is treated as an elementary particle. We calculate the invariant matrix element $\mathscr{M}$ in the frame $\boldsymbol{P}_A + \boldsymbol{P}_B = 0$ very like the Breit frame. If the heavy quark is the spectator, the relevant kinematics are

$$\boldsymbol{P}_A = \frac{1}{2}\boldsymbol{p}'_C, \quad (18)$$

$$\boldsymbol{P}_B = -\frac{1}{2}\boldsymbol{p}'_C, \quad (19)$$

$$\boldsymbol{p}_{\bar{q},A} = \boldsymbol{k} + \frac{1}{2}\boldsymbol{p}'_C, \quad (20)$$

$$\boldsymbol{p}_{\bar{q},B} = \boldsymbol{k} - \frac{1}{2}\boldsymbol{p}'_C, \quad (21)$$

$$\boldsymbol{p}_{Q,A} = -\boldsymbol{k}, \quad (22)$$

$$\boldsymbol{p}_{Q,B} = -\boldsymbol{k}, \quad (23)$$

$$\boldsymbol{k}_A = \frac{m_Q \boldsymbol{p}_{\bar{q},A} - m_q \boldsymbol{p}_{Q,A}}{m_q + m_Q} = \boldsymbol{k} + \frac{1}{2}\eta_Q \boldsymbol{p}'_C, \quad (24)$$

$$\boldsymbol{k}_B = \frac{m_Q \boldsymbol{p}_{\bar{q},B} - m_q \boldsymbol{p}_{Q,B}}{m_q + m_Q} = \boldsymbol{k} - \frac{1}{2}\eta_Q \boldsymbol{p}'_C, \quad (25)$$

where

$$\eta_Q = \frac{m_Q}{m_Q + m_q}, \quad (26)$$

$$\eta_q = 1 - \eta_Q, \quad (27)$$

and the momentum of C particle in this frame is

$$\boldsymbol{p}'_C = \frac{(M_A^2 - (M_B + M_C)^2)(M_A^2 - (M_B - M_C)^2)}{2M_A^2 + 2M_B^2 - M_C^2}. \quad (28)$$

The matrix elements are listed below. Following

Ref. [24], the formulae are all written in the way similar to the non-relativistic formulae except the overlapping integrals which approach unity in the non-relativistic limit $m \to \infty$ and deviate from unity significantly for the light quarks.

For the pionic decay of the D^* meson, we have

$$\frac{\langle P\pi|\mathscr{L}(0)|V\rangle}{\sqrt{4E_V E_P}} = i\frac{g_A^q}{2f_\pi}F_1(\boldsymbol{p}_\pi^2, m_q, \eta_Q)\boldsymbol{p}_\pi \cdot \boldsymbol{\epsilon}_V. \quad (29)$$

Its radiative decay amplitude reads

$$\frac{\langle P\pi|\mathscr{L}_\gamma(0)|V\rangle}{\sqrt{4E_V E_P}} = i\boldsymbol{\epsilon}_V \cdot \boldsymbol{p}_\gamma \times \boldsymbol{\epsilon}_\gamma \Big[\mu_{\bar{q}}F_3(\boldsymbol{p}_\gamma^2, m_q, \eta_Q) - \mu_Q F_3(\boldsymbol{p}_\gamma^2, m_Q, \eta_q)\Big]. \quad (30)$$

In the case of D_s^*, the following substitution is understood

$$\frac{g_A^q}{2f_\pi} \to -\theta_m \frac{g_A^q}{\sqrt{3}f_\eta}. \quad (31)$$

$D^*_{sj}(2317)$ can decay into D_s through the emission of one π. The decay matrix element contains two terms

$$\frac{\langle P\pi|\mathscr{L}(0)|S\rangle}{\sqrt{4E_S E_P}} = -i\sqrt{\frac{3}{2}}\frac{g_A}{2f_\pi}\frac{E_\pi\beta}{m_q}F_2(\boldsymbol{p}_\pi^2, m_q, \eta_Q) + i\eta_Q\sqrt{\frac{1}{6}}\frac{g_A}{2f_\pi}\frac{\boldsymbol{p}_\pi^2}{\beta}F_1(\boldsymbol{p}_\pi^2, m_q, \eta_Q). \quad (32)$$

