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Structure of Nuclei in Lead Region (I) Spectra of $^{207-210}$Tl and ^{207,208}Hg

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The spectra of $^{207-210}$Tl and ^{207,208}Hg are calculated in terms of the shell model with different interactions. The one-particle one-hole ($1p$–$1h$) excitation is taken into account for ^{207}Tl and ^{208}Tl. The calculated results for low-lying states agree with the available experimental data very well, which shows that those effective interactions are still valid to the nuclei such as ^{208}Tl. Some levels of ^{208,209}Tl whose spins and parties have not been determined in the experiment are predicted. The spectra of ^{207}Hg and ^{208}Hg are calculated and can be used to guide the feature experiments.

Key words: shell model, effective interaction, nuclear structure, β decay.

1. INTRODUCTION

The properties of nuclei around double magic nucleus ^{208}Pb can be described successfully in the shell-model. The lead isotopes with $A < 208$ have been studied successfully by McGrory and Kuo [1] by using Kuo-Herling interaction (KHI) [2]. Warburton and Brown performed calculations for $^{210-212}$Pb and other isotopes with $A = 210$–212 by using an improved Kuo-Herling interaction (KHHe and KHPe), recently [3]. Poppelier and Glaudemans gave a consistent description of ^{205,207}Tl, $^{206-209}$Pb, and ^{209}Bi [4] using a simple surface delta interaction (SDI) [5] whose parameters were determined by a least-square fit to 74 experimental energies in $A = 207$–209 nuclei. This interaction which includes $1p$–$1h$ excitation components in the wave functions has been proved to be a successful one. However, only few results are available for the nuclei such as $^{207-210}$Tl and ^{207,208}Hg. Shell model calculations are

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more difficult for this kind of nuclei because the valence particles (neutron) and hole (protons) fill in two different major shells which are separated by 82–126 major shells. Recently, the properties of this kind of nuclei invoke interests in experiments. For example, the half-life of ^{208}Hg measured by Zhang Li *et al.* [6] was much longer than that of ^{206}Hg. However, experimental data on these nuclei are very scarce. As an example, we consider ^{208}Tl. There are only six energy levels which were determined in the experiment, so that analyzing the β decay of ^{208}Hg becomes difficult. It is significant to predict theoretically the spectra and wave functions of these nuclei. This calculation can also be used to test the effective interactions which are used in the lead region. The β decay could be a powerful tool to test the interactions. The states of the daughter nuclei via the β decay of their parent nuclei are lying in relatively high energy and are very sensitive to the interactions. In this paper the calculations is performed by using the shell model code OXBASH [7].

2. MODEL SPACE AND THE EFFECTIVE INTERACTIONS

There are two kinds of effective interactions used in this mass region. As the first choice, we take the modified surface delta potential $V^{\rm SDI}$ of the form [5]

$$V^{\rm SDI}(\boldsymbol{r}_1, \boldsymbol{r}_2) = -4\pi A_T \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\delta(\boldsymbol{r}_1 - R), \tag{1}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors of the interacting particles, respectively; $R = r_0 A^{1/3}$ is nuclear radius and the quantity A_T is the strength parameter. This potential is very successful in describing lead isotopes with $A < 208$ [8]. The other effective interaction used is the Kuo-Herling interaction which was derived by the reaction matrix technique from a free nucleon-nucleon potential with renormalization [2]. The corresponding two-body matrix elements (TBME) is a sum of a leading bare term and a modified term due to the one-particle one-hole core polarization, i.e.,

$$TBME(\text{K–H}) = ME_{\rm bare} + K_{ph} ME_{1p\text{-}1h}, \tag{2}$$

where K_{ph} is the strength of the $ME_{1p\text{-}1h}$ term.

