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Monopole Dynamics of Yang-Mills Theory without Gauge-Fixing*

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Abstract A new off-shell decomposition of $SU(2)$ gauge field without any gauge fixing is proposed. This decomposition yields, for an appropriate gauge-fixing, a Skyrme-Faddeev-like Wilsonian action and confirms the presence of high-order derivatives of a color-unit-vector at the classical level. The 't Hooft's conjecture that "monopole" dynamics of infrared Yang-Mills theory is projection independent is also independently demonstrated.

Key words Yang-Mill theory, off-shell decomposition, monopole dynamics

In an appealing attempt to interpret color confinement Polyakov[1] has constructed string variables as relevant degrees of freedom for describing the four dimensional Yang-Mills(YM) theory at large distances. Recently, a proposal for identifying the infrared variables of the YM theory has been made by Faddeev and Niemi[2]. They parameterized $SU(2)$ connection into an unit vector $\boldsymbol{n}$ and a set of dual variables, and conjectured that the infrared dynamics of YM theory is effectively described by the Skyrme-Faddeev model of the infrared order parameter $\boldsymbol{n}$

$$S^{SF}(\boldsymbol{n}) = \int d^4 x\left[m^2(\partial_\mu \boldsymbol{n})^2 + \frac{1}{e^2}(\boldsymbol{n}\cdot\partial_\mu \boldsymbol{n}\times\partial_\nu \boldsymbol{n})^2\right]. \tag{1}$$

Here, the first mass term was introduced so as to include the quantum effect of $\boldsymbol{n}$ fluctuations through Wilsonian renormalization group arguments and the first hint of this effect have been observed in one-loop approximations[3,4]. The action(1) with a mass term supports knot-like configuration as a stable soliton[5] and is then supposed to be consistent with the commonly accepted picture of confinement. The idea of decomposing the YM connection in terms of an unit isovector originated in the works[6,7] of non-Abelian monopoles, and recently, has been extended to the connection decomposition in terms of spinor for describing the topological effect of instantons due to the gauge-fixed defects[8].

The presence of gauge symmetry in YM theory complicates this ambitious conjecture in two ways: firstly, in order to formulate a quantum theory, the decomposition of YM connection has to also include the overabundant gauge degrees of freedom; and secondly, the gauge has to be fixed in a prescribed way, not only to be able to perform functional integration, but also to arrive at a unique effective action for $\boldsymbol{n}$ field.

The first problem was discussed by Shabanov[9], who established an one-to-one correspondence between the un-fixed gauge field and the some certain decomposition. The second problem of gauge fixing implies that a successful realization of the idea of Faddeev and Niemi will only be meaningful in a certain gauge, under which the infrared degrees of freedom of the theory can be determined by $\boldsymbol{n}$ field in a simple effective action. In a different gauge, these infrared variables may be hidden in a complicated effective action given by $\boldsymbol{n}$ field and other fields. Recent calculations[3,10] of the effective action based on Faddeev-Niemi decomposition and its extension[9] qualitatively confirm the generation of a mass scale and necessity of higher-derivative terms of $\boldsymbol{n}$ field for describing the large-distance degrees of freedom.

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However, the complexity of the final results involving higher-derivative terms of $\boldsymbol{n}$ field and the perturbative nature of the derivation makes this calculation hardly suffice to confirm the action (1) as the infrared limit of YM theory and further complicate the question of predicted stability of the string-like solition. Therefore, the intrinsic features of the Abelian projection is essential for confirmation of the dual-superconductor picture of confinement. In particular, it is essential to prove 't Hooft conjecture[11] that the separation of monopole degree of freedom from original connection is independent of the gauge-fixing.

