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Z_n 格点模型的量子仿射变形代数

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摘 要

应用两种不同的星三角关系及其对应的 Boltzmann 面权，通过反对称聚合，构造出了在椭圆情形下的 q 变形仿射代数.

关键词 仿射代数，Boltzmann 权，椭圆.

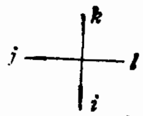
1 引 言

七十年代分别蓬勃发展的经典完全可积场[1]，精确可解统计模型[2]，与无穷维仿射代数[3]三领域，丰富和深化了对无穷多自由度体系的认识. 八十年代对完全可积经典场无穷维代数[4]的研究开始逐渐揭示出三领域交叠蕴含有深刻的联系. 首先引起普遍注意的是量子化共形场[5]与统计模型都有仿射代数的特征[6]. 随后又发现共形场与统计模型转移矩阵有同样的杨-Baxter 关系[7]. 这意味着二者均具有 q 变形代数（量子代数）的对称性[8].

探讨无穷多自由度体系规律的这三领域仍在相互渗透促进下继续向前发展. 最近的重要进展之一是京都组[9]发现了半无穷格点上三角函数类六顶角模型(或 XXZ 链)在主 regime 下具有仿射（即中心扩张）q 变形代数对称[10,11]. 而不只是如同在一定边界条件下的有限链那样[12]只具有不含中心扩张的 q 变形代数对称性. 他们利用晶体化基[13]讨论了如何用物理模型实现 q 仿射代数的无穷维表示空间，并应用允许组态表示空间的顶角算子(动力学算子)给出了三角类统计模型以及相应的 Thirring, Sine-Gordan 场动力学机制的完美表述. 从而由 q 变形 KZ 方程推出关联函数的差分方程，系统而有效地解出了相应的物理量. 他们还得到了用椭圆函数表达 Boltzmann 权的更普遍一般的八顶角(对应于 XYZ 链)顶点模型与面模型的差分方程及其解. 我们发现这一普遍情况所蕴含的动力学规律可表示为椭圆型 q 变形的仿射代数、即 sklyanin 代数[14-16]的中心扩张.

2 模型与背景组态

Belavin[17] $Z_n \otimes Z_n$ 顶角模型指晶格各边上的组态变量 k 取值于 Z_n, $0 \leqslant k \leqslant n$, 而

顶点  的 Boltzmann 权为:

$$R_{ij,kl}(z)=\begin{cases}h(z)\theta^{(i-j)}(z+\omega)/\theta^{(i-j)}(\omega)\theta^{(k-j)}(z) \\ \qquad\qquad \text{如 } i+j=k+l \bmod n \\ 0 \qquad\qquad \text{如 } i+j\neq k+l \bmod n\end{cases} \tag{1}$$

式中

$$h(z)=\prod_{j=0}^{n-1}\theta^{(j)}(z)\Big/\prod_{j=1}^{n-1}\theta^{(j)}(0)$$

$$\theta^{(j)}(z)=\theta\begin{bmatrix}\frac{1}{2}-\frac{j}{n}\\ \frac{1}{2}\end{bmatrix}(z,n\tau)$$

$$\theta\begin{bmatrix}r\\ s\end{bmatrix}(z,\tau)=\sum_{m\in Z}\exp(\pi i\tau(m+r)^2+2\pi i(m+r)(z+s)). \tag{2}$$

z 为谱参数，w 为交叉参数。注意本文的 R 与 Jimbo、Miwa、Okado[18,19] 相同，取为 Richey、Tracy[20] 的转置。$R(z)$ 满足杨-Baxter 方程:

$$\begin{aligned}&R_{12}(z_1-z_2)R_{13}(z_1-z_3)R_{23}(z_2-z_3)\\ &=R_{23}(z_2-z_3)R_{13}(z_1-z_3)R_{12}(z_1-z_2).\end{aligned} \tag{3}$$

