# Study on Strong Decays of $\mathrm{D}_{\mathrm{sJ}}(2632)^{*}$ 

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#### Abstract

The resonance $\mathrm{D}_{\mathrm{sJ}}$（2632）observed by SELEX，has attracted great interests and meanwhile brought up serious dispute．Its spin－parity，so far has not finally determined and if it exists，its quark－structure might be exotic．Following the previous literature where $\mathrm{D}_{\mathrm{SJ}}(2632)$ is assumed to be a radial－excited state of $1^{-}$， we consider the possibilities that it might be a $q \bar{q}$ ground state of $2^{+}$or the first radial－excited state of $0^{+}$ $\mathrm{D}_{\mathrm{sJ}}(2317)$ and re－calculate its strong decay widths in terms of the Bethe－Salpeter equation．Our results indicate that there still is a sharp discrepancy between the theoretical evaluation and data．


Key words $\mathrm{D}_{\mathrm{sJ}}(2632)$ ，strong decay，Bethe－Salpeter equation

## 1 Introduction

In 2004，the SELEX collaboration reported that a charmed meson $\mathrm{D}_{\mathrm{sJ}}(2632)$ with narrow width was observed ${ }^{[1]}$ ，its mass is $2632.5 \pm 1.7 \mathrm{MeV}$ at $90 \% \mathrm{CL}$ ． This state was observed in two channels $D_{s}^{+} \eta$ and $\mathrm{D}^{0} \mathrm{~K}^{+}\left(\mathrm{D}^{+} \mathrm{K}^{0}\right)$ which are OZI allowed processes．The measured ratio of their branching ratios $R=\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow\right.$ $\left.\mathrm{D}^{0} \mathrm{~K}^{+}\right) / \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \eta\right)=0.14 \pm 0.06$ shows a strange pattern．Because the phase space available for $\mathrm{D}^{0} \mathrm{~K}^{+}$ is almost 1.5 times larger than that for $D_{s}^{+} \eta$ ，but the final products possess the same quark contents，there－ fore one is tempted to believe that this discrepancy implies that the quark structure of $\mathrm{D}_{\mathrm{sJ}}$（2632）might be exotic．To eventually confirm the allegation，one needs to carefully seek for solutions in the traditional theoretical framework．In fact，the spin－parity of the resonance $\mathrm{D}_{\mathrm{sJ}}(2632)$ has not finally determined yet．

According to the final states，it may be $0^{+}, 1^{-}$or even $2^{+[2]}$ which can decay into two pseudoscalar mesons via $S$－，$P$－and $D$－waves respectively．

Actually，before considering the exotic structure， such as hybrid，four－quark state etc．，one should seri－ ously investigate if it can be embedded in the frame of regular mesons which contain only one quark and one anti－quark ${ }^{[3-5]}$ ．Chang et al．${ }^{[6]}$ assumed that $D_{\mathrm{sJ}}(2632)$ is the first radial excited state of $D_{s}^{*}$ and has spin－parity as $1^{-}$，and then they evaluated its decay widths in terms of the Bethe－Salpeter equa－ tion．Here we consider alternative possibilities that $\mathrm{D}_{\mathrm{sJ}}(2632)$ may be $0^{+}$or $2^{+}$resonances of cs ．Namely， we suppose that $\mathrm{D}_{\mathrm{sJ}}(2632)$ is the first radial excited state of $\mathrm{D}_{\mathrm{sJ}}(2317)$ whose spin－parity is well deter－ mined by several collaborations ${ }^{[7]}$ ，or the $2^{+}$radial ground state of $\mathrm{c} \overline{\mathrm{s}}$ ．Then following Chang et al．${ }^{[6]}$ ，we also evaluate the decay widths of $\left.\mathrm{D}_{\mathrm{sJ}}^{+}(2632) \rightarrow \mathrm{D}^{0} \mathrm{~K}^{+}\right)$

[^0]and $\left(D_{s J}^{+}(2632) \rightarrow D_{s}^{+} \eta\right)$ by assuming it to be of a ra－ dial excited state of $0^{+}$or a radial ground state of $2^{+}$ with only $\overline{c s}$ quark contents．

In the derivation，we have also considered two ap－ proaches for the $0^{+}$case．First，we assume that it is the first radial excited state of the observed $D_{\text {sJ }}(2317)$ whose mass is 2317 MeV ，and then we obtain a mass of the excited state as $2700 \pm 20 \mathrm{MeV}$（see the fol－ lowing text for details）．Alternatively，if we assume 2632 MeV as the mass of an $0^{+}$radial excited state， then we obtain the mass of its corresponding radial ground state of $0^{+}$as $2245 \pm 25 \mathrm{MeV}$ ．That is in analog to the approach of Ref．［6］，where the authors calcu－ lated the mass of the first radial excited state of $D_{s}^{*}$ as $2658 \pm 15 \mathrm{MeV}$ ．In the second approach，the newly ob－ tained mass $2245 \pm 25 \mathrm{MeV}$ obviously deviates from the observed 2317 MeV which is well measured，therefore unless a new resonance $D_{s J}$ of $0^{+}$were experimentally observed，this scenario is not favored by the present data．However，considering the experimental errors， it may still be possible，and we will further discuss it in the last section．

This work is organized as follows，after this intro－ duction，we discuss all the aforementioned possibili－ ties and by solving the B－S equation，we obtain the mass spectrum and the OZI－allowed decay widths．In Sec．II，we deal with the case where $\mathrm{D}_{\mathrm{sJ}}(2632)$ is as－ sumed to be the radial ground state of $2^{+}$，while in Sec．III，we assume that it is a radial excited state of $0^{+}$．All the numerical results along with all the in－ put parameters are presented in the sections．The last section is devoted to the discussions and a brief conclusion．

## $2 \quad \mathrm{D}_{\mathrm{sJ}}(2632)$ as the ground state of $2^{+}$ $\mathrm{c} \overline{\mathrm{S}}$

The B－S equation with instantaneous approxima－ tion about the $0^{-}, 1^{-}$mesons has been thoroughly studied ${ }^{[8]}$ ．Following the method and technical de－ tails introduced in Ref．［8］，we solve the B－S equation for the mesons of $2^{+}$under the instantaneous approx－ imation．