In this paper, we propose a new off-shell decomposition of $SU(2)$ gauge field without any gauge-fixing. This decomposition can lead to a partially dual dynamics of YM theory which qualitatively confirms, for an appropriate gauge-fixing, a Skyrme-Faddeev-like action through Wilsonian renormalization group arguments and it confirms the presence of high-order derivatives of $\boldsymbol{n}$ field at the classical level. By discussing the monopole mechanism of the YM theory and the intrinsic relation between Abelian projection and the rotation-symmetry breaking of topological variable, we independently prove 't Hooft conjecture that the monopole dynamics of YM theory is projection (gauge) independent.

We begin by decomposing the $SU(2)$ YM connection in terms of the color unit vector $\boldsymbol{n}$. We hope that this decomposition of YM field includes full field degrees of freedom so that it remains complete at the level of quantum theory. For this, we need an off-shell decomposition, which contains the $\boldsymbol{n}$-field and describes $4d = 12$ field components in total, where $d = N^2 - 1 = 3$ is the dimension of gauge group. In quantum theory, $d = 3$ components can be removed by gauge condition to guarantee convergence of functional integration.

Firstly, we relate three-component unit vector $\boldsymbol{n}$ to $\boldsymbol{A}_\mu$ by employing the definition $D_\mu \boldsymbol{n} - \partial_\mu \boldsymbol{n} = e\boldsymbol{A}_\mu \times \boldsymbol{n}$ of covariant derivative of $\boldsymbol{n}$, where e is coupling constant. Direct calculation of $\boldsymbol{n} \times (\boldsymbol{A}_\mu \times \boldsymbol{n})$ shows[6]

$$\boldsymbol{A}_\mu = A_\mu \boldsymbol{n} + e^{-1} \partial_\mu \boldsymbol{n} \times \boldsymbol{n} - e^{-1} D_\mu \boldsymbol{n} \times \boldsymbol{n}, \quad (2)$$

where $\boldsymbol{b}_\mu = -e^{-1} D_\mu \boldsymbol{n} \times \boldsymbol{n}$ should describe six variables. In (2), the Abelian component $A_\mu = \boldsymbol{A}_\mu \cdot \boldsymbol{n}$ of original connection $\boldsymbol{A}_\mu$ describes four variables, corresponding to $U(1)$ symmetry. It is clear that the first part $A_\mu \boldsymbol{n}$ in (2) is valued in subgroup $H = U(1)$ while the second and third terms, both of which are orthogonal to the direction $\boldsymbol{n}$, is valued in orbit $SU(2)/H = S^2$.

One of schemes[9] for parametrization of $\boldsymbol{b}_\mu$ is to unify these six variables into an antisymmetric tensor $u_{\mu\nu} = -u_{\nu\mu}$ and re-parameterize it into $\boldsymbol{b}_\mu = [u_{\mu\nu} + W_{\mu\nu}(u, \boldsymbol{n})]\partial_\nu \boldsymbol{n} \times \boldsymbol{n}$, where $W_{\mu\nu}$ is a symmetric tensor which depends on $u_{\mu\nu}$ and $\boldsymbol{n}$. Observed that the two vectors $\partial_\mu \boldsymbol{n}$ and $\partial_\mu \boldsymbol{n} \times \boldsymbol{n}$ form a basis of the orbit space $SU(2)/U(1)$, the degrees of freedom along the internal direction $\partial_\mu \boldsymbol{n}$ should also be included. Here, we combine the two basis vectors by antisymmetric tensors $u_{\mu\nu}$ and $\epsilon_{\mu\nu}$, and parameterize $\boldsymbol{b}_\mu$ as

$$\boldsymbol{b}_\mu = u_{\mu\nu} \partial_\nu \boldsymbol{n} + \epsilon_{\mu\nu} \partial_\nu \boldsymbol{n} \times \boldsymbol{n}. \quad (3)$$