RSOS[19] 与循环[21]面模型的组态用与顶角模型对偶的晶格上各格点的(λ)来标记，

$$\begin{aligned}&(\lambda)=(l_0,\cdots,l_{n-1})\in Z_N^{\otimes n}\\ &(\lambda)=(\lambda')\text{mod}(l,\cdots,1)\\ &w=\frac{1}{N},\ N=\text{Coxeter 数 } n+\text{水平 } l\end{aligned} \tag{4}$$

RSOS 的 λ 为权格 $P^+(n,l)$ 上的矢量，即:

$$\begin{aligned}\lambda&=\sum_{\mu=1}^{n-1}(l_{\mu-1}-l_\mu)\Lambda_\mu+(-l_0+l_{n-1}+l)\Lambda_0\\ &=l\Lambda_0+\sum_{\mu=0}^{n-1}l_\mu\hat{\mu}.\end{aligned} \tag{5}$$

式中 Λ_μ 为 $A_{n-1}^{(1)}$ 的基本表示的支配权，$\hat{\mu}$ 为 A_{n-1} 的矢量表示的各权。

$$\begin{aligned}&\Lambda_{\mu+1}=\Lambda_\mu+\hat{\mu},\\ &\hat{\mu}=\varepsilon_\mu-\frac{1}{n}\sum_{\nu=0}^{n-1}\varepsilon_\nu,\\ &\langle\varepsilon_\mu,\varepsilon_\nu\rangle=\delta_{\mu\nu},\\ &\Lambda_\mu=\bar{\Lambda}_\mu+\Lambda_0,\\ &\langle\Lambda_0,\varepsilon_\mu\rangle=0.\end{aligned} \tag{6}$$

$\varepsilon_\mu(0\leqslant\mu\leqslant n-1)$ 为正交归一基矢。注意到在(4)式的标记:

$$
\begin{aligned}
&\Lambda_\mu = (1,\cdots,1,0,\cdots,0) \\
&l_\nu = 1 \qquad\qquad \nu < \mu \\
&l_\nu = 0 \qquad\qquad \nu \geqslant \mu
\end{aligned}
\tag{7}
$$

而 $\Lambda_\mu + \hat{\nu}$ 指

$$
\underset{0,\cdots,\mu-1,\mu,\cdots,n-1}{(1,\cdots,\ 1,\ 0,\cdots,\ 0)} + \underset{0,\cdots,\nu-1,\nu,\nu+1,n-1}{(0,\cdots,\ 0,1,\ \ 0,\cdots,0)}
$$

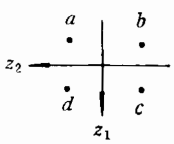
对于 RSOS 时，l_μ 限在 $l_1 \geqslant l_2 \geqslant \cdots \geqslant l_n$，$l \geqslant l_0 - l_{n-1}$ 即 $l_\mu(\mu > 0)$ 对应于杨图的行长．为了给出面 Boltzmann 权，Jimbo 等引入[18,19]：

$$
\begin{aligned}
\lambda_\mu &= l_\mu - \frac{1}{n}\sum_{\nu=0}^{n-1} l_\nu + w_\mu, \\
\lambda_{\mu\nu} &= \lambda_\mu - \lambda_\nu.
\end{aligned}
\tag{8}
$$

当 w_μ 取 generic 值时，对应于循环面模型；当 $w_\mu = \dfrac{n-1}{2} - \mu$，对 应 于 RSOS 模型．这时 λ_μ 可表为

$$
\lambda_\mu = \langle \lambda + \rho, \hat{\mu} \rangle,
$$

其中 $\rho = \sum\limits_{\mu=0}^{n-1} \Lambda_\mu$，$\widehat{\mu + n} = \hat{\mu} - \delta$，$\langle \Lambda_0, \delta \rangle = 0$．

在对偶晶格上 

态的面 Boltzmann 权为 $W\begin{pmatrix} a & b \\ d & c \end{pmatrix}\left|\dfrac{z_1 - z_2}{w}\right)$．当