Generally，the B－S wavefunction for a $2^{+}$meson
can be wrtten as ${ }^{[9]}$ ：

$$
\begin{align*}
\varphi_{P_{i}}(\boldsymbol{q})= & \epsilon_{i j} q_{\perp}^{j}\left\{q _ { \perp } ^ { i } \left[\varphi_{1}(\boldsymbol{q})+\gamma_{0} \varphi_{2}(\boldsymbol{q})+q_{\perp} \varphi_{3}(\boldsymbol{q})+\right.\right. \\
& \left.\gamma_{0} q_{\perp} \varphi_{4}(\boldsymbol{q})\right]+\gamma^{i}\left[\varphi_{5}(\boldsymbol{q})+\gamma_{0} \varphi_{6}(\boldsymbol{q})+\right. \\
& \left.\left.\not q_{\perp} \varphi_{7}(\boldsymbol{q})\right]+\mathrm{i} \epsilon^{0 i l k} q_{\perp l} \gamma_{k} \gamma_{5} \varphi_{8}(\boldsymbol{q})\right\} . \tag{1}
\end{align*}
$$

where $\varphi_{i}(\boldsymbol{q})$ is the component function，$q_{\perp}=(0, \boldsymbol{q})$ ， and $\boldsymbol{q}$ is the relative three－momentum of the quark－ anti－quark in the meson，$\epsilon^{0 i l k}$ is the fully antisym－ metric tensor and $\epsilon_{i j}$ is the polarization tensor of $2^{+}$． For the convenience，we redefine $\psi_{1}=\varphi_{1}, \psi_{2}=\varphi_{2}$ ， $\psi_{3}=\boldsymbol{q}^{2} \varphi_{3}, \psi_{4}=\boldsymbol{q}^{2} \varphi_{4}, \psi_{5}=\varphi_{5}, \psi_{6}=-\varphi_{6}, \psi_{7}=\varphi_{7}$, $\psi_{8}=\varphi_{8}$.

By the well－known constraint conditions for the projected wavefunctions $\varphi_{P_{i}}^{+-}=0$ and $\varphi_{P_{i}}^{-+}=0^{[8,10]}$ ， one has

$$
\begin{align*}
& \psi_{1}(\boldsymbol{q})=\frac{-\left(\left(\omega_{1}+\omega_{2}\right) \psi_{3}(\boldsymbol{q})-2 \omega_{2} \psi_{5}(\boldsymbol{q})\right)}{\omega_{2} m_{1}+\omega_{1} m_{2}} \\
& \psi_{2}(\boldsymbol{q})=\frac{-\left(\omega_{1}-\omega_{2}\right)\left(\psi_{4}(\boldsymbol{q})-\psi_{6}(\boldsymbol{q})\right)}{\omega_{2} m_{1}+\omega_{1} m_{2}}  \tag{2}\\
& \psi_{7}(\boldsymbol{q})=\frac{\left(\omega_{1}-\omega_{2}\right) \psi_{5}(\boldsymbol{q})}{\omega_{2} m_{1}+\omega_{1} m_{2}} \\
& \psi_{8}(\boldsymbol{q})=-\frac{\left(\omega_{1}+\omega_{2}\right) \psi_{6}(\boldsymbol{q})}{\omega_{2} m_{1}+\omega_{1} m_{2}}
\end{align*}
$$

where $\omega_{1}=\sqrt{m_{1}^{2}+\boldsymbol{q}^{2}}, \omega_{2}=\sqrt{m_{2}^{2}+\boldsymbol{q}^{2}}$ ，and in this text $m_{1}$ and $m_{2}$ stand as $m_{\mathrm{c}}$ and $m_{\mathrm{q}}$ ，which are masses of charm quark and light flavor $q(q=\mathrm{u}, \mathrm{d}, \mathrm{s})$ ．

Thus the wavefunction of a $2^{+}$meson can be fur－ ther written as

$$
\begin{align*}
\varphi_{P_{i}}(\boldsymbol{q})= & \epsilon_{i j} q_{\perp}^{j}\left\{q _ { \perp } ^ { i } \left[\psi_{3}(\boldsymbol{q})\left(\frac{\not q_{\perp}}{\boldsymbol{q}^{2}}-\frac{\left(\omega_{1}+\omega_{2}\right)}{m_{2} \omega_{1}+m_{1} \omega_{2}}\right)+\right.\right. \\
& \psi_{4}(\boldsymbol{q}) \gamma_{0}\left(\frac{\phi_{\perp}}{\boldsymbol{q}^{2}}-\frac{\left(\omega_{1}-\omega_{2}\right)}{m_{2} \omega_{1}+m_{1} \omega_{2}}\right)+ \\
& \psi_{5}(\boldsymbol{q}) \frac{2 \omega_{2}}{m_{2} \omega_{1}+m_{1} \omega_{2}}+ \\
& \left.\psi_{6}(\boldsymbol{q}) \gamma_{0} \frac{\left(\omega_{1}-\omega_{2}\right)}{m_{2} \omega_{1}+m_{1} \omega_{2}}\right]+ \\
& \gamma^{i}\left[\psi_{5}(\boldsymbol{q})\left(1+\frac{\not q_{\perp}\left(\omega_{1}-\omega_{2}\right)}{m_{2} \omega_{1}+m_{1} \omega_{2}}\right)-\gamma_{0} \psi_{6}(\boldsymbol{q})\right]- \\
& \left.\mathrm{i} \epsilon^{0 i l k} q_{\perp l} \gamma_{k} \gamma_{5} \psi_{6}(\boldsymbol{q}) \frac{\left(\omega_{1}+\omega_{2}\right)}{m_{2} \omega_{1}+m_{1} \omega_{2}}\right\} . \tag{3}
\end{align*}
$$

Then，we obtain an equation group which con－ tains four mutually coupled equations，the detailed expressions are collected in appendix．

In this work we adopt the values given in Ref．［6］ for the concerned parameters，but only change $V_{0}$ to obtain the mass of $2632 \pm 16 \mathrm{MeV}$ for the ground state
of $2^{+}$．By solving the equation group，numerical solu－ tions for the component functions $\psi_{3}, \psi_{4}, \psi_{5}, \psi_{6}$ are achieved，these functions are shown in Fig．1．Actu－ ally，$\psi_{3} \approx \psi_{4}$ ，and $\psi_{5} \approx \psi_{6}$ ，therefore，in the figure they seem to overlap together．


Fig．1．The component functions of $\mathrm{D}_{\mathrm{sJ}}(2632)$ which is assumed to be the radial ground state of $2^{+} c \bar{q}, \psi_{3}, \psi_{4}, \psi_{5}, \psi_{6}$ ．

Now，we can use the formula given by Ref．［6］to evaluate the widths of the strong decays．

$$
\begin{equation*}
\Gamma=\frac{\left|\boldsymbol{P}_{f 1}\right|}{8 \pi M^{2}}|T|^{2} \tag{4}
\end{equation*}
$$

where $M$ is the mass of the initial meson $\mathrm{D}_{\mathrm{sJ}}(2632)$ ， $\boldsymbol{P}_{f 1}$ is the three momentum of the produced mesons $D\left(\right.$ or $\left.D_{s}\right)$ in the center of mass frame of $D_{s J}(2632)$ ． For $\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{0} \mathrm{~K}^{+}$and $\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{+} \mathrm{K}^{0}$ ，the matrix ele－ ment $T$ is

$$
\begin{align*}
T= & \frac{P_{f 2}^{\mu}}{f_{\mathrm{K}}} \int \frac{\mathrm{~d} \boldsymbol{q}}{(2 \pi)^{3}} \operatorname{Tr}\left[\bar{\varphi}_{P_{f 1}}^{++}\left(\boldsymbol{q}-\frac{m_{1}}{m_{1}+m_{2}} \boldsymbol{p}_{f 1}\right) \times\right. \\
& \left.\frac{P_{i}}{M} \varphi_{P_{i}}^{++}(\boldsymbol{q}) \gamma_{\mu} \gamma_{5}\right] \tag{5}
\end{align*}
$$