Its radiative decay matrix elements also contain two pieces

$$\langle V\gamma|\mathscr{L}_\gamma(0)|S\rangle = \langle V\gamma|\mathscr{L}_\gamma(0)|S\rangle_E + \langle V\gamma|\mathscr{L}_\gamma(0)|S\rangle_M, \quad (33)$$

$$\frac{\langle V\gamma|\mathscr{L}_\gamma(0)|S\rangle_E}{\sqrt{4E_V E_S}} = \sqrt{\frac{2}{3}}\beta\boldsymbol{\epsilon}_V^* \cdot \boldsymbol{\epsilon}_\gamma \Big[\mu_{\bar{q}}F_4(\boldsymbol{p}_\gamma^2, m_q, \eta_Q) - \mu_Q F_4(\boldsymbol{p}_\gamma^2, m_Q, \eta_q)\Big], \quad (34)$$

$$\frac{\langle V\gamma|\mathscr{L}_\gamma(0)|S\rangle_M}{\sqrt{4E_V E_S}} = \sqrt{\frac{1}{6}}\frac{\boldsymbol{p}_\gamma^2}{\beta}\boldsymbol{\epsilon}_V^* \cdot \boldsymbol{\epsilon}_\gamma \Big[\mu_{\bar{q}}\eta_Q F_3(\boldsymbol{p}_\gamma^2, m_q, \eta_Q) - \mu_Q \eta_q F_3(\boldsymbol{p}_\gamma^2, m_Q, \eta_q)\Big]. \quad (35)$$

The decay matrix elements of $D_{sj}(2460)$ are complicated, which are listed according to its decay modes below.

• $A \to V + \pi$

$$\frac{\langle V\pi|\mathscr{L}(0)|A\rangle}{\sqrt{4E_V E_A}} = -i\sqrt{\frac{3}{2}}\frac{g_A}{2f_\pi}\frac{E_\pi\beta}{m_q}F_2(\boldsymbol{p}_\pi^2, m_q, \eta_Q)\boldsymbol{\epsilon}_V^* \cdot \boldsymbol{\epsilon}_A + i\eta_Q\sqrt{\frac{1}{6}}\frac{g_A}{2f_\pi}\frac{\boldsymbol{p}_\pi^2}{\beta}F_1(\boldsymbol{p}_\pi^2, m_q, \eta_Q)\boldsymbol{\epsilon}_V^* \cdot \boldsymbol{\epsilon}_A. \quad (36)$$

- $A \to P + \gamma$

$$\langle P\gamma|\mathscr{L}_\gamma(0)|A\rangle = \langle P\gamma|\mathscr{L}_\gamma(0)|A\rangle_E + \langle P\gamma|\mathscr{L}_\gamma(0)|A\rangle_M, \tag{37}$$

$$\frac{\langle P\gamma|\mathscr{L}_\gamma(0)|A\rangle_E}{\sqrt{4E_P E_A}} = \sqrt{\frac{2}{3}}\beta\boldsymbol{\epsilon}_A\cdot\boldsymbol{\epsilon}_\gamma\big[\mu_{\bar{q}}F_4(\boldsymbol{p}_\gamma^2,m_q,\eta_Q)- \\ \mu_Q F_4(\boldsymbol{p}_\gamma^2,m_Q,\eta_q)\big], \tag{38}$$

$$\frac{\langle P\gamma|\mathscr{L}_\gamma(0)|A\rangle_M}{\sqrt{4E_P E_A}} = \sqrt{\frac{1}{6}}\frac{\boldsymbol{p}_\gamma^2}{\beta}\boldsymbol{\epsilon}_A\cdot\boldsymbol{\epsilon}_\gamma\big[\mu_{\bar{q}}\eta_Q F_3(\boldsymbol{p}_\gamma^2,m_q,\eta_Q)+ \\ \mu_Q \eta_q F_3(\boldsymbol{p}_\gamma^2,m_Q,\eta_q)\big]. \tag{39}$$