$(a,b),(b,c),(a,d),(d,c)$ 都为允许对，即 $b-a, c-b, d-a, c-d$ 为某一$\hat{\mu}$时有：

$$
\begin{aligned}
&W\left(\begin{matrix} a & a+\hat{\mu} \\ a+\hat{\mu} & a+2\hat{\mu} \end{matrix}\middle|\,\mu\right) = \frac{[1+u]}{[1]} \\
&W\left(\begin{matrix} a & a+\hat{\mu} \\ a+\hat{\mu} & a+\hat{\mu}+\hat{\nu} \end{matrix}\middle|\,\mu\right) = \frac{[a_{\mu\nu} - u]}{[a_{\mu\nu}]} \quad \mu \neq \nu \\
&W\left(\begin{matrix} a & a+\hat{\mu} \\ a+\hat{\nu} & a+\hat{\mu}+\hat{\nu} \end{matrix}\middle|\,\mu\right) = \frac{[u]}{[1]}\frac{[a_{\mu\nu}+1]}{[a_{\mu\nu}]} \quad \mu \neq \nu
\end{aligned}
\tag{9}
$$

其它情况下$W\left(\begin{matrix} a & b \\ d & c \end{matrix}\middle|\, u\right) = 0$．其中 $[u] = \theta\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(uw, \tau)$．注意本文标记与参考文献[18]中的标记东北与西南角对掉．面权满足三角星关系：

$$
\begin{aligned}
&\sum_g W\left(\begin{matrix} b & g \\ c & d \end{matrix}\middle|\, z_1 - z_2\right) W\left(\begin{matrix} a & f \\ b & g \end{matrix}\middle|\, z_1 - z_3\right) W\left(\begin{matrix} f & e \\ g & d \end{matrix}\middle|\, z_2 - z_3\right) \\
&= \sum_g W\left(\begin{matrix} a & g \\ b & c \end{matrix}\middle|\, z_2 - z_3\right) W\left(\begin{matrix} g & e \\ c & d \end{matrix}\middle|\, z_1 - z_3\right) W\left(\begin{matrix} a & f \\ g & e \end{matrix}\middle|\, z_1 - z_2\right).
\end{aligned}
\tag{10}
$$

面顶角互换关系为：

$$\sum_{\gamma,\delta} R(z_1 - z_2)_{\alpha\beta,\gamma\delta} \varphi^{\gamma}_{a,a+\hat{\mu}}(z_1) \otimes \varphi^{\delta}_{a+\hat{\mu},a+\hat{\mu}+\hat{\nu}}(z_2)$$

$$= \sum_{k} \varphi^{\alpha}_{a+\hat{k},a+\hat{\mu}+\hat{\nu}}(z_1) \otimes \varphi^{\beta}_{a,a+\hat{k}}(z_2) W \begin{pmatrix} a & a+\hat{\mu} \\ a+\hat{k} & a+\hat{\mu}+\hat{\nu} \end{pmatrix} \left| \frac{z_1 - z_2}{w} \right). \tag{11}$$

式中 $\varphi^{\alpha}_{a,a+\hat{\mu}}(z) = \theta^{(\alpha)}(z - nwa_{\mu})$。

如果存在 $\bar{\varphi}_{a,a+\hat{\mu}}(z)$，它与 $\varphi_{a,a+\hat{\mu}}(z)$ 满足正交归一关系[21—23]:

$$\sum_{\alpha} \bar{\varphi}^{\alpha}_{a,a+\hat{\mu}}(z) \varphi^{\alpha}_{a,a+\hat{\nu}}(z) = \delta_{\mu\nu}$$

$$\sum_{\mu} \varphi^{\alpha}_{a,a+\hat{\mu}}(z) \bar{\varphi}^{\beta}_{a,a+\hat{\mu}}(z) = \delta^{\alpha\beta}. \tag{12}$$