For $D_{\mathrm{sJ}}^{+} \rightarrow D_{\mathrm{s}}^{+} \eta$ ，it is

$$
\begin{align*}
T= & P_{f 2}^{\mu}\left[\frac{-2 M_{\eta}^{2} \cos \theta}{\sqrt{6} M_{\eta_{8}}^{2} f_{\eta_{8}}}+\frac{M_{\eta}^{2} \sin \theta}{\sqrt{3} M_{\eta_{0}}^{2} f_{\eta_{0}}}\right] \times \\
& \int \frac{\mathrm{d} \boldsymbol{q}}{(2 \pi)^{3}} \operatorname{Tr}\left[\bar{\varphi}_{P_{f 1}}^{++}\left(\boldsymbol{q}-\frac{m_{1}}{m_{1}+m_{2}} \boldsymbol{p}_{f 1}\right) \frac{P_{i}}{M} \varphi_{P_{i}}^{++}(\boldsymbol{q}) \gamma_{\mu} \gamma_{5}\right] . \tag{6}
\end{align*}
$$

Here $\boldsymbol{q}$ is the inner relative three－momentum in the initial meson $\mathrm{D}_{\mathrm{sJ}}(2632), P_{f 2}$ is the four－ momentum of the produced meson K （or $\eta$ ），$P_{i}$ is the
four－momentum of $\mathrm{D}_{\mathrm{sJ}}$（2632），$M_{\eta}$ is the mass of $\eta$ ． $\varphi_{P_{i}}^{++}$and $\varphi_{P_{f_{1}}}^{++}$is the positive－energy wavefunction of the initial or final meson，and $\bar{\varphi}_{P_{f 1}}^{++}=-\gamma_{0}\left(\varphi_{P_{f 1}}^{++}\right)^{+} \gamma_{0}$ ． The factor in Eq．（6）$\left[\frac{-2 M_{\eta}^{2} \cos \theta}{\sqrt{6} M_{\eta_{8}}^{2} f_{\eta_{8}}}+\frac{M_{\eta}^{2} \sin \theta}{\sqrt{3} M_{\eta_{0}}^{2} f_{\eta_{0}}}\right]$ takes into account the $\eta-\eta^{\prime}$ mixing，the readers are recom－ mended to refer to Ref．［6］for some details．$f_{\mathrm{K}}$ is the decay constant of K meson，$f_{\eta_{8}}, f_{\eta_{0}}$ are the decay constants of $\eta_{8}$ and $\eta_{0}$ respectively．

As the decay products are pseudoscalar mesons， their positive－energy wavefunctions are ${ }^{[8]}$

$$
\begin{align*}
\varphi_{P_{f 1}}^{++}(\boldsymbol{q})= & \frac{M_{f 1}}{2}\left(\varphi_{1}(\boldsymbol{q})+\varphi_{2}(\boldsymbol{q}) \frac{m_{1} m_{2}}{\omega_{1} \omega_{2}}\right)\left[\frac{\omega_{1}+\omega_{2}}{m_{1}+m_{2}}+\right. \\
& \left.\gamma_{0}-\frac{\not q_{\perp}\left(m_{1}-m_{2}\right)}{m_{2} \omega_{1}+m_{1} \omega_{2}}+\frac{\gamma_{0} \phi_{\perp}\left(m_{1}-m_{2}\right)}{m_{2} \omega_{1}+m_{1} \omega_{2}}\right] \gamma_{5} \tag{7}
\end{align*}
$$

The relation of the positive－energy wavefunction of the initial meson and the its wavefunction $\varphi_{P_{i}}(\boldsymbol{q})$ reads ${ }^{[6]}$

$$
\begin{align*}
\varphi_{P_{i}}^{++}(\boldsymbol{q})= & \frac{1}{2 \omega_{1}}\left(\omega_{1} \gamma_{0}+m_{1}+q_{\perp}\right) \gamma_{0} \varphi_{P_{i}}(\boldsymbol{q}) \gamma_{0} \times \\
& \frac{1}{2 \omega_{2}}\left(\omega_{2} \gamma_{0}-m_{2}-q_{\perp}\right) \tag{8}
\end{align*}
$$

Using the wavefunctions obtained by solving the equations，we evaluate the partial widths as

$$
\begin{align*}
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{0} \mathrm{~K}^{+}\right) & =2.10 \pm 0.30 \mathrm{MeV}  \tag{9}\\
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{+} \mathrm{K}^{0}\right) & =2.22 \pm 0.31 \mathrm{MeV}  \tag{10}\\
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \eta\right) & =0.23 \pm 0.02 \mathrm{MeV} \tag{11}
\end{align*}
$$

The corresponding ratio of the branching ratios is

$$
\begin{align*}
& \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{0} \mathrm{~K}^{+}\right) / \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \eta\right) \approx \\
& \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{+} \mathrm{K}^{0}\right) / \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \eta\right) \approx 9.2 \pm 0.9 \tag{12}
\end{align*}
$$

If we assume that the observed $\mathrm{D}_{\mathrm{sJ}}(2632)$ is a ra－ dial ground state of $2^{+}$，the obtained total width is consistent with the data，but the ratio of the branch－ ing ratios obviously differs from the observation．

In the numerical computations，we choose the input parameters as Ref．［6］，$m_{\mathrm{c}}=1755.3 \mathrm{MeV}$ ， $m_{\mathrm{s}}=487 \mathrm{MeV}, \quad m_{\mathrm{d}}=311 \mathrm{MeV}, \quad m_{\mathrm{u}}=305 \mathrm{MeV}, \quad f_{\pi}=$ $130.7 \mathrm{MeV}, f_{\eta_{8}}=1.26 f_{\pi}, f_{\eta_{8}}=1.07 f_{\pi}, f_{\mathrm{K}}=159.8 \mathrm{MeV}$ ， $M_{n_{8}}=564.3 \mathrm{MeV}, M_{\eta_{0}}=948.1 \mathrm{MeV}$ and $\theta=-9.95^{\circ}$ ． The theoretical uncertainties are estimated by vary－ ing all the parameters simultaneously within $\pm 5 \%$ ．