One can see that this parameterization naturally satisfies the constrain $\boldsymbol{b}_\mu \cdot \boldsymbol{n} = 0$ and contains six desired variables. In principle, the six field variables $u_{\mu\nu}$ can be used to construct three pairs of scalar fields ρ^i and σ^i ($i = 1,2,3$), forming three dual complex fields $\phi^i = \rho^i + i\sigma^i$. In this work, we will use the variables $u_{\mu\nu}$ instead of three charged fields. Therefore, our suggestion for off-shell decomposition of the $SU(2)$ connection is

$$\boldsymbol{A}_\mu = A_\mu \boldsymbol{n} + e^{-1} \partial_\mu \boldsymbol{n} \times \boldsymbol{n} + u_{\mu\nu} \partial_\nu \boldsymbol{n} + \epsilon_{\mu\nu} \partial_\nu \boldsymbol{n} \times \boldsymbol{n}, \quad (4)$$

The reason for this proposal is that a parameterized part along the direction $\partial_\mu \boldsymbol{n} \times \boldsymbol{n}$ is already present in (2) and this change of variables contains desired twelve independent field components, corresponding to the four components of $U(1)$ connection A_μ, the two independent components of n^a and the six components $u_{\mu\nu}$. Among twelve degrees of freedom in (4), three can be fixed by the gauge condition, leaving $3d = 9$ independent field degrees of freedom. The Gauss law can be used to further remove another three so that the decomposition (4) describes six physical degrees of freedom of the connection $\boldsymbol{A}_\mu$. Therefore, this parameterization is an off-shell extension of Faddeev-Niemi decomposition. Observed that $\boldsymbol{n}$ represents a direction which is purely associated with orientation of the moving frame $\{\boldsymbol{n}, \partial_\mu \boldsymbol{n}, \partial_\mu \boldsymbol{n} \times \boldsymbol{n}\}$, one can identify $\boldsymbol{n}$ as a topological degree of freedom.

An off-shell decomposition[12] of YM field with a partial gauge-fixing has been proposed recently by

Faddeev and Niemi for describing the quantum structure of infrared YM theory. This decomposition is elegantly formulated by Maurer-Cartan one-form of an $SU(2)$ matrix variables and contains ten degrees of freedom, corresponding to a partial gauge-fixing implicitly employed for two independent variables. Unfortunately, in this improved parameterization, the color $\boldsymbol{n}$-field, which was originally conjectured to be the relevant degree of freedom for describing the knot-like soliton, has been replaced by the $SU(2)$ matrix and the simplicity of the original on-shell decomposition [2] in terms of $\boldsymbol{n}$-field has lost in a complex mathematical form of the new matrix variables. In contrast, our decomposition has full (twelve) degrees of freedom of the original connection and it keeps $\boldsymbol{n}$ field being un-replaced so as to find the effective action of this topological field at the quantum level. Our decomposition (4) also differs from the scheme[9] in that the degrees of freedom along the internal direction $\partial_\mu \boldsymbol{n}$ have also been included so that the orientation of $\boldsymbol{b}_\mu$ in (4) can be in all directions in orbit space $SU(2)/U(1)$ as $u_{\mu\nu}$ vary.

Although the fourth term in (4) is not linearly independent on the connection part $\boldsymbol{C}_\mu = e^{-1}\partial_\mu \boldsymbol{n} \times \boldsymbol{n}$, we still separate it from $\boldsymbol{C}_\mu$ since it can not transform as a connection. A single magnetic monopole can be given by with the singular "hedgehog" configuration $n^a = x^a/r$ [13]. That is why we identify $\boldsymbol{C}_\mu$ as a monopole degree of freedom instead of including the fourth term in (4). To see the magnetic mechanism of YM theory, we consider its dual dynamics in independent-particle and mean-field approximations.