则有以下的互换关系式:

$$\bar{\varphi}_{a+\hat{\mu},a+\hat{\mu}+\hat{\nu}}(z_1) \otimes \bar{\varphi}_{a,a+\hat{\mu}}(z_2) R(z_1 - z_2)$$

$$= \sum_{k} W \begin{pmatrix} a & a+\hat{k} \\ a+\hat{\mu} & a+\hat{\mu}+\hat{\nu} \end{pmatrix} \left| \frac{z_1 - z_2}{w} \right) \bar{\varphi}_{a,a+\hat{k}}(z_1) \otimes \bar{\varphi}_{a+\hat{k},a+\hat{\mu}+\hat{\nu}}(z_2) \tag{13}$$

在循环模型，w_j 为 generic，满足(12)式的 φ 是存在的，

$$\bar{\varphi}^{(i)}_{a,a+j}(z) = \frac{B_{ij}}{\det M}.$$

B_{ij} 是矩阵M的伴随矩阵

$$M_{ij} = \varphi^{(i)}_{a,a+j}(z), \quad \det M = c\sigma_0\left(\sum_{j=0}^{n-1} z_j - \frac{n-1}{2}\right) \prod_{i<k} \sigma_0(z_i - z_k) \tag{13a}$$

则(13)式恒成立。RSOS 如果(13)式中出现的 λ 取值于 $P^{+}(n,l)$ 内时，则(13)式互换关系式仍成立。否则，例如附录所示。

对偶表示空间上顶角型 R 矩阵给出如下[10,11]

$$R^{V,V^*}(z) = R^{-1,t_2}(z), \qquad R^{V,{}^*V}(z) = R^{t_2,-1}(z),$$

$$R^{{}^*V,V}(z) = R^{t_1,-1}(z), \qquad R^{V^*,V}(z) = R^{-1,t_1}(z),$$

$$R^{V^*,V^*}(z) = R^{t_1 t_2}(z).$$

根据文献[24—26]的结果可以得到:

$$R^{V,V^*}(z) = R^{-1,t_2}(z) = -\frac{\sigma_0(w)}{\sigma_0(z+w)} \left[\prod_{p=1}^{n-1} \frac{\sigma_0(z-pw)}{\sigma_0(w)}\right]^{-1}$$

$$|\otimes P^{(-)}_{n-1} R^{1,2}(z-w) \cdots R^{1,n-1}(z-(n-1)w)| \otimes P^{(-)}_{n-1},$$

$$R^{V,{}^*V}(z) = R^{t_2,-1}(z)$$

$$= -\frac{\sigma_0(w)}{\sigma_0(z+(n+1)w)} \prod_{p=1}^{n-1} \frac{\sigma_0(w)}{\sigma_0(z+pw)} |\otimes P^{(-)}_{n-1} R^{1,2}(z+(n-1)w) \cdots$$

$$R^{1,n-1}(z+w)| \otimes P^{(-)}_{n-1},$$

$$R^{{}^*V,V}(z) = R^{t_1,-1}(z)$$

$$= -\frac{\sigma_0(w)}{\sigma_0(z+(n+1)w)} \prod_{p=1}^{n-1} \frac{\sigma_0(w)}{\sigma_0(z+pw)} P^{(-)}_{n-1} \otimes | R^{1,n}(z+(n-1)w) \cdots$$

$$R^{n-1,n}(z+w) P^{(-)}_{n-1} \otimes |,$$

$$R^{V^*,V}(z)=R^{-1,t_1}(z)$$
$$=-\frac{\sigma_0(w)}{\sigma_0(z+w)}\prod_{p=1}^{n-1}\frac{\sigma_0(w)}{\sigma_0(z-pw)}P_{n-1}^{(-)}\otimes|R^{1,n}(z-w)\cdots$$
$$R^{n-1,n}(z-(n-1)w)P_{n-1}^{(-)}\otimes|,\tag{14}$$
$$R^{V^*,V^*}(z)=R^{t_1t_2}(z)=\prod_{p=1}^{n-2}\frac{\sigma_0(w)}{\sigma_0(z+pw)}|\otimes P_{n-1}^{(-)}R^{V^*,2}(z+(n-3)w)\cdots$$
$$R^{V^*,n}(z-w)|\otimes P_{n-1}^{(-)}.$$