One must be very careful in selecting the model space to describe these nuclei because of the difficulty of calculation. To calculate ^{208}Tl, Kim and Rasmussen truncated the model space to the two lowest single-particle orbits for proton hole and neutron particle, which is too small for our opinion. The Kuo-Herling model space consists of full 50–82 major shells for the proton hole and 126–184 major shells for the neutron particle [2]. We carry out a calculation for all these nuclei in the Kuo-Herling model space (refer to Eq.(1)). However, this model space cannot incorporate particle hole (p–h) excitations. In order to incorporate the one-particle one-hole ($1p$–$1h$) and two-particle two-hole ($2p$–$2h$) excitations in the calculation for the nuclei with A = 207–209, Poppelier *et al.* selected a model space consisting 14 orbits, i.e., 6 nearest particle orbits and 8 hole orbits with respect to ^{208}Pb core [4]. We calculate ^{207,208}Tl in this model space by considering at most $1p$–$1h$ excitation (refer to Eq.(2)).

For Eq.(1), we combine KHHe and KHPe [3] interactions as well as their connection the modified surface delta interaction (MSDI) [5] whose parameters are determined by fitting the energy levels of ^{208}Tl [10]. This combined interaction is denoted as KHMSDI. For Eq. (2), we use two kinds of interactions, i.e., SDI of Poppelier *et al.* [4] and PKH in Ref. [11]. In order to incorporate with $1p$–$1h$ excitations, the single-particle energies of PKH are adjusted to provide the best descriptions of the low-lying energy levels of ^{207}Tl, ^{207}Pb, ^{209}Pb, ^{209}Bi, and ^{208}Pb. As pointed out by Warburton [11] that the Coulomb energy was not treated consistently in the SDI of Poppelier [4] so that interaction cannot be used for the β decays of ^{208}Tl, ^{207}Hg, and ^{208}Hg, since in the wave functions of ^{208}Pb, ^{207}Tl, and ^{208}Tl, the $1p$–$1h$ excitation components depend sensitively on the relative energies of proton and neutron excitations.

3. CALCULATED SPECTRA AND WAVE FUNCTIONS

The resultant spectra are given in Figs. 1–5. The corresponding experimental data are also shown in the figures. The discussions for individual nuclei are shown below.

3.1. Nucleus ^{207}Tl

The low-lying states of ^{207}Tl were discussed by Poppelier *et al.* in Ref. [4] with SDI. The low-lying states can be considered as a proton-hole state of $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, or $1h_{11/2}$. This nucleus is a suitable candidate in investigating the $1p$–$1h$ effect on the low-lying states. There are 91%, 93%, 85%, and 84% single-proton hole components [3] in the $1/2^+$, $3/2^+$, $5/2^+$, and $11/2^-$ in the calculation with SDI. The corresponding results with PKH are 95%, 96%, 90%, and 95%, respectively. The single-proton hole components calculated with SDI and PKH are very close for the positive parity states and both results can be considered as pure single-proton hole states. The $1p$–$1h$ excitations are not important for these low-lying normal parity states. But for the negative parity state $11/2^-$, two results have an 11% difference. We will give more discussions in the following.

The ground-state of ^{207}Hg can decay via first-forbidden β transition into the states $7/2^-$, $9/2^-$, or $11/2^-$ of ^{207}Tl [12]. The calculated spectra with SDI and PKH are given in Fig. 1. The experimental data are also shown in Fig. 1. The positions of these states calculated with SDI and PKH are similar and close to the experimental data. However, the wave functions obtained by using two different interactions deviate greatly. For the $7/2^-$ state, both the wave functions calculated with SID and PKH are dominated by $1p$–$2h$ components, but the result with PKH has a larger neutron $1p$–$1h$ component than that with SDI. For the wave function of the first $9/2^-$ state, two interactions give very different results. The wave function obtained by using SDI has almost equal weighted $1p$–$2h$ and neutron $1p$–$1h$ components, while the wave function calculated with PKH is dominated by $1p$–$2h$ components. The wave function of the first $11/2^-$ state calculated with SDI has a larger neutron $1p$–$1h$ component than that calculated with PKH. These different structures of the negative parity wave functions arise from the different treatments of the Coulomb energy in the interactions with SDI and PKH. The $1p$–$1h$