## $3 \quad \mathrm{D}_{\mathrm{sJ}}(2632)$ as the first radial excited state of $0^{+}$cs

In our earlier work，we obtained the expression of the wavefunction for $0^{+}$diquark ${ }^{[11]}$ ，for a $0^{+}$meson which is composed of a quark and an antiquark，the B－S equation is the same，but the integration kernel is different．

The wavefunction of a $0^{+}$scalar meson is written as

$$
\begin{equation*}
\varphi_{P_{i}}(\boldsymbol{q})=\left[\varphi_{1}(\boldsymbol{q})+\gamma_{0} \varphi_{2}(\boldsymbol{q})+g_{\perp} \varphi_{3}(\boldsymbol{q})+\gamma_{0} g_{\perp} \varphi_{4}(\boldsymbol{q})\right] . \tag{13}
\end{equation*}
$$

For the convenience，we redefine $\psi_{1}=\varphi_{1}, \psi_{2}=\varphi_{2}$ ， $\psi_{3}=|\boldsymbol{q}| \varphi_{3}, \psi_{4}=|\boldsymbol{q}| \varphi_{4}$.

By the constraint condition of projected wave function，one has

$$
\begin{align*}
& \psi_{1}(\boldsymbol{q})=-\frac{\left(\omega_{1}-\omega_{2}\right) \psi_{4}(\boldsymbol{q})|\boldsymbol{q}|}{\omega_{2} m_{1}+\omega_{1} m_{2}} \\
& \psi_{3}(\boldsymbol{q})=\frac{\left(\boldsymbol{q}^{2}-m_{1} m_{2}-\omega_{1} \omega_{2}\right) \psi_{2}(\boldsymbol{q})}{\left(m_{1}+m_{2}\right)|\boldsymbol{q}|} \tag{14}
\end{align*}
$$

With the constraints，one can further write the wavefunction as

$$
\begin{gather*}
\varphi_{P_{i}}(\boldsymbol{q})=\psi_{2}(\boldsymbol{q})\left[\gamma_{0}+q_{\perp} \frac{\boldsymbol{q}^{2}-m_{1} m_{2}-\omega_{1} \omega_{2}}{\left(m_{1}+m_{2}\right) \boldsymbol{q}^{2}}\right]+ \\
\psi_{4}(\boldsymbol{q})\left(\frac{\gamma_{0} \not \phi_{\perp}}{|\boldsymbol{q}|}-\frac{\left(\omega_{1}-\omega_{2}\right)|\boldsymbol{q}|}{m_{1} \omega_{2}+m_{2} \omega_{1}}\right) . \tag{15}
\end{gather*}
$$

Substituting this wavefunction into Eq．（8），we obtain the positive－energy wavefunction of $0^{+}$scalar meson．

We can simplify the B－S equation and obtain an equation group which only contains two coupled equa－ tions，the detailed expressions are presented in the appendix．

As we indicated in the introduction，we take two approaches．

1）Adjusting parameters while 2317 MeV is taken as the basic input parameter，namely the observed meson（may correspond to $\mathrm{D}_{\mathrm{sJ}}(2632)$ ）is supposed to be the first excited state of the well measured $\mathrm{D}_{\mathrm{sJ}}$（2317）：

Adopting the parameter given in the Ref．［6］，but varying $V_{0}$ to fit the mass of the ground state of $0^{+}$ which is set as 2317 MeV ，we obtain the mass of the
first radial excited state as $2700 \pm 20 \mathrm{MeV}$ ，and the corresponding component wavefunctions．$\psi_{2}, \psi_{4}$ are shown in Fig． 2.


Fig．2．Case 1．we adjust parameters when 2317 MeV serves as the basic input parame－ ter，the curves correspond to the component wavefunctionss of the $0^{+}$ground state and first radial excited state $\psi_{2}, \psi_{4}$ respectively．

Substituting the wavefunctions into Eq．（4），we obtain the numerical values

$$
\begin{align*}
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{0} \mathrm{~K}^{+}\right) & =9.58 \pm 1.33 \mathrm{MeV},  \tag{16}\\
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{+} \mathrm{K}^{0}\right) & =9.36 \pm 1.30 \mathrm{MeV},  \tag{17}\\
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \mathfrak{\eta}\right) & =3.64 \pm 0.31 \mathrm{MeV} . \tag{18}
\end{align*}
$$

The corresponding ratio of the branching ratios would be

$$
\begin{align*}
& \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{0} \mathrm{~K}^{+}\right) / \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \mathfrak{\eta}\right) \approx \\
& \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{+} \mathrm{K}^{0}\right) / \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \mathfrak{\eta}\right) \approx 2.6 \pm 0.3 . \tag{19}
\end{align*}
$$

2）Numerical results while 2632 MeV is taken as the input parameter for the mass of the first radial excited state of an undiscovered $0^{+}$meson（it might be the measured $\mathrm{D}_{\mathrm{sJ}}(2317)$ if the experimental errors are indeed large）．

While we adjust $V_{0}$ to fit 2632 MeV which is taken as the input parameter for the mass of the first radial excited state，we obtain the mass of the ground state of $0^{+}$as $2245 \pm 25 \mathrm{MeV}$ ．The corresponding component wavefunctions $\phi_{2}, \phi_{4}$ are shown in Fig． 3.


Fig．3．Instead，the corresponding component wavefunctions of the $0^{+}$ground state and first radial excited state $\psi_{2}, \psi_{4}$ ，as we take 2632 MeV as the input parameter of the mass of the first radial excited state to adjust other parameters．

Substituting the wavefunctions into Eq．（4），we obtain

$$
\begin{align*}
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{0} \mathrm{~K}^{+}\right) & =7.58 \pm 1.06 \mathrm{MeV}  \tag{20}\\
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{+} \mathrm{K}^{0}\right) & =7.25 \pm 1.01 \mathrm{MeV}  \tag{21}\\
\Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \eta\right) & =2.37 \pm 0.20 \mathrm{MeV} \tag{22}
\end{align*}
$$

and the corresponding ratio of the branching ratios

$$
\begin{align*}
& \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{0} \mathrm{~K}^{+}\right) / \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \eta\right) \approx \\
& \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}^{+} \mathrm{K}^{0}\right) / \Gamma\left(\mathrm{D}_{\mathrm{sJ}}^{+} \rightarrow \mathrm{D}_{\mathrm{s}}^{+} \eta\right) \approx 3.1 \pm 0.4 \tag{23}
\end{align*}
$$

## 4 Discussions and conclusion

In terms of the B－S equation，we evaluate the mass spectra and the decay rates when assuming $\mathrm{D}_{\mathrm{sJ}}(2632)$ observed by the SELEX collaboration to be a $0^{+}$or $2^{+}$meson of $\mathrm{c} \overline{\mathrm{s}}$ ．