对偶表示面模型的组态与权给出如下,其中 RSOS 与循环面模型的组态取值在于 (λ)对偶的 (λ^*) 中:

$$\begin{aligned}&(\lambda^*)=(l_0^*,\cdots,l_{n-1}^*)\in Z_N^{\otimes n},\\&(\lambda^*)=(\lambda'^*)\quad \mathrm{mod}(1,\cdots,1).\end{aligned}\tag{15}$$

而 λ^* 可以由对偶的支配权 Λ_μ^* 表达为:

$$\lambda^*=\sum_{\mu=1}^{n-1}(l_{\mu-1}^*-l_\mu^*)\Lambda_\mu^*+(-l_0^*+l_{n-1}^*+l^*)\Lambda_0$$
$$=l^*\Lambda_0+\sum_{\mu=0}^{n-1}l_\mu^*\hat{\mu}^*,$$
$$\Lambda_\mu^*\equiv\Lambda_0+\sum_{\nu=0}^{\mu(n-1)-1}\hat{\nu}\quad(\mathrm{mod}\delta)$$
$$\equiv\Lambda_0+\sum_{\nu=0}^{n-(\mu+1)}\hat{\nu}$$
$$=\Lambda_0+\sum_{\nu=0}^{\mu-1}\hat{\nu}^*,\tag{16}$$
$$\hat{\mu}^*\equiv\sum_{\substack{\nu=0\\ \nu\neq n-\mu-1}}^{n-1}\hat{\nu}=-(\widehat{n-\mu-1}).$$

可见 $\Lambda_{\mu+1}^*=\Lambda_\mu^*+\hat{\mu}^*$,

即:

$$(\Lambda_\mu^*)=\underset{0,\cdots,\mu-1,\mu,\cdots,n-1}{(0,\cdots,\ 0,\ -1,\cdots,\ -1)}\equiv\underset{0,\cdots,\mu-1,\mu,\cdots,n-1,}{(1,\cdots,\ 1,\ 0,\cdots,\ 0)}$$
$$(\mu^*)=\underset{0,\ \cdots,\mu-1,\mu,\mu+1,\cdots,n-1}{(-1,\cdots,\ -1,\ 0,\ -1,\ \cdots,\ -1)}\equiv\underset{0,\cdots,\mu-1,\mu,\mu+1,\cdots n-1.}{(0,\cdots,\ 0,\ 1,\ 0,\ \cdots,\ 0)}$$

令对偶表示允许 path 为 $(\lambda^*,\hat{\mu}^*)$,当 $\lambda^*-\hat{\mu}^*$ 等于某一 $\hat{\nu}^*$,则(16)式定义的 Λ_μ^* 保证 $\Lambda_0,\cdots\Lambda_{\mu(n-1)},\cdots$ 的长 $\mu(n-1)$的允许 path,反对称聚合为 $\Lambda_0^*,\Lambda_{-1}^*,\cdots\Lambda_{-\mu}^*,\cdots$ 其中 $\Lambda_0=\Lambda_0^*$,第 ν 个是固定 $\Lambda_{\nu(n-1)}$,在 $\Lambda_{\nu(n-1)}$ 与 $\Lambda_{(\nu+1)(n-1)}$ 内取反对称聚合得到. 当 w_j 为 generic 时,可由 $(n-1)\times(n-1)$ 的面权W反对称聚合后将东北与西南对调得到对偶的面权 W^+,