- $A \to V + \gamma$

$$\langle V\gamma|\mathscr{L}_\gamma(0)|A\rangle = \langle V\gamma|\mathscr{L}_\gamma(0)|A\rangle_E + \langle V\gamma|\mathscr{L}_\gamma(0)|A\rangle_M, \tag{40}$$

$$\frac{\langle V\gamma|\mathscr{L}_\gamma(0)|A\rangle_E}{\sqrt{4E_V E_A}} = i\sqrt{\frac{2}{3}}\beta\boldsymbol{\epsilon}_V^*\times\boldsymbol{\epsilon}_A\cdot\boldsymbol{\epsilon}_\gamma\big[\mu_{\bar{q}}F_4(\boldsymbol{p}_\gamma^2,m_q,\eta_Q)- \\ \mu_Q F_4(\boldsymbol{p}_\gamma^2,m_Q,\eta_q)\big], \tag{41}$$

$$\frac{\langle V\gamma|\mathscr{L}_\gamma(0)|A\rangle_M}{\sqrt{4E_V E_A}} = i\eta_Q\sqrt{\frac{1}{6}}\frac{\boldsymbol{p}_\gamma^2}{\beta}\mu_{\bar{q}}\boldsymbol{\epsilon}_V^*\times\boldsymbol{\epsilon}_A\cdot\boldsymbol{\epsilon}_\gamma F_3\times \\ (\boldsymbol{p}_\gamma^2,m_q,\eta_Q)-i\eta_q\sqrt{\frac{2}{3}}\frac{\mu_Q}{\beta}F_3\times \\ (\boldsymbol{p}_\gamma^2,m_Q,\eta_q)\boldsymbol{\epsilon}_V^*\cdot\Big(\boldsymbol{p}_\gamma\boldsymbol{p}_\gamma-\frac{1}{2}\boldsymbol{p}_\gamma^2\mathbb{1}\Big)\cdot \\ \boldsymbol{\epsilon}_A\times\boldsymbol{\epsilon}_\gamma. \tag{42}$$

- $A \to S + \gamma$

$$\langle S\gamma|\mathscr{L}_\gamma(0)|A\rangle = \langle S\gamma|\mathscr{L}_\gamma(0)|A\rangle_E + \langle S\gamma|\mathscr{L}_\gamma(0)|A\rangle_M, \tag{43}$$

$$\frac{\langle S\gamma|\mathscr{L}_\gamma(0)|A\rangle_E}{\sqrt{4E_A E_S}} = i\Big[\frac{2}{3}(1+\eta_Q)\mu_{\bar{q}}F_4(\boldsymbol{p}_\gamma^2,m_q,\eta_Q)- \\ \mu_{\bar{q}}F_5(\boldsymbol{p}_\gamma^2,m_q,\eta_Q)+ \\ \frac{2}{3}\eta_q\mu_Q F_4(\boldsymbol{p}_\gamma^2,m_Q,\eta_q)- \\ \mu_Q F_5(\boldsymbol{p}_\gamma^2,m_Q,\eta_q)\Big]\boldsymbol{p}_\gamma\times\boldsymbol{\epsilon}_\gamma\cdot\boldsymbol{\epsilon}_A, \tag{44}$$

$$\frac{\langle S\gamma|\mathscr{L}_\gamma(0)|A\rangle_M}{\sqrt{4E_A E_S}} = i\frac{\boldsymbol{p}_\gamma^2}{6\beta^2}[\eta_Q^2\mu_{\bar{q}}F_3(\boldsymbol{p}_\gamma^2,m_q,\eta_Q)+ \\ \eta_q^2\mu_Q F_3(\boldsymbol{p}_\gamma^2,m_Q,\eta_q)]\boldsymbol{p}_\gamma\times\boldsymbol{\epsilon}_\gamma\cdot\boldsymbol{\epsilon}_A. \tag{45}$$

3 Numerical results

In our calculation, the wave function parameter β and the quark masses are taken from Ref. [3]:

$$\beta = 400\text{MeV}, \quad m_u = m_d = 220\text{MeV}, \\ m_s = 419\text{MeV}, \quad m_c = 1628\text{MeV}. \tag{46}$$

The value of $g_A^q = 0.87$ is taken from Ref. [24]. We present the pionic decay widths of $D^{(*)}$ mesons in Table 2. Our results agree with the experimental data very well.