Our observation is that if $\mathrm{D}_{\mathrm{sJ}}(2632)$ is a $2^{+} \mathrm{c} \overline{\mathrm{S}}$ state，the total width obtained in terms of the B－S equation can be consistent with data，but the pre－ dicted branching ratios obviously conflict with data． If it is a $0^{+} c \bar{s}$ state，our results can be summarized as follows．In approach（1），we adjust $V_{0}$ to fit the mass

2317 MeV which is the mass of the $0^{+}$ground state of $\bar{c} \bar{s}$ ，and obtain the mass of the first radial excited state as $2700 \pm 20 \mathrm{MeV}$ ．Instead，in approach（2），we adjust $V_{0}$ to fit 2632 MeV which is taken as the mass of the first radial excited state of $0^{+} c \bar{s}$ ，then we get the mass of the ground state as $2245 \pm 25 \mathrm{MeV}$ ．It is noted that the wavefunctions obtained in the two cases are very close as shown in Figs． 2 and 3．The decay rates calculated in approach（1）are a bit larger than that in approach（2）due to a larger phase space．Thus in approach（2），the obtained total width is consis－ tent with data，but not the ratio of branching ratios within the experimental tolerance range，whereas in approach（1），both the total width and the ratio do not coincide with the present data．

Moreover，if $\mathrm{D}_{\mathrm{sJ}}(2632)$ is a $1^{-} \mathrm{c} \overline{\mathrm{s}}$ vector meson and is the first radial excited state of $\mathrm{D}_{\mathrm{s}}^{*}$ ，as Chang et al．evaluated ${ }^{[6]}$ ，the ratio of branching ratios is also inconsistent with data．

Definitely，the calculations are model－dependent， especially the linear confinement part of the kernel in the B－S equation is phenomenologically introduced and the coefficient $\kappa$ is determined by fitting data． Thus the numerical results obtained in this theoret－ ical framework cannot be very accurate，a factor of as large as $2-3$ may be expected，however，the or－ der of magnitude and the qualitative behavior of the solution do not change．

Therefore，there may be some possible explana－ tions for obvious discrepancy between data and the－ oretical result．First，from the theoretical aspect，the discrepancy may indicate that $\mathrm{D}_{\mathrm{sJ}}(2632)$ possess a large exotic component，namely it may be a four－ quark state ${ }^{[12]}$ ，molecular state，or a hybrid ${ }^{[13-15]}$ ．

Another possibility is，as Swanson et al． suggested ${ }^{[16,17]}$ ，it could be the＂artefact＂of experi－ ments．Moreover，the other prestigious experimental groups ${ }^{[16,18]}$ do not see evidence of $\mathrm{D}_{\mathrm{sJ}}(2632)$ ，there－ fore，its existence is still suspicious．Further and more precise measurements are necessary to clarify this mystery，great efforts from both sides of theory and experiments must be made．

We thank C．H．Chang for helpful discussions．

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## Appendix A

## $2^{+}$

## Normalization condition

$$
\begin{equation*}
\int \frac{\mathrm{d} \boldsymbol{q}}{2 \pi^{3}} \frac{16 \boldsymbol{q}^{2}\left(2 \psi_{3} \psi_{4}+2 \psi_{3} \psi_{6}-2 \psi_{4} \psi_{5}-5 \psi_{5} \psi_{6}\right) \omega_{1} \omega_{2}}{3\left(\omega_{1} m_{2}+\omega_{2} m_{1}\right)}=2 P_{i 0} \tag{A1}
\end{equation*}
$$

## Coupled equations

$$
\begin{align*}
& \frac{\left(M-\omega_{1}-\omega_{2}\right) 4 \boldsymbol{q}^{4}\left(\left(m_{1}+m_{2}\right) \psi_{4}(\boldsymbol{q})+\left(\omega_{1}+\omega_{2}\right) \psi_{3}(\boldsymbol{q})-\left(m_{1}+m_{2}\right) \psi_{6}(\boldsymbol{q})-\left(\omega_{1}+\omega_{2}\right) \psi_{5}(\boldsymbol{q})\right)}{3\left(m_{2} \omega_{1}+m_{1} \omega_{2}\right)}= \\
& \int \frac{\mathrm{d} \boldsymbol{k}}{(2 \pi)^{3}}\left\{-3\left(V_{\mathrm{s}}-V_{\mathrm{v}}\right)\left[\left(m_{1}+m_{2}\right) \psi_{3}(\boldsymbol{q})+\left(\omega_{1}+\omega_{2}\right) \psi_{4}(\boldsymbol{q})\right]\left(m_{2} \omega_{1 k}+m_{1} \omega_{2 k}\right)(\boldsymbol{k} \cdot \boldsymbol{q})^{3} / \boldsymbol{k}^{2}-\right. \\
& 3\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right)\left[-\psi_{6}(\boldsymbol{q})\left(m_{2} \omega_{1} \omega_{1 k}-m_{1} \omega_{2} \omega_{1 k}-m_{2} \omega_{1} \omega_{2 k}-m_{1} \omega_{2} \omega_{2 k}\right)+\right. \\
& \psi_{5}(\boldsymbol{q})\left(\omega_{1 \mathrm{k}} \boldsymbol{q}^{2}+\omega_{2 \mathrm{k}} \boldsymbol{q}^{2}-m_{1} m_{2} \omega_{1 \mathrm{k}}+\omega_{1} \omega_{2} \omega_{1 \mathrm{k}}-m_{1} m_{2} \omega_{2 \mathrm{k}}+\omega_{1} \omega_{2} \omega_{2 \mathrm{k}}\right)+ \\
& \left.\psi_{4}(\boldsymbol{q})\left(m_{1} \omega_{2}-m_{2} \omega_{1}\right)\left(\omega_{2 \mathrm{k}}-\omega_{1 \mathrm{k}}\right)+\psi_{3}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}+m_{1} m_{2}-\omega_{1} \omega_{2}\right)\left(\omega_{1 \mathrm{k}}+\omega_{2 \mathrm{k}}\right)\right] \times \\
& (\boldsymbol{k} \cdot \boldsymbol{q})^{2} \boldsymbol{q}^{2}\left(V_{\mathrm{s}}-V_{\mathrm{v}}\right)\left[\psi_{4}(\boldsymbol{q})\left(\omega_{1}+\omega_{2}\right)+\psi_{3}(\boldsymbol{q})\left(m_{1}+m_{2}\right)+2 \psi_{5}(\boldsymbol{q})\left(m_{1}+m_{2}\right)+\right. \\
& \left.2 \psi_{6}(\boldsymbol{q})\left(\omega_{1}+\omega_{2}\right)\right]\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right) \boldsymbol{k} \cdot \boldsymbol{q}+ \\
& \boldsymbol{k}^{2} \boldsymbol{q}^{2}\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right)\left[-\psi_{6}(\boldsymbol{q})\left(m_{2} \omega_{1} \omega_{1 \mathrm{k}}-m_{1} \omega_{2} \omega_{1 \mathrm{k}}-m_{2} \omega_{1} \omega_{2 \mathrm{k}}-m_{1} \omega_{2} \omega_{2 \mathrm{k}}\right)+\right. \\
& \psi_{5}(\boldsymbol{q})\left(\omega_{1 \mathrm{k}} \boldsymbol{q}^{2}+\omega_{2 \mathrm{k}} \boldsymbol{q}^{2}-m_{1} m_{2} \omega_{1 \mathrm{k}}+\omega_{1} \omega_{2} \omega_{1 \mathrm{k}}-m_{1} m_{2} \omega_{2 \mathrm{k}}+\omega_{1} \omega_{2} \omega_{2 \mathrm{k}}\right)+ \\
& \psi_{4}(\boldsymbol{q})\left(m_{1} \omega_{2}-m_{2} \omega_{1}\right)\left(\omega_{2 \mathrm{k}}-\omega_{1 \mathrm{k}}\right)+ \\
& \left.\left.\psi_{3}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}+m_{1} m_{2}-\omega_{1} \omega_{2}\right)\left(\omega_{1 \mathrm{k}}+\omega_{2 \mathrm{k}}\right)\right]\right\} /\left[3 \omega_{1} \omega_{2}\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right] \tag{A2}
\end{align*}
$$