$$W^+\begin{pmatrix}a^* & a^*-\hat{\mu}^*\\ a^*-\hat{\mu}^* & a^*-2\hat{\mu}^*\end{pmatrix}\Bigg|u\Bigg)=\frac{[1+u]}{[1]},$$

$$W^+\begin{pmatrix} a^* & a^*-\hat{\mu}^* \\ a^*-\hat{\mu}^* & a^*-\hat{\mu}^*-\hat{\nu}^* \end{pmatrix}\Big| u\Big) = \frac{[u+a^*_{\mu\nu}]}{[a^*_{\mu\nu}]},$$

$$W^+\begin{pmatrix} a^* & a^*-\hat{\mu}^* \\ a^*-\hat{\nu}^* & a^*-\hat{\mu}^*-\hat{\nu}^* \end{pmatrix}\Big| u\Big) = \frac{[u]}{[1]}\frac{[a^*_{\mu\nu}-1]}{[a^*_{\mu\nu}]}. \tag{17}$$

其中 $a^*_{\mu\nu} = a^*_\mu - a^*_\nu$.

$$a^*_\mu = \langle \Lambda^* + \rho, \hat{\mu}^* \rangle = l^*_\mu - \frac{1}{n}\sum_{\nu=0}^{n-1} l^*_\nu + \left(\mu + 1 - \frac{n-1}{2}\right). \tag{18}$$

并易见 W^+ 满足三角星关系.

定义
$$\varphi^{+k}_{a^*, a^*-\mu^*}(z) \equiv \theta^{(k)}(z + nwa^*_\mu). \tag{19}$$

容易验证 R, W^+ 满足以下的 intertwiner 关系,

$$\sum_{k,l} R(z_1-z_2)_{ij,kl}\varphi^{+k}_{a^*,a^*-\mu^*}(z_1)\otimes\varphi^{+l}_{a^*-\mu^*,a^*-\mu^*-\hat{\nu}^*}(z_2)$$
$$= \sum_k \varphi^{+i}_{a^*-\hat{k}^*,a^*-\mu^*-\hat{\nu}^*}(z_1)\otimes\varphi^{+j}_{a^*,a^*-\hat{k}^*}(z_2)W^+\begin{pmatrix} a^* & a^*-\hat{\mu}^* \\ a^*-\hat{k}^* & a^*-\hat{\mu}^*-\hat{\nu}^* \end{pmatrix}\Big| \frac{z_1-z_2}{w}\Big). \tag{20}$$

如存在 $\bar{\varphi}^+$ 与 φ^+ 正交归一，则有 $\bar{\varphi}^+\bar{\varphi}^+ R = \Sigma W\bar{\varphi}^+\bar{\varphi}^+$. 实际上可证此式在以下的意义上为(10)式的对偶. 为此只要注意到 $R^{i_1 i_2}$ 为 R 的反对称聚合, W^+ 为 W 的反对称聚合.

首先, 定义 $\bar{\varphi}^+_{a^*,a^*+\mu^*}(z)$,

$$\bar{\varphi}^+_{a^*,a^*+\mu^*}(z) = P^{(-)}_{n-1}\varphi^+_{a^*,a^*-\hat{0}^*}(z)\otimes\cdots\otimes\varphi^+_{a^*,a^*-\widehat{\mu-1}^*}(z)$$
$$\otimes\varphi^+_{a^*,a^*-\widehat{\mu+1}^*}(z)\otimes\cdots\otimes\varphi^+_{a^*,a^*-\widehat{n-1}^*}(z).$$

利用 R 与 W^+ 之间的 intertwiner 关系(20)和 R 的反对称聚合结果可得:

$$R^{V^*,V^*}(z_1-z_2)\bar{\varphi}^+_{a^*,a^*+\hat{\mu}^*}(z_1)\otimes\bar{\varphi}^+_{a^*+\hat{\mu}^*,a^*+\hat{\mu}^*+\hat{\nu}^*}(z_2)$$
$$= \sum_k W\begin{pmatrix} a^* & a^*+\hat{\mu}^* \\ a^*+\hat{k}^* & a^*+\hat{\mu}^*+\hat{\nu}^* \end{pmatrix}\Big| \frac{z_1-z_2}{w}\Big)\bar{\varphi}^+_{a^*+\hat{k}^*,a^*+\hat{\mu}^*+\hat{\nu}^*}(z_1)\otimes\bar{\varphi}^+_{a^*,a^*+\hat{k}^*}(z_2). \tag{21}$$