Table 2. Decay widths of $D^* \to D + \pi$ in unit of MeV and the relevant F_1 values.

$V \to P + \pi$	$\boldsymbol{p}_\pi$	Exp. [25]	F_1	present work	Ref. [24]
$D^{*\pm} \to D^\pm + \pi^0$	38	0.030	0.65	0.028	0.029
$D^{*\pm} \to D^0 + \pi^\pm$	40	0.065	0.65	0.061	0.064
$D^{*0} \to D^0 + \pi^0$	43	<1.3	0.65	0.039	0.041

In Table 3, we collect the numerical results of the isospin violating pionic decays of D_s^* and $D_{sj}^{(*)}$ states. We used the commonly accepted value $\theta_m = 0.010$ for π^0-η mixing. The radiative decay widths are listed in Table 4 together with some results from other groups. The relevant overlapping integrals are collected in Table 5, where

$$F_i^q = F_i(\boldsymbol{p}_\gamma^2,m_q,\eta_Q), \tag{47}$$

$$F_i^Q = F_i(\boldsymbol{p}_\gamma^2,m_Q,\eta_q). \tag{48}$$

Table 3. Pionic decay widths of D_s^* and $D_{sj}^{(*)}$ in unit of keV and relevant F_i values.

	$\boldsymbol{p}_\pi$/MeV	F_1	F_2	present work
$D_s^* \to D_s + \pi^0$	49	0.80		7.4×10^{-3}
$D_{sj}^*(2317) \to D_s + \pi^0$	298	0.70	0.52	1.9
$D_{sj}(2460) \to D_s^* + \pi^0$	297	0.70	0.52	1.9

Table 4. Radiative decay widths in unit of keV.

	present work	QM[26]	QM[27]	QM[28]	QSR[29]	LCSR[30]	VMD[11]
$D^{*\pm} \to D^\pm\gamma$	0.25	0.36	0.050		0.23		
$D^{*0} \to D^0\gamma$	14.5	17.9	7.3		12.9		
$D_s^* \to D_s\gamma$	0.065	0.118	0.101		0.13		
$D_{sj}^*(2317) \to D_s^*\gamma$	1.5			1.9		$4-6$	0.85
$D_{sj}(2460) \to D_s\gamma$	6.3			6.2		$19-29$	3.3
$D_{sj}(2460) \to D_s^*\gamma$	3.7			5.5		$0.6-1.1$	1.5
$D_{sj}(2460) \to D_{sj}^*(2317)$	0.18			0.012		$0.5-0.8$	

From the table, we see clearly that the overlapping integrals, which are related to the relativistic effects, are very important for the light quarks. For the heavy quarks, the overlapping integrals always approach unity.

Table 5. Overlapping integrals related to radiative decays.

	F_3^{q}	F_3^{Q}	F_4^{q}	F_4^{Q}	F_5^{q}	F_5^{Q}
$D^{*\pm} \to D^{\pm}\gamma$	0.39	0.94				
$D^{*0} \to D^0\gamma$	0.39	0.94				
$D_s^* \to D_s\gamma$	0.62	0.94				
$D_{sj}^*(2317) \to D_s^*\gamma$	0.60	0.94	0.56	0.93		
$D_{sj}(2460) \to D_s\gamma$	0.47	0.92	0.44	0.91		
$D_{sj}(2460) \to D_s^*\gamma$	0.54	0.93	0.51	0.92		
$D_{sj}(2460) \to$ $D_{sj}^*(2317)\gamma$	0.62	0.94	0.58	0.93	0.50	0.91

4　Summary

At present, only the decay widths of D^* mesons have been measured. For $D_s^*, D_{sj}(2317), D_{sj}(2460)$ states, the ratios between their radiative and pionic decay widths are available experimentally, which are collected in Table 6. The experimental data are taken from "Review of Particle Physics" by Particle Data Group (PDG)[25] and its online update server: http://pdg.lbl.gov/pdg.html.

