Influence of Multiplicity Fluctuation on the Erraticity Behaviour in High Energy Collisions [*]

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Abstract The influence of multiplicity distribution (fluctuation of multiplicity in the event space) on the erraticity behaviour in high energy collisions is investigated via Monte Carlo simulation and compared with the experimental results from NA27 data. It is shown that, the erraticity phenomenon is insensitive to the multiplicity fluctuation. When the average multiplicity is low this phenomenon is dominated by the statistical fluctuations, also in the case of multiplicities fluctuating from event to event.

Key words erraticity, multiplicity distribution, dynamical fluctuations

Since the discovery of unexpectedly large local fluctuations in a high multiplicity event recorded by the JACEE collaboration[1], the investigation of non-linear phenomena in high energy collisions has attracted much attention[2]. An anomalous scaling of factorial moments, defined as

$$F_q = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m (n_m - 1) \cdots (n_m - q + 1) \rangle}{\langle n_m \rangle^q} \quad , \tag{1}$$

at diminishing phase space scale or increasing partition number M of phase space[3]

$$F_q \propto M^{-\phi_q}, \tag{2}$$

called intermittency (or fractal) has been proposed for this purpose in multiparticle system and has been observed successfully in various experiments[4,5]. The average $\langle \cdots \rangle$ in Eq. (1) is over the whole event sample and n_m is the number of particle falling into the mth bin.

Recently, Cao and Hwa[6] argued that besides the factorial moments F_q averaged over event sample, the fluctuation of single event factorial moments

$$F_q^{(e)} = \frac{\frac{1}{M} \sum_{m=1}^{M} n_m (n_m - 1) \cdots (n_m - q + 1)}{\left(\frac{1}{M} \sum_{m=1}^{M} n_m \right)^q} \tag{3}$$

in the event space can also be ultilized to characterize the dynamical property of the hadronic system produced in the collisions.

The fluctuations of F_q^e from event to event can be quantified by its normalized moments as

$$C_{p,q} = \langle \Phi_q^p \rangle, \quad \Phi_q = F_q^{(e)} / \langle F_q^{(e)} \rangle, \tag{4}$$

and by $dC_{p,q}/dp$ at $p = 1$,

$$\Sigma_q \equiv \frac{dC_{p,q}}{dp}\bigg|_{p=1} = \langle \Phi_q \ln\Phi_q \rangle . \tag{5}$$

If there is a power-law behavior of the fluctuation as the partition number goes to infinity (or as the resolution $\delta = \Delta/M$ becomes very small), i.e. : $C_{p,q}(M) \propto M^{\psi_q(p)}$, $\tag{6}$
the phenomenon is referred to as erraticity[7]. The derivative of the exponent $\psi_q(p)$ at $p = 1$

$$\mu_q = \frac{d}{dp}\psi_q(p)\bigg|_{p=1} = \frac{\partial\Sigma_q}{\partial\ln M} . \tag{7}$$

describes the width of the fluctuation and is called the entropy index.

Wang et al. [8] have used this method to analyse the 400 GeV/c pp collision data from NA27 and really found evidences for the predicted erraticity phenomenon. Some physical conclusions have been drawn tentatively from this experimental finding.

However, it is well known that the obstacle of event-by-event analysis is the influence of statistical fluctuations caused by the insufficient amount of particles. The big advantage[3] of sample factorial moments, Eq. (1), in being able to eliminate the statistical fluctuations comes from the average over event sample. The same procedure cannot be applied to the event factorial moments of Eq. (3)[9,10]. Since the number of particles in one event is always finite, event factorial monents cannot completely eliminate statistical fluctuations and therefore cannot represent the dynamical probability moments associated with it.

It has been shown[10] that a flat probability distribution with 9 particles in each event, distributed according to the Bernoulli distribution.

$$B(n_1,\cdots,n_M \mid p_1,\cdots,p_M) = \frac{N!}{n_1!\cdots n_M!}\, p_1^{n_1}\cdots p_M^{n_M}, \quad \sum_{m=1}^{M} n_m = N, \tag{8}$$

can reproduce the phenomenon observed in NA27 data[8]. This indicates that when multiplicity is low the fluctuation of factorial moments in event space is dominated by statistical fluctuations[9]. The erraticity phenomenon observed in NA27 data is mainly due to statistical fluctuations and has little to do with real physics.
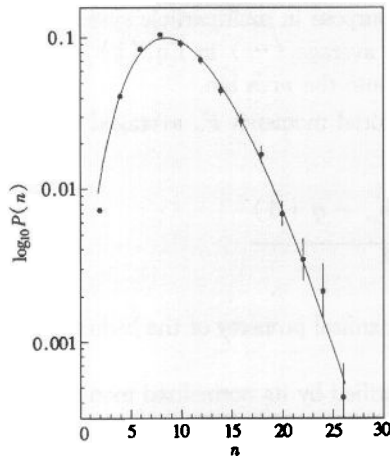

Fig. 1. The negative-binomial distribution ($\langle n \rangle = 9.84$, $k = 12.76$) fitting to the multiplicity distribution of 400 GeV/c pp collision. Data taken from Ref. [11].

However, in order to draw a more reliable conclusion, it should be noticed that the multiplicity in NA27 experiment is not a constant, but is fluctuating from event to event. How will this additional fluctuation in the event space influence the fluctuation in the same space of the factorial moment? To answer to this question is the main goal of this letter.

Let us parametrize the multiplicity distribution in 400 GeV/c pp collision from NA27 experiment[11] by means of a negative − binomial distribution (NBD)[12]

$$P_n = \binom{n+k-1}{n}\left(\frac{\langle n \rangle/k}{1+\langle n \rangle/k}\right)^n \frac{1}{(1+\langle n \rangle/k)^k} . \tag{9}$$

In Eq. (9) the average multiplicity $\langle n \rangle$ is taken as 9.84, the parameter k is fitted to be 12.76, (cf. Fig. 1).