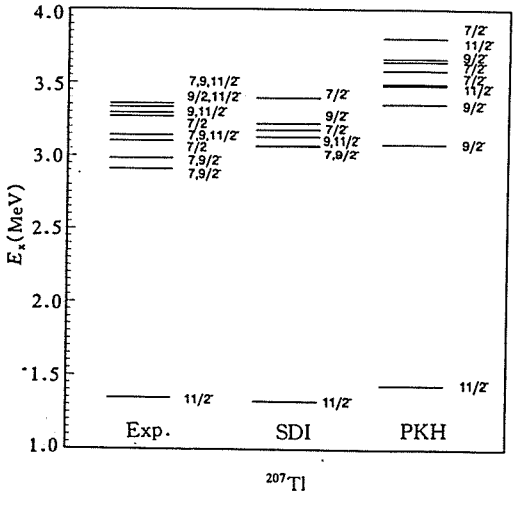

Fig. 1
The spectra of ^{207}Tl for the negative parity states.

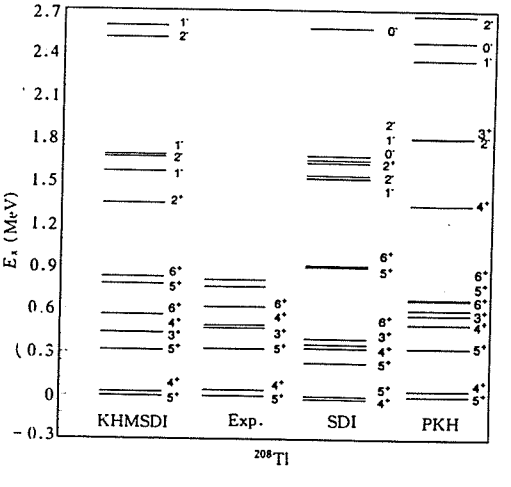

Fig. 2
The spectra of ^{208}Tl.

excitation component is very important for the negative parity state and very sensitive to the relative proton and neutron excited energies. The first-forbidden β transition matrix elements can give a more sensitive test to the wave functions obtained by different interactions.

3.2. Nucleus ^{208}Tl

^{208}Tl is a typical nucleus discussed in this paper. It has one proton hole in the 50–82 shell and one neutron in the 126–184 shell. The experimental [13] and calculated spectra are given in Fig. 2. In the experiment, eight low-lying levels and two top levels have no spin-parity (J^π) assignments. As far as the low-lying states, all the calculations agree with the experimental data well, especially the results with KHMSDI and PKH. According to the calculation, the spin-parities (J^π) of the two unassigned states should be 5^+ and 6^+, respectively. Although the spectra are similar, the resultant wave functions with different interactions have some differences. The wave functions calculated with KHMSDI and PKH have similar structures. For example, the ground-state 5^+ and the first excited state 4^+ are dominated by the configuration $\pi 3s_{1/2}^{-1}\nu 2g_{9/2}$ in the calculations with KHMSDI and PKH, but have large differences from those with SDI which gives strong configuration mixing. This is not surprising, because the KHMSDI and PKH have the same parts of the pp (hole) and nn interactions. It is difficult to say whether SDI or PKH is better. The electromagnetic moments and transitions could be a sensitive test for the interaction. However, there are not enough experimental data on these properties. According to the wave functions calculated with KHMSDI and PKH, 5^+ and 4^+ are the splitting of the $(\pi s_{1/2}^{-1}\nu g_{9/2})$ and $(\pi d_{3/2}^{-1}\nu g_{9/2})$, the 3^+ and 6^+ are stretch states of $(\pi d_{3/2}^{-1}\nu g_{9/2})$. The wave functions obtained by SDI have complex structures, and the configuration mixings are strong. All the interactions give small portions of $1p$–$1h$ excitation components in these states, so $1p$–$1h$ excitations are not important for these low-lying states.