Our calculated ratios $\dfrac{D^{*0} \to D^0\gamma}{D^{*0} \to D^0\pi^0}$ and $\dfrac{D_s^* \to D_s\pi^0}{D_s^* \to D_s\gamma}$ agree with the experimental ratio within a factor of two. Such an agreement is quite interesting if we keep in mind both our model wave function and the strong decay Hamiltonian are so simple.

Our calculated ratio $\dfrac{D^{*\pm} \to D^{\pm}\gamma}{D^{*\pm} \to D^{\pm}\pi^0}$ is nearly six times smaller than the experimental data. This may be partly attributed to the uncertainty of our naive SHO wave functions. In the $V \to P + \gamma$ formula Eq. (30), there exists a strong cancellation between the light and heavy quark contributions. We have

$$\frac{\mu_{\bar{\mathrm{d}}}}{\mu_{\mathrm{c}}} = \frac{m_{\mathrm{c}}}{2m_{\mathrm{d}}} \approx -\frac{1600}{440}, \qquad (49)$$

and

$$\frac{F_3^{\mathrm{q}}}{F_3^{\mathrm{Q}}} \approx 0.4, \qquad (50)$$

i.e., $\mu_{\bar{\mathrm{d}}}F_3^{\mathrm{q}} \approx -\mu_{\mathrm{c}}F_3^{\mathrm{Q}}$ which leads to the strong cancellation. The sensitivity of the overlapping integrals to

the uncertainty of the wave function is amplified in this case. For example, if we change the β parameter to $\beta = 300\mathrm{MeV}$, we have $F_3^{\mathrm{q}} \approx 0.48$. Then the resulting ratio will increase to ~ 0.02.

Table 6. Branching ratios between radiative and pionic decays.

	Exp.	present work
$\dfrac{D^{*0} \to D^0\gamma}{D^{*0} \to D^0\pi^0}$	0.62	0.35
$\dfrac{D_s^* \to D_s\pi^0}{D_s^* \to D_s\gamma}$	0.062	0.11
$\dfrac{D^{*\pm} \to D^{\pm}\gamma}{D^{*\pm} \to D^{\pm}\pi^0}$	0.052	0.009
$\dfrac{D_{sj}^*(2317) \to D_s^*\gamma}{D_{sj}^*(2317) \to D_s\pi^0}$	<0.059	0.79
$\dfrac{D_{sj}(2460) \to D_s\gamma}{D_{sj}(2460) \to D_s^*\pi^0}$	0.31	3.3
$\dfrac{D_{sj}(2460) \to D_s^*\gamma}{D_{sj}(2460) \to D_s^*\pi^0}$	<0.16	1.9
$\dfrac{D_{sj}(2460) \to D_{sj}^*(2317)\gamma}{D_{sj}(2460) \to D_s^*\pi^0}$	<0.22	0.094

As can be seen in Table 4, the radiative decay widths of different channels in this work are comparable with those from other groups (see also Ref. [31]). However, the pionic decay widths of $D_{sj}(2317)$ and $D_{sj}(2460)$ from the simple chiral quark model in Table 3 are ten times smaller than those from light-cone QCD sum rules approach[14] and the $^3P_0$ decay model[20]. Hence our calculated ratios between EM and pionic decay widths of $D_{sj}(2317)$ and $D_{sj}(2460)$ mesons are systematically larger than the experimental data by a factor of 10. Such a large systematic discrepancy cannot easily be ascribed to either the uncertainty of the meson wave function or the uncertainty of the value of the η-π^0 mixing amplitude[21, 32]. We tend to conclude that the simple strong decay mechanism based on the pion and chiral quark axial vector coupling is not realistic if $D_{sj}(2317)$ and $D_{sj}(2460)$ mesons are conventional $c\bar{s}$ states.