$$
\begin{align*}
& \frac{\left(M+\omega_{1}+\omega_{2}\right) 4 \boldsymbol{q}^{4}\left(\left(m_{1}+m_{2}\right) \psi_{4}(\boldsymbol{q})-\left(\omega_{1}+\omega_{2}\right) \psi_{3}(\boldsymbol{q})-\left(m_{1}+m_{2}\right) \psi_{6}(\boldsymbol{q})+\left(\omega_{1}+\omega_{2}\right) \psi_{5}(\boldsymbol{q})\right)}{3\left(m_{2} \omega_{1}+m_{1} \omega_{2}\right)}= \\
& \int \frac{\mathrm{d} \boldsymbol{k}}{(2 \pi)^{3}}\left\{-3\left(V_{\mathrm{s}}-V_{\mathrm{v}}\right)\left[\left(m_{1}+m_{2}\right) \psi_{3}(\boldsymbol{q})-\left(\omega_{1}+\omega_{2}\right) \psi_{4}(\boldsymbol{q})\right]\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)(\boldsymbol{k} \cdot \boldsymbol{q})^{3} / \boldsymbol{k}^{2}-\right. \\
& 3\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right)\left[\psi_{6}(\boldsymbol{q})\left(m_{2} \omega_{1} \omega_{1 \mathrm{k}}-m_{1} \omega_{2} \omega_{1 \mathrm{k}}-m_{2} \omega_{1} \omega_{2 \mathrm{k}}-m_{1} \omega_{2} \omega_{2 \mathrm{k}}\right)+\right. \\
& \psi_{5}(\boldsymbol{q})\left(\omega_{1 \mathrm{k}} \boldsymbol{q}^{2}+\omega_{2 \mathrm{k}} \boldsymbol{q}^{2}-m_{1} m_{2} \omega_{1 \mathrm{k}}+\omega_{1} \omega_{2} \omega_{1 \mathrm{k}}-m_{1} m_{2} \omega_{2 \mathrm{k}}+\omega_{1} \omega_{2} \omega_{2 \mathrm{k}}\right)- \\
& \left.\psi_{4}(\boldsymbol{q})\left(m_{1} \omega_{2}-m_{2} \omega_{1}\right)\left(\omega_{2 \mathrm{k}}-\omega_{1 \mathrm{k}}\right)+\psi_{3}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}+m_{1} m_{2}-\omega_{1} \omega_{2}\right)\left(\omega_{1 \mathrm{k}}+\omega_{2 \mathrm{k}}\right)\right] \times \\
& (\boldsymbol{k} \cdot \boldsymbol{q})^{2} \boldsymbol{q}^{2}\left(V_{\mathrm{s}}-V_{\mathrm{v}}\right)\left[-\psi_{4}(\boldsymbol{q})\left(\omega_{1}+\omega_{2}\right)+\psi_{3}(\boldsymbol{q})\left(m_{1}+m_{2}\right)+2 \psi_{5}(\boldsymbol{q})\left(m_{1}+m_{2}\right)-\right. \\
& \left.2 \psi_{6}(\boldsymbol{q})\left(\omega_{1}+\omega_{2}\right)\right]\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right) \boldsymbol{k} \cdot \boldsymbol{q}+ \\
& \boldsymbol{k}^{2} \boldsymbol{q}^{2}\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right)\left[\psi_{6}(\boldsymbol{q})\left(m_{2} \omega_{1} \omega_{1 \mathrm{k}}-m_{1} \omega_{2} \omega_{1 \mathrm{k}}-m_{2} \omega_{1} \omega_{2 \mathrm{k}}-m_{1} \omega_{2} \omega_{2 \mathrm{k}}\right)+\right. \\
& \psi_{5}(\boldsymbol{q})\left(\omega_{1 \mathrm{k}} \boldsymbol{q}^{2}+\omega_{2 \mathrm{k}} \boldsymbol{q}^{2}-m_{1} m_{2} \omega_{1 \mathrm{k}}+\omega_{1} \omega_{2} \omega_{1 \mathrm{k}}-m_{1} m_{2} \omega_{2 \mathrm{k}}+\omega_{1} \omega_{2} \omega_{2 \mathrm{k}}\right)- \\
& \psi_{4}(\boldsymbol{q})\left(m_{1} \omega_{2}-m_{2} \omega_{1}\right)\left(\omega_{2 \mathrm{k}}-\omega_{1 \mathrm{k}}\right)+ \\
& \left.\left.\psi_{3}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}+m_{1} m_{2}-\omega_{1} \omega_{2}\right)\left(\omega_{1 \mathrm{k}}+\omega_{2 \mathrm{k}}\right)\right]\right\} /\left[3 \omega_{1} \omega_{2}\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right], \tag{A3}
\end{align*}
$$