The simplest dual dynamics of YM theory can be given by the independent particle approximation, where one can ignore the correlation effect between monopoles and color sources. In this case, we take the Abelian component $\boldsymbol{G}_{\mu\nu} \cdot \boldsymbol{n} = F_{\mu\nu} + H_{\mu\nu} + e^{-1}\boldsymbol{n} \cdot (D_\mu \boldsymbol{n} \times D_\mu \boldsymbol{n})$ of the full field strength $\boldsymbol{G}_{\mu\nu}$ as the dominant variable, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $H_{\mu\nu} = -\frac{1}{e}\boldsymbol{n} \cdot (\partial_\mu \boldsymbol{n} \times \partial_\nu \boldsymbol{n}) = \partial_\mu C_\nu - \partial_\nu C_\mu$ stands, respectively, for the "electric" and magnetic field strengths. Here, C_μ is the magnetic potential. As a physical field, the total field exhibiting "electronic"-magnetic duality should be given by the gauge-invariant part

$$f_{\mu\nu} = \boldsymbol{G}_{\mu\nu} \cdot \boldsymbol{n} - \frac{1}{e}\boldsymbol{n} \cdot (D_\mu \boldsymbol{n} \times D_\mu \boldsymbol{n}) = \quad + H_{\mu\nu} \tag{5}$$

which is the 't Hooft tensor[14] in $SU(2)$ gauge theory. Substituting (5) into the standard YM action we get a fully dual and Abelized dynamics

$$S = \frac{1}{4e^2}\int d^4x (F_{\mu\nu} + H_{\mu\nu})^2. \tag{6}$$

in which the second term in the action (1) is already present. As the case of superconductor phase-transition below the critical temperature, one can assume that at low-energy limit the gauge symmetry of ground state of hardrons has been dynamically broken so that the magnetic potential effectively acquires a mass, that is, a mass term $m^2 \boldsymbol{C}_\mu^2/2 = m^2(\partial_\mu \boldsymbol{n})^2/2e^2$ should be added to the action (6). Therefore, the model (1) can be observed at the level of the least approximation.

When all degrees of freedom is taken into account, the components of full field strength in the $SU(2)/H$ direction will appear. With (4), one can find

$$\boldsymbol{G}_{\mu\nu} = \boldsymbol{n}[F_{\mu\nu} + \widetilde{H}_{\mu\nu} - e\epsilon_{\mu\lambda} u_{\nu\rho}(\partial_\lambda \boldsymbol{n} \cdot \partial_\rho \boldsymbol{n})] + \boldsymbol{N}_{\mu\nu}, \tag{7}$$

where $\widetilde{H}_{\mu\nu} = H_{\mu\nu} - e\epsilon_{\mu\lambda} H_{\lambda\nu} + e\epsilon_{\nu\lambda} H_{\lambda\mu} - e^2 \epsilon_{\mu\lambda}\epsilon_{\nu\rho} H_{\lambda\rho}$ and the non-Abelian component $\boldsymbol{N}_{\mu\nu}$ is given by

$$\begin{aligned}\boldsymbol{N}_{\mu\nu} = {} & (\partial_{[\mu} u_{\nu],k} + e\epsilon_{k[\mu} A_{\nu]})\partial_k \boldsymbol{n} + eA_{[\mu} u_{\nu]} \boldsymbol{n} \times \\ & \partial_k \boldsymbol{n} - 2e^{-1}\partial_\mu \boldsymbol{n} \times \partial_\nu \boldsymbol{n} - \epsilon_{k\nu}\partial_\mu \partial_k \boldsymbol{n} + \\ & u_{k[\mu}\partial_{\nu]}\partial_k \boldsymbol{n} + \epsilon_{k\nu}\partial_\mu \boldsymbol{n} \times \partial_k \boldsymbol{n} - \epsilon_{k\mu}\partial_\nu \boldsymbol{n} \times \\ & \partial_k \boldsymbol{n} + eu_{\mu k} u_{\nu\rho}(\partial_k \boldsymbol{n} \times \partial_\rho \boldsymbol{n}) + \\ & \epsilon_{k\mu}\partial_\nu \partial_k \boldsymbol{n} \times \boldsymbol{n},\end{aligned} \tag{8}$$