To date, there are no experimental data available for the negative states of ^{208}Tl. However, the negative states 0^-, 1^-, and 2^- are essential to explain the β decay of ^{208}Hg, just like the states $7/2^-$, $9/2^-$, and $11/2^-$ of ^{207}Tl which are essential to explain the β decay of ^{207}Hg as mentioned in the first part of this section. We predict these levels with three interactions. The 1^- and 2^- states calculated with KHMSDI are the splitting of the configurations $(\pi 1h_{11/2}^{-1}\nu 1i_{11/2})$ and $(\pi 1h_{11/2}^{-1}\nu 2g_{9/2})$ and the 0^- state is dominated by the configuration $(\pi 1h_{11/2}^{-1}\nu 1i_{11/2})$. Although the 0^- states calculated with SDI and PKH are dominated by neutron $1p$–$1h$ excitations, the structures of wave function are quiet different. Similar to the result with KHMSDI, the first 1^- and 2^- states calculated by PHK are dominated by the $(\pi h_{11/2}^{-1}\nu g_{9/2})$ configurations, while those calculated with SDI are dominated by neutron $1p$-$1h$ excitation components. For many higher-lying states, all the wave functions have complex structures.

3.3. Nuclei $^{209\text{-}210}$Tl

It is more difficult to carry out calculations for $^{209\text{-}210}$Tl with the $1p$–$1h$ excitation effect accounted. It has been shown above that the $1p$–$1h$ effect is not important for the low-lying normal parity states of ^{207}Tl and ^{208}Tl. We neglect this $1p$–$1h$ excitation effect in calculating the spectra of ^{209}Tl and ^{210}Tl with KHMSDI. The calculated results are given in Figs. 3 and 4.

For ^{209}Tl, the energy levels whose spin-parities have been assigned in the experiment are similar to the single-particle states of ^{207}Tl. This suggests that the these states should be caused by the excitations of the single-proton hole. According to the calculated wave functions, the $1/2^+$ state has a 97% probability to be $\pi s_{1/2}^{-1}$, and the $3/2^+$ state has an 82% probability to be $\pi d_{3/2}^{-1}$. They are quite pure single-proton hole excited states. For $5/2^+$ states, the first resultant state is a state coupling a $\pi s_{1/2}^{-1}$ with 2^+ of ^{206}Hg; the second $5/2^+$ state corresponds to the experimental single-proton hole state $\pi d_{5/2}^{-1}$. These show that the p-n interaction can modify the wave function structures. $11/2^-$ is a good pure $\pi h_{11/2}^{-1}$ state because $h_{11/2}$ is an intruded single-particle orbit.

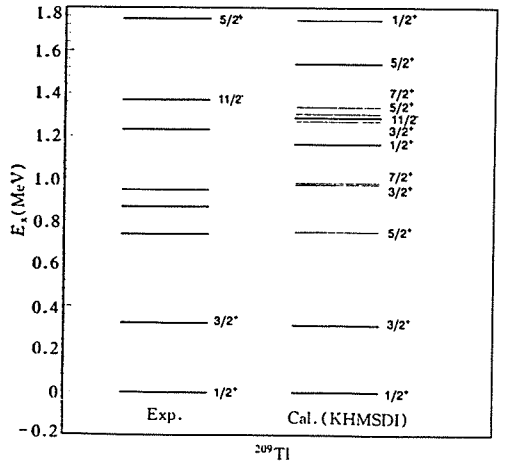

Fig. 3
The spectra of ^{209}Tl.

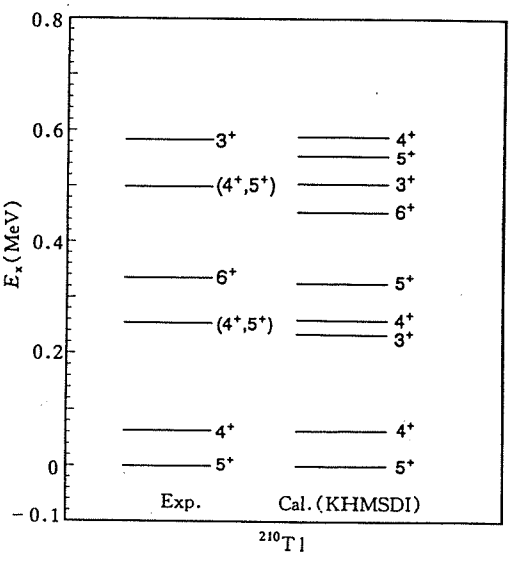

Fig. 4
The spectra of ^{210}Tl.