$$
\begin{align*}
& \frac{\left(M-\omega_{1}-\omega_{2}\right)}{3\left(m_{2} \omega_{1}+m_{1} \omega_{2}\right)} 2 \boldsymbol{q}^{2}\left[-5 \psi_{5}(\boldsymbol{q})\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)+2 \psi_{3}(\boldsymbol{q})\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)-\right.  \tag{A4}\\
& \left.\psi_{6}(\boldsymbol{q})\left(\boldsymbol{q}^{2}+5 m_{1} \omega_{2}+5 m_{2} \omega_{1}\right)+2 \psi_{4}(\boldsymbol{q})\left(-q^{2}+m_{1} m_{2}+\omega_{1} \omega_{2}\right)\right]= \\
& \int \frac{\mathrm{d} \boldsymbol{k}}{(2 \pi)^{3}}\left\{12\left(V_{\mathrm{s}}-V_{\mathrm{v}}\right) \psi_{3}(\boldsymbol{q})\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)(\boldsymbol{k} \cdot \boldsymbol{q})^{3} / \boldsymbol{k}^{2}+3\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right) \times\right. \\
& {\left[-\psi_{6}(\boldsymbol{q})\left(5 \omega_{1} \omega_{1 \mathrm{k}}+\omega_{2} \omega_{1 \mathrm{k}}+\omega_{1} \omega_{2 \mathrm{k}}+\omega_{2} \omega_{2 \mathrm{k}}\right)+\psi_{5}(\boldsymbol{q})\left(-5 m_{1} \omega_{1 \mathrm{k}}+m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}-5 m_{2} \omega_{2 \mathrm{k}}\right)+\right.} \\
& \left.2 \psi_{4}(\boldsymbol{q})\left(\omega_{1}-\omega_{2}\right)\left(\omega_{1 k}-\omega_{2 k}\right)+\psi_{3}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}+m_{1} m_{2}-\omega_{1} \omega_{2}\right)\left(\omega_{1 k}+\omega_{2 k}\right)\right](\boldsymbol{k} \cdot \boldsymbol{q})^{2} \times \\
& 2\left(V_{\mathrm{s}}-V_{\mathrm{v}}\right)\left[-\psi_{6}(\boldsymbol{q})\left(5 m_{2} \omega_{1}+5 m_{1} \omega_{2}\right)+2 \psi_{4}(\boldsymbol{q})\left(m_{2} \omega_{1}+m_{1} \omega_{2}\right)+\psi_{5}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}-5 m_{1} m_{2}-5 \omega_{1} \omega_{2}\right)+\right. \\
& \left.2 \psi_{3}(\boldsymbol{q})\left(2 \boldsymbol{q}^{2}+m_{1} m_{2}+\omega_{1} \omega_{2}\right)\right]\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right) \boldsymbol{k} \cdot \boldsymbol{q}- \\
& \boldsymbol{k}^{2} \boldsymbol{q}^{2}\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right)\left[-\psi_{6}(\boldsymbol{q})\left(5 \omega_{1} \omega_{1 \mathrm{k}}+\omega_{2} \omega_{1 \mathrm{k}}+\omega_{1} \omega_{2 \mathrm{k}}+\omega_{2} \omega_{2 \mathrm{k}}\right)+\right. \\
& \psi_{5}(\boldsymbol{q})\left(-5 m_{1} \omega_{1 \mathrm{k}}+m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}-5 m_{2} \omega_{2 \mathrm{k}}\right)+2 \psi_{4}(\boldsymbol{q})\left(\omega_{1}-\omega_{2}\right)\left(\omega_{1 \mathrm{k}}-\omega_{2 \mathrm{k}}\right)+ \\
& \left.\left.\psi_{3}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}+m_{1} m_{2}-\omega_{1} \omega_{2}\right)\left(\omega_{1 \mathrm{k}}+\omega_{2 \mathrm{k}}\right)\right]\right\} /\left[6 \omega_{1} \omega_{2}\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right],
\end{align*}
$$

$$
\frac{\left(M+\omega_{1}+\omega_{2}\right)}{3\left(m_{2} \omega_{1}+m_{1} \omega_{2}\right)} 2 \boldsymbol{q}^{2}\left[-5 \psi_{5}(\boldsymbol{q})\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)+2 \psi_{3}(\boldsymbol{q})\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)+\right.
$$

$$
\left.\psi_{6}(\boldsymbol{q})\left(\boldsymbol{q}^{2}+5 m_{1} \omega_{2}+5 m_{2} \omega_{1}\right)-2 \psi_{4}(\boldsymbol{q})\left(-q^{2}+m_{1} m_{2}+\omega_{1} \omega_{2}\right)\right]=
$$

$$
\int \frac{\mathrm{d} \boldsymbol{k}}{(2 \pi)^{3}}\left\{-12\left(V_{\mathrm{s}}-V_{\mathrm{v}}\right) \psi_{3}(\boldsymbol{q})\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)(\boldsymbol{k} \cdot \boldsymbol{q})^{3} / \boldsymbol{k}^{2}-3\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right) \times\right.
$$

$$
\left[\psi_{6}(\boldsymbol{q})\left(5 \omega_{1} \omega_{1 \mathrm{k}}+\omega_{2} \omega_{1 \mathrm{k}}+\omega_{1} \omega_{2 \mathrm{k}}+\omega_{2} \omega_{2 \mathrm{k}}\right)+\psi_{5}(\boldsymbol{q})\left(-5 m_{1} \omega_{1 \mathrm{k}}+m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}-5 m_{2} \omega_{2 \mathrm{k}}\right)-\right.
$$

$$
\left.2 \psi_{4}(\boldsymbol{q})\left(\omega_{1}-\omega_{2}\right)\left(\omega_{1 \mathrm{k}}-\omega_{2 \mathrm{k}}\right)+\psi_{3}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}+m_{1} m_{2}-\omega_{1} \omega_{2}\right)\left(\omega_{1 \mathrm{k}}+\omega_{2 \mathrm{k}}\right)\right](\boldsymbol{k} \cdot \boldsymbol{q})^{2}
$$

$$
2\left(V_{\mathrm{s}}-V_{\mathrm{v}}\right)\left[\psi_{6}(\boldsymbol{q})\left(5 m_{2} \omega_{1}+5 m_{1} \omega_{2}\right)-2 \psi_{4}(\boldsymbol{q})\left(m_{2} \omega_{1}+m_{1} \omega_{2}\right)+\psi_{5}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}-5 m_{1} m_{2}-5 \omega_{1} \omega_{2}\right)+\right.
$$

$$
\left.2 \psi_{3}(\boldsymbol{q})\left(2 \boldsymbol{q}^{2}+m_{1} m_{2}+\omega_{1} \omega_{2}\right)\right]\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right) \boldsymbol{k} \cdot \boldsymbol{q}-
$$

$$
\boldsymbol{k}^{2} \boldsymbol{q}^{2}\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right)\left[\psi_{6}(\boldsymbol{q})\left(5 \omega_{1} \omega_{1 k}+\omega_{2} \omega_{1 \mathrm{k}}+\omega_{1} \omega_{2 \mathrm{k}}+\omega_{2} \omega_{2 \mathrm{k}}\right)+\right.
$$

$$
\psi_{5}(\boldsymbol{q})\left(-5 m_{1} \omega_{1 \mathrm{k}}+m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}-5 m_{2} \omega_{2 \mathrm{k}}\right)-2 \psi_{4}(\boldsymbol{q})\left(\omega_{1}-\omega_{2}\right)\left(\omega_{1 \mathrm{k}}-\omega_{2 \mathrm{k}}\right)+
$$

$$
\begin{equation*}
\left.\left.\psi_{3}(\boldsymbol{q})\left(-\boldsymbol{q}^{2}+m_{1} m_{2}-\omega_{1} \omega_{2}\right)\left(\omega_{1 \mathrm{k}}+\omega_{2 \mathrm{k}}\right)\right]\right\} /\left[6 \omega_{1} \omega_{2}\left(m_{2} \omega_{1 k}+m_{1} \omega_{2 k}\right)\right] \tag{A5}
\end{equation*}
$$