Here, λ, ρ and k are the Lorenz indices. The field strength (7) gives rise to the following partially dual dynamics

$$\begin{aligned}\mathscr{S}_{\text{dual}} \quad & \frac{1}{4e^2}\Big\{[F_{\mu\nu}^2 + 2F_{\mu\nu}(\widetilde{H}_{\mu\nu} - e\epsilon_{\mu\lambda} u_{\nu\rho}(\partial_\lambda \boldsymbol{n} \\ & \partial_\rho \boldsymbol{n})) + (\widetilde{H}_{\mu\nu} - e\epsilon_{\mu\lambda} u_{\nu\rho}(\partial_\lambda \boldsymbol{n} \cdot \partial_\rho \boldsymbol{n}))^2 + \\ & 2[F_{\mu\nu} + \widetilde{H}_{\mu\nu} - e\epsilon_{\mu\lambda} u_{\nu\rho}(\partial_\lambda \boldsymbol{n} \cdot \partial_\rho \boldsymbol{n})][2H_{\mu\nu} + \\ & e\epsilon_{\mu\lambda} H_{\lambda\mu} - e\epsilon_{\nu\lambda} H_{\lambda\nu} - e^2 u_{\mu\lambda} u_{\nu\rho} H_{\lambda\rho} + (\epsilon_{k\nu} + u_{k\nu}) \\ & \partial_k \boldsymbol{n} \cdot \partial_\mu \boldsymbol{n} - u_{k\mu}\partial_k \boldsymbol{n} \cdot \partial_\nu \boldsymbol{n}] + (\boldsymbol{N}_{\mu\nu})^2\Big\}.\end{aligned} \tag{9}$$

where $\boldsymbol{N}_{\mu\nu}$ is given in (8).

It has been seen that various terms of the high-order derivatives of $\boldsymbol{n}$-field has already presented at the classical level and partial duality of "electronic"-magnetic variables appears as a complicated fashion in contrast with full duality in the action (6). To find a qualitative picture of monopole mechanism of confinement, let us assume that the vacuum state of infrared quantum YM theory for (9) breaks the gauge symmetry $SU(2)$ down to $U(1)$ and the mean-field limit permits such a simple expectation $\langle \partial_{\mu} u_{\nu k} \rangle \propto m\delta_{\mu\nu}$ for any k. This can be realized, e.g., in such a gauge condition, $u_{\nu k} = u(\nu)$, for $k = 1, 2, 3$. Then, by averaging (9) over the dual variables $(A_\mu, u_{\nu k})$, we can get an classical action with leading term

$$S(\boldsymbol{n}) = \int d^4 x [m^2 (\partial_\mu \boldsymbol{n})^2 + \widetilde{H}_{\mu\nu}^2], \qquad (10)$$

where $\langle F_{\mu\nu} \rangle = 0$ due to the residual $U(1)$ invariance. This is a Skyrme-Faddeev-like action, where $H_{\mu\nu}$ is replaced by $\widetilde{H}_{\mu\nu}$ and it includes the action (1) as a leading term. The explicit form of the resulting action contains the high-order derivatives of $\boldsymbol{n}$-field, such as $(\partial_\nu \partial_k \boldsymbol{n})^2$ and $(\partial_k \boldsymbol{n} \cdot \partial_\mu \boldsymbol{n})^2$, and we do not write out explicitly here. Similar terms of higher derivatives of $\boldsymbol{n}$-field was also seen in the Wilsonian effective action based on the decomposition proposed by Shabanov[9] and they are argued to be necessary for the description of topologically stable knotlike solitons.

To see the monopole mechanism of YM theory, we proceed to discuss the relation of Abelian projection with the rotation-symmetry breaking of $\boldsymbol{n}$. As suggested by 't Hooft et al[11,15], if the dual Meissner effect do occur in the infrared regime of the theory so that dual-superconductor picture of confinement is qualitatively true, the effective model of the confining phase should be described by the order parameter involving the magnetic monopoles. This is dual to the case of superconductor where order parameter is given by the condensate of the electron pairs. For this reason, one can assumes that the confining phase of infrared YM theory, where the original symmetry is broken down to $U(1)$, is described by the order parameter $\langle \boldsymbol{n} \rangle$, a nonvanishing expectation value of $\boldsymbol{n}$, whose stationary subgroup exactly corresponds to the residual $U(1)$ symmetry and for which the broken symmetry corresponds to the gauge rotation $\delta \boldsymbol{n} = \boldsymbol{n} \times \boldsymbol{\varepsilon}$.