Spin-parity assignments have not been given of the energy levels of ^{209}Tl lying at 0.739, 0.868, 0.947, and 1.230 MeV. According to our calculation, the state lying at 0.739 MeV might be $5/2^+$, 0.868 might be $3/2^+$ or $7/2^+$, 0.947 MeV might be $1/2^+$, $3/2^+$, or $7/2^+$, and 1.230 MeV might be $1/2^+$, $3/2^+$, $5/2^+$, or $7/2^+$. More experiments should be done to determine the spins and parties of these states.

The spectra of ^{210}Tl is shown in Fig. 4. The two lowest states calculated (5^+ and 4^+) are in noticeable agreement with the experiments. The wave functions of these two states are dominated by the $\pi s_{1/2}^{-1}\nu g_{9/2}^3$, their probabilities are 61 and 56%, respectively, and these states have the similar structures to those of ^{208}Tl. Though the higher excited states calculated deviate from experiments, the agreement is still sufficiently good.

3.4. Nuclei ^{207,208}Hg

There are few experimental data available for ^{207}Hg and ^{208}Hg. By using the interaction KHMSDI, we give the calculated results of these two nuclei in Fig. 5. The spectra of ^{207}Hg are quite similar to those of ^{209}Tl. The states $9/2^+$ and $15/2^-$ are good pure single-neutron states $\nu g_{9/2}$ and $\nu j_{15/2}$ with corresponding probabilities being 88 and 80%, respectively. But for the $1/2^+$, $3/2^+$, $5/2^+$, and $11/2^+$ states, the wave functions have more configuration mixings.

The p–n interaction is much stronger in ^{208}Hg than that in other nuclei mentioned in this paper. This causes strong configuration mixings even in the ground-state wave function. An accurate ground-state wave function is essential for determining the half-life of ^{208}Hg. The strong p–n interaction also pushes down the first 2^+ state to 0.640 MeV which is much lower than that of ^{206}Hg which lies at 1.06 MeV.

We have discussed the low-lying states of $^{207\text{-}210}$Tl and ^{207,208}Hg and their wave functions. The calculated results for ^{207,208}Tl show that the particle-hole excitations have little effect on the low-lying normal parity states. However, in the case of the negative parity states, the particle-hole excitations play an important role. These states are essential to the first forbidden β decay of ^{207,208}Tl. The wave

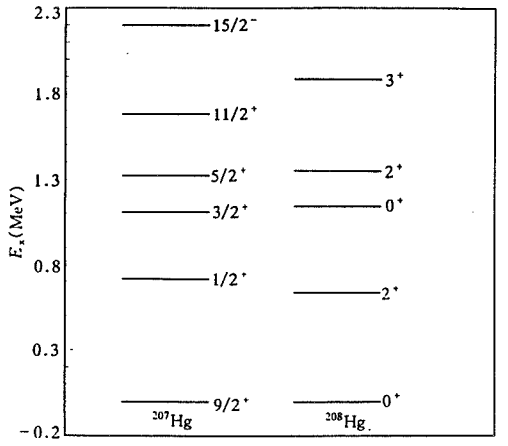

Fig. 5
The spectra of ^{207}Hg and ^{208}Hg calculated with KHMSDI.

function structures calculated with PKH and KHMSDI are similar but have large differences from those obtained with SDI, especially for the negative parity states. These differences come from the different treatments of the Coulomb interaction. The components in the negative parity states depend sensitively on the relative energies of proton and neutron excitations. So SDI is not suitable to describe the negative parity states and the first forbidden β decays of ^{208}Tl and ^{207,208}Hg.

4. CONCLUSIONS

We calculate the spectra of $^{207\text{-}210}$Tl and ^{207,208}Hg by using some interactions which are used in the lead region. In general, all these interactions can give good agreements with experiments for the low-lying states, but the KHMSDI and PKH are even more reasonable in describing these nuclei. We also predict the spins and parities of some states whose spins and parities are not determined in the experiment. These predictions can be used as guides in the experimental feature. We calculate the negative states of ^{207}Tl and ^{208}Tl. These states are important to the β decays of their parent nuclei. The discussion of the first-forbidden β decay is shown in Ref. [14].

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