## Appendix B

$0^{+}$

## Normalization condition

$$
\begin{equation*}
\int \frac{\mathrm{d} \boldsymbol{q}}{2 \pi^{3}} \frac{16 \psi_{2} \psi_{4} \omega_{1} \omega_{2}\left(-\boldsymbol{q}^{2}+m_{1} m_{2}+\omega_{1} \omega_{2}\right)}{\left(m_{1}+m_{2}\right)\left(\omega_{1} m_{2}+\omega_{2} m_{1}\right)|\boldsymbol{q}|}=2 P_{i 0} \tag{B1}
\end{equation*}
$$

## Coupled equations

$$
\begin{align*}
& \frac{2\left[\psi_{2}(\boldsymbol{q})\left(m_{2} \omega_{1}+m_{1} \omega_{2}\right)-\psi_{4}(\boldsymbol{q})\left(m_{2}+m_{1}\right)|\boldsymbol{q}|\right]}{m_{2} \omega_{1}+m_{1} \omega_{2}}=\int \frac{\mathrm{d} \boldsymbol{k}}{(2 \pi)^{3}}\left\{\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right) \times\right. \\
& {\left[\psi_{2}(\boldsymbol{k})\left(-\boldsymbol{q}^{2}+m_{2} m_{1}-\omega_{1} \omega_{2}\right)\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)-\psi_{4}(\boldsymbol{k}) \boldsymbol{k}\left(m_{2} \omega_{\mathrm{c}}-m_{1} \omega_{\mathrm{s}}\right)\left(\omega_{1 \mathrm{k}}-\omega_{2 \mathrm{k}}\right)\right] \boldsymbol{k}^{2}+} \\
& \left(V_{\mathrm{s}}-V_{\mathrm{v}}\right)\left[\psi_{2}(\boldsymbol{k})\left(\left(m_{1}-m_{2}\right)\left(\omega_{1 \mathrm{k}}-\omega_{2 \mathrm{k}}\right) \boldsymbol{k}^{2}+2 m_{1} m_{2}\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right)-\right. \\
& \left.\left.\psi_{4}(\boldsymbol{k})|\boldsymbol{k}|\left(\omega_{1}+\omega_{2}\right)\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right] \boldsymbol{k} \cdot \boldsymbol{q}\right\} /\left[\boldsymbol{k}^{2} \omega_{1} \omega_{2}\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right], \tag{B2}
\end{align*}
$$

$$
\begin{align*}
& \frac{2\left[\psi_{2}(\boldsymbol{q})\left(m_{2} \omega_{1}+m_{1} \omega_{2}\right)+\psi_{4}(\boldsymbol{q})\left(m_{2}+m_{1}\right)|\boldsymbol{q}|\right]}{m_{2} \omega_{1}+m_{1} \omega_{2}}=\int \frac{\mathrm{d} \boldsymbol{k}}{(2 \pi)^{3}}\left\{\left(V_{\mathrm{s}}+V_{\mathrm{v}}\right) \times\right. \\
& {\left[\psi_{2}(\boldsymbol{k})\left(-\boldsymbol{q}^{2}+m_{2} m_{1}-\omega_{1} \omega_{2}\right)\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)+\psi_{4}(\boldsymbol{k})|\boldsymbol{k}|\left(m_{2} \omega_{\mathrm{c}}-m_{1} \omega_{\mathrm{s}}\right)\left(\omega_{1 \mathrm{k}}-\omega_{2 \mathrm{k}}\right)\right] \boldsymbol{k}^{2}+} \\
& \left(V_{\mathrm{s}}-V_{\mathrm{v}}\right)\left[\psi_{2}(\boldsymbol{k})\left(\left(m_{1}-m_{2}\right)\left(\omega_{1 \mathrm{k}}-\omega_{2 \mathrm{k}}\right) \boldsymbol{k}^{2}+2 m_{1} m_{2}\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right)+\right. \\
& \left.\left.\psi_{4}(\boldsymbol{k})|\boldsymbol{k}|\left(\omega_{1}+\omega_{2}\right)\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right] \boldsymbol{k} \cdot \boldsymbol{q}\right\} /\left[\boldsymbol{k}^{2} \omega_{1} \omega_{2}\left(m_{2} \omega_{1 \mathrm{k}}+m_{1} \omega_{2 \mathrm{k}}\right)\right] . \tag{B3}
\end{align*}
$$

where $\omega_{1 \mathrm{k}}=\sqrt{m_{1}^{2}+\boldsymbol{k}^{2}}$ and $\omega_{2 \mathrm{k}}=\sqrt{m_{q}^{2}+\boldsymbol{k}^{2}} ; V_{\mathrm{v}}$ and $V_{\mathrm{s}}$ were given by Ref．［8］．

## $\mathrm{D}_{\mathrm{sJ}}(2632)$ 强衰变的研究 ${ }^{*}$



摘要 SELEX 合作组发现的 $\mathrm{D}_{\mathrm{sJ}}(2632)$ 引起了很多的讨论，同时也带来了激烈的争论。它的自旋宇称还没有最后确定，如果它真的存在，它的夸克结构可能是奇特的。以前有的文献假定 $\mathrm{D}_{\mathrm{sJ}}(2632)$ 是 $1^{-}$的径向激发态，我们假定它可能是夸克结构为 $\mathrm{q} \overline{\mathrm{q}}, \mathrm{J}^{P}$ 为 $2^{+}$的基态或者 $0^{+} \mathrm{D}_{\mathrm{sJ}}(2317)$ 的第一径向激发态，用 Bethe－Salpeter 方程重新计算了它的衰变宽度。计算结果表明，理论值和实验数据还是存在尖锐的矛盾。

关键词 $\mathrm{D}_{\mathrm{sJ}}(2632)$ 强衰变 Bethe－Salpeter 方程

2006－03－15 收稿，2006－04－19 收修改稿
＊国家自然科学基金（10475042）资助
1）E－mail：khw020056＠mail．nankai．edu．cn


[^0]:    Received 15 March 2006，Revised 19 April 2006
    ＊Supported by NSFC（10475042）
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