We write $\boldsymbol{n} = U\tau^3 U^\dagger = n^a \tau^a$ and substitute the explicit express $U = \exp(-\tau^3 \gamma) \exp(-\tau^2 \alpha) \exp(-\tau^3 \beta)$ of U into $\boldsymbol{n}$. Then, one finds that the direction $\langle \boldsymbol{n} \rangle$ of $\boldsymbol{n}$ is determined by two Euler angles α and β

$$\langle \boldsymbol{n} \rangle = (\sin\alpha\cos\beta, \sin\alpha\sin\beta, \cos\alpha) \neq 0, \qquad (11)$$

and magnetic potential becomes $C_\mu = e^{-1}(\cos\alpha\ \partial_\mu \beta + \partial_\mu \gamma)$, which still shares an $U(1)$ gauge degree of freedom $C_\mu \to C_\mu + \partial_\mu \gamma$. This implies, a specific Abelian projection $\boldsymbol{n} \to \boldsymbol{n}_0 = \langle \boldsymbol{n} \rangle$ corresponds to a fixed gauge U with α and β fixed, leaving γ as a residual $U(1)$ degree of freedom. Therefore, the different Abelian projections is one-to-one corresponding to the different directions (11) of $\langle \boldsymbol{n} \rangle$, and further one-to-one corresponding to the different matrices U. This property can be used to prove the 't Hooft's conjecture[11].

In spite of presence of some numerical works[16,17] confirming monopole-like defects in Abelian components of the YM theory and several derivations of effective actions for infrared YM theory in different gauges[3,4,10], the stability of the knotted string-like soliton[2] that is proposed to be the candidate for gluenball is not completely confirmed due to the limit of number of the Abelian gauge used by lattice calculations and the variation of the derived marginal contributions to the action (1). Hence, before deriving the quantum dynamics of infrared YM theory, it is essential to prove the equivalence of the final effective model for different gauge-fixing. For this, we need to prove the 't Hooft's conjecture that all Abelian projection are equivalent. Here, we demonstrate this conjecture based on the intrinsic relation between Abelian projection and the rotation-symmetry breaking of topological variable that is manifested in our decomposition.

We consider the quantum functional for (9). The Jacobian J for the variable change (4) of $\boldsymbol{A}_\mu$ into $(A_\mu, \boldsymbol{n}, u)$ can be given by calculating the distance element $d\boldsymbol{A}_\mu \cdot d\boldsymbol{A}_\mu$ in connection space. Given a fixed frame $\left\{\boldsymbol{n}, \partial_\mu \boldsymbol{n}, \partial_\mu \boldsymbol{n} \times \boldsymbol{n}\right\}$, one can explicitly calculate the Jacobian $J = J(\boldsymbol{n})$ as a functional of $\boldsymbol{n}$-field and write the functional integration as

$$Z \propto \int D\boldsymbol{A}_\mu e^{-S_{YM}} \propto \int DA_\mu Du_{\nu k} e^{-S_{dual}}.$$

To remove the divergence of this integration arising from gauge symmetry, we need to choose a gauge condition. One can choose any physical gauge-fixing $\boldsymbol{F}_\mu(A_\mu, u, \boldsymbol{n}) = 0$, e.g., covariant gauge condition $\partial_\mu \boldsymbol{b}_\mu + A_\mu \boldsymbol{n} \times$