According to the NBD, Eq. (9), 60000 events are obtained using Monte Carlo method. In each event the phase space region Ω is divided into M bins and the n particles of this event are allocated in these bins according to the Bernoulli distribution, Eq. (8). The event factorial moments $F_q^{(e)}$ and the corresponding $C_{p,q}$, Σ_q are then calculated according to Eqs. (3)—(5).

The results of $\ln C_{p,q}$ versus $\ln M(q = 2,3,4)$ are plotted in Fig. 2 together with the experi-

mental results[8] from NA27 data. Comparing the results of the present model with both statistical and multiplicity fluctuations with the results in Ref. [10] for fixed multiplicity ($n = 9$) having only statistical fluctuations, we can see that the influence of multiplicity fluctuation is weak. When the average multiplicity is low, the erraticity phenomenon is still dominated by the statistical fluctuations, even in the case of multiplicities fluctuating from event to event.
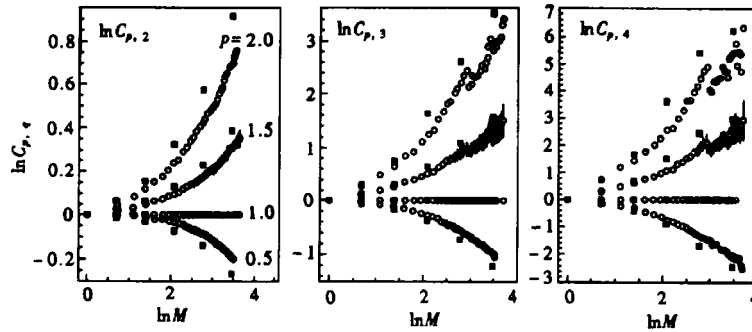

Fig. 2.　The $C_{p,q}$ of flat probability distribution with both statistical and multiplicity fluctuations as compared with that of 400 GeV/c pp collision. Data taken from Ref. [11].
■$\langle n \rangle = 9.84$, ○NA27.

In order to show the dependence of erraticity behaviour on the width of multiplicity distribution, i. e. the strength of multiplicity fluctuation in the event space, in more details, we have done the same calculation by fixing the average multiplicity to $\langle n \rangle = 9$ while varying the distribution width through changing the NBD parameter k from 0.1 to 18. The corresponding multiplicity distributions for different k is shown in Fig. 3. The results for $\ln C_{p,q}$ and Σ are shown in Figs 4(a) and (b) respectively. It can be seen from Fig. 3
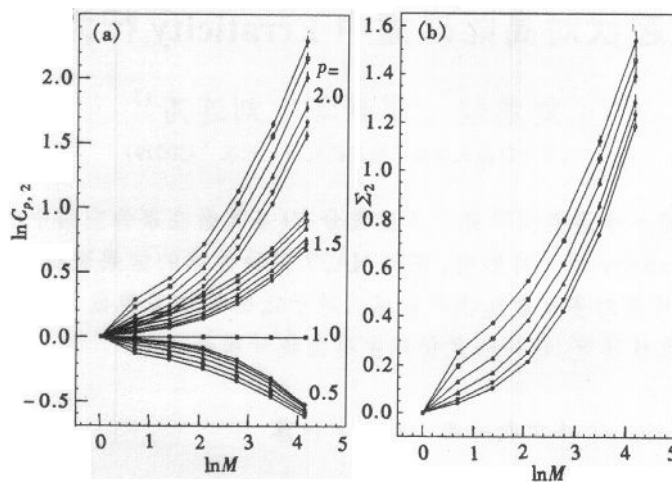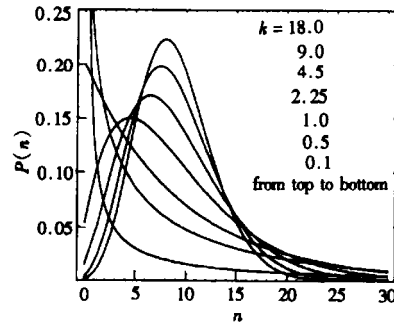

Fig. 4.　The $C_{p,q}$ (a) and Σ_q (b) from flat probability distribution with statistical fluctuation and negative-binomial multiplicity distribution (with various values of parameter k).
$\langle n \rangle = 9$, ●$k = 0.5$, ⌐$k = 1.0$, ▲$k = 2.25$, △$k = 4.5$, ▼$k = 9.0$, ○$k = 18.0$.

bution changes considerably while the erraticity phenomenon remains qualitatively the same. The corresponding curves are nearly parallel to each other at large M, showing that the value of entropy index defined in Eq. (7) is insensitive to the width of multiplicity distribution.

In conclusion, we have been able to reproduce the erraticity phenomenon observed in the experimental data from NA27, using a flat probability distribution with both statistical and multiplicity fluctuations. The result shows that when the average multiplicity is low, the erraticity phenomenon is dominated by the statistical fluctuations also in the case of multiplicities fluctuating from event to event. Through varying the NBD parameter k over a large range, we have shown that the erraticity phenomenon is insensitive to the width of multiplicity distribution.

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多重数起伏对高能碰撞中 Erraticity 行为的影响 *

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摘要 用蒙特-卡洛模拟研究了多重数分布(多重数在事件空间中的起伏)对高能碰撞中 erraticity 行为的影响,并和 NA27 实验数据的结果进行了比较. 证明了 erraticity 现象对多重数起伏不敏感. 对于低的平均多重数,即使各个事件之间的多重数起伏不定,这一现象仍然主要由统计涨落决定.

关键词 erraticity 多重数分布 动力学涨落