In summary, we perform a systematic calculation of the decay widths of D_s^*, $D_{sj}^*(2317)$, and $D_{sj}(2460)$. The EM radiative decay widths agree with the available experimental data and other model results reasonably well. But the isospin violating pionic decay widths of $D_{sj}^*(2317)$ and $D_{sj}(2460)$ are too small to

fit the experimental data. This disagreement cannot easily be resolved by changing the wave functions or the η-π⁰ mixing amplitude in the chiral quark model. One may wonder whether other possible theoretical schemes such as the coupled-channel effects, hybrid meson, molecule state or tetraquark interpretations of

these two resonances may resolve the above discrepancy. However, there is no clear evidence in favor of these exotic schemes from BABAR's most recent extensive measurement[33]. Therefore we strongly suspect the chiral quark pion interaction Hamiltonian may be too simple to describe strong decays reliably.

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Appendix A

The overlapping integrals

All $F_i(\boldsymbol{p}^2, m, \eta)$ can be expressed as

$$F_i(\boldsymbol{p}^2, m, \eta) = e^{-\frac{1}{4}\eta \boldsymbol{p}^2/\beta^2} \left(\frac{2\sqrt{\pi}}{\beta}\right)^3 \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} e^{-\boldsymbol{k}^2/\beta^2} \sqrt{\frac{q^0+m}{2q^0}} \sqrt{\frac{q'^0+m}{2q'^0}} G_i \ , \tag{A1}$$

where

$$G_1 = 1 + \frac{2(\boldsymbol{k}\cdot\hat{\boldsymbol{p}})^2 - \left(\boldsymbol{k}^2 + \frac{1}{4}\boldsymbol{p}^2\right)}{(q^0+m)(q'^0+m)}, \tag{A2}$$

$$G_2 = \frac{2\boldsymbol{k}^2}{3\beta^2}\left[\frac{m(q^0+q'^0+2m)}{(q^0+m)(q'^0+m)} - \frac{m(\hat{\boldsymbol{k}}\cdot\boldsymbol{p})^2}{(q^0+m)(q'^0+m)(q^0+q'^0)}\right], \tag{A3}$$

$$G_3 = \frac{m(q^0 + q'^0 + 2m)}{(q^0 + m)(q'^0 + m)} - \frac{4m(\hat{\boldsymbol{p}} \cdot \boldsymbol{k})^2}{(q^0 + m)(q'^0 + m)(q^0 + q'^0)}, \tag{A4}$$

$$G_4 = \frac{m(q^0 + q'^0 + 2m)}{(q^0 + m)(q'^0 + m)} \frac{\boldsymbol{k}^2 - (\boldsymbol{k} \cdot \hat{\boldsymbol{p}})^2}{\beta^2}, \tag{A5}$$

$$G_5 = \frac{2\boldsymbol{k}^2}{3\beta^2} \left[\frac{m(q^0 + q'^0 + 2m)}{(q^0 + m)(q'^0 + m)} - \frac{4m(\hat{\boldsymbol{p}} \cdot \boldsymbol{k})^2}{(q^0 + m)(q'^0 + m)(q^0 + q'^0)} \right], \tag{A6}$$

and $\boldsymbol{q}$ and $\boldsymbol{q}'$ are quark's momenta

$$\boldsymbol{q} = \boldsymbol{k} + \frac{1}{2}\boldsymbol{p}, \tag{A7}$$

$$\boldsymbol{q}' = \boldsymbol{k} - \frac{1}{2}\boldsymbol{p}. \tag{A8}$$

夸克模型中的 $D_{sj}^*(2317)$, $D_{sj}(2460)$ 衰变[*]

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摘要 用夸克模型结合 D_s, D_s^*, 讨论窄共振态 $D_{sj}^*(2317)$, $D_{sj}(2460)$. 除 η-π⁰ 混合角由 D_s^* 衰变宽度给出, 所有的参数均取自 Godfrey 和 Isgur 的夸克模型. 得到的电磁衰变宽度与实验数据和其他作者的结果一致. 但是, 与现有的实验结果比较, $D_{sj}^*(2317)$, $D_{sj}(2460)$ 的 π 介子衰变宽度要小一个数量级. 我们怀疑简单的手征夸克–π 轴矢相互作用的哈密顿量不适合于 $D_{sj}^*(2317)$, $D_{sj}(2460)$ 的强衰变。

关键词 夸克模型　窄共振态　衰变宽度