$\boldsymbol{b}_\mu + \boldsymbol{n}(\boldsymbol{b}_\mu \cdot \partial_\mu \boldsymbol{n}) = 0$, where $\boldsymbol{b}_\mu$ is given in (2) and coupling constant e is omitted. Inserting the identity $1 = \int D\boldsymbol{n}\ \Delta_F(A_\mu, u, \boldsymbol{n})\ \delta(\boldsymbol{F}_\mu)$ into the above functional, we obtain

$$Z \propto \int D\boldsymbol{n} DA_\mu Du J(\boldsymbol{n}) \Delta_F(A_\mu, u, \boldsymbol{n}) \delta(\boldsymbol{F}_\mu) e^{-S_{\rm dual}}$$

$$\propto \int D\boldsymbol{n} DA_\mu Du J(\boldsymbol{n}) e^{-\int d^4 x \mathscr{L}_{\rm eff}} \qquad (12)$$

where $\Delta_F(A_\mu, u, \boldsymbol{n}) = \det[\delta \boldsymbol{F}_\mu / \delta \boldsymbol{n}]$ can be determined by $\boldsymbol{n}$ field and dual variables (A_μ, u), and the effective Lagrangian is given by

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm dual} + c^{a*} M_F^{ab} c^b + \frac{1}{2\xi}(\boldsymbol{F}_\mu)^2. \qquad (13)$$

Since different choices of Abelian projection correspond to the different decompositions (4) with fixed gauge given in new frame $\{\boldsymbol{n}^g, \partial_\mu \boldsymbol{n}^g, \partial_\mu \boldsymbol{n}^g \times \boldsymbol{n}^g\}$, the 't Hooft's conjecture can be proved by the invariance of the Faddeev-Popov integration (12) under the variation of $\boldsymbol{n}$ and induced change of other variables. The change of the parameterization (4) can be realized by a left-action transformation $U \to gU$ in $\boldsymbol{n} = U\tau^3 U^+$, where $g \in SU(2)$. For small gauge transformation $U \to e^{\epsilon} U$ ($\epsilon = \epsilon^a \tau^a$), one can check that the variables $(A_\mu, \boldsymbol{n}, u)$ transform as $A_\mu \to A_\mu + \partial_\mu \theta$, $\delta \boldsymbol{n} = \boldsymbol{n} \times \boldsymbol{\varepsilon}$, $u_{\mu\nu} \to e^{i\beta} u_{\mu\nu}$ and the ghost field as $c \to e^{i\delta} c$, which is a subset of the BRST transformation of $(A_\mu, \boldsymbol{n}, u)$ and the ghost field c. Here, θ, β_i and δ are corresponding small parameters of ϵ. When $u_{\mu\nu}$ are integrated out, the invariance of the ensuing effective action under the transformation $U \to e^{\epsilon} U$ can be guaranteed by the invariance of the Faddeev-Popov integration under the BRST transformation of $(A_\mu, \boldsymbol{n}, c)$. This implies, a different gauge-fixed decomposition corresponds to another subset of the BRST transformation with different parameters θ, β and δ of ϵ. We note that the invariance of the measure $D\boldsymbol{n} DA_\mu DuJ(\boldsymbol{n})$ can be guaranteed by the invariance of the differential form. Then, we conclude that all Abelian projections are gauge equivalent. That means, the partially dual dynamics (9) of the YM theory does not depend on the choices of the Abelian projection, i.e., it is gauge independent. This supports the original suggestion[11] that confinement may originate from a topological feature of the non-Abelian gauge theory.

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杨-密耳斯理论的单极机制*

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摘要 对于未加规范条件的杨-密耳斯理论,提出了 $SU(2)$ 规范场一种新的离壳分解.在一定的规范条件下,该分解可以导出类似于Skyrme-Faddeev形式的Wilson作用量并在经典层次上肯定了色单位矢量场的高阶导数的存在.进一步独立地证明了't Hooft关于低能杨-密耳斯理论的单极动力学是投射无关的猜想.

关键词 杨-密尔斯理论 离壳分解 单极动力学

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