其中*:

$$W\begin{pmatrix} a^* & a^*+\hat{\mu}^* \\ a^*+\hat{k}^* & a^*+\hat{\mu}^*+\hat{\nu}^* \end{pmatrix}\Big| u\Big) = \frac{[u]}{[1]}\frac{[a^*_{\mu k}+1]}{[a^*_{\mu k}]}F_{\mu k} \quad \mu\neq k,$$

$$W\begin{pmatrix} a^* & a^*+\hat{\mu}^* \\ a^*+\hat{\mu}^* & a^*+2\hat{\mu}^* \end{pmatrix}\Big| u\Big) = \frac{[1+u]}{[1]}F_{\mu\mu},$$

$$W\begin{pmatrix} a^* & a^*+\hat{\mu}^* \\ a^*+\hat{\mu}^* & a^*+\hat{\mu}^*+\hat{k}^* \end{pmatrix}\Big| u\Big) = \frac{[a^*_{\mu k}-u]}{[a^*_{\mu k}]}f_{\mu k} \quad \mu\neq k. \tag{22}$$

在(20)式中, 取 $a^* = \Lambda^*_{i+2}, \hat{\mu}^* = \widehat{i+1}^*, \hat{\nu}^* = \hat{i}^*$, 并注意到:

$$W^+\begin{pmatrix} \Lambda^*_{i+2} & \Lambda^*_{i+1} \\ \Lambda^*_{i+2}-\hat{i}^* & \Lambda^*_i \end{pmatrix}\Big| u\Big) = \frac{[u]}{[1]}\frac{[(\Lambda^*_{i+2})_{i+1,i}-1]}{[(\Lambda^*_{i+2})_{i+1,i}]} = \frac{[u][1-1]}{[1]\ [1]} = 0,$$

可以得到:

---

* 其中 $F_{\mu\nu}$, $F_{\mu\mu}$, $f_{\mu\nu}$ 是规范变换的矩阵元, 见附录2.

$$R(z_1 - z_2)\varphi^+_{\Lambda^*_{i+2},\Lambda^*_{i+1}}(z_1)\otimes\varphi^+_{\Lambda^*_{i+1},\Lambda^*_i}(z_2)$$
$$= \frac{\sigma_0(z_1 - z_2 + w)}{\sigma_0(w)}\,\varphi^+_{\Lambda^*_{i+1},\Lambda^*_i}(z_1)\otimes\varphi^+_{\Lambda^*_{i+2},\Lambda^*_{i+1}}(z_2). \tag{23}$$

在(21)式中取 $a^* = \Lambda^*_i, \hat{\mu}^* = \hat{i}^*,\ \hat{\nu}^* = \widehat{i+1}^*$,且注意到:

$$W\begin{pmatrix} \Lambda^*_i & \Lambda^*_{i+1} \\ \Lambda^*_i + \widehat{i+1}^* & \Lambda^*_{i+2} \end{pmatrix}\!\!\left|u\right) = \frac{[u]}{[1]}\frac{[(\Lambda^*_i)_{i,i+1}+1]}{[\Lambda^*_{i,i+1}]} = \frac{[u][-1+1]}{[1]\ [-1]} = 0,$$

可以得到

$$R^{V^*,V^*}(z_1 - z_2)\bar{\varphi}^+_{\Lambda^*_i,\Lambda^*_{i+1}}(z_1)\otimes\bar{\varphi}^+_{\Lambda^*_{i+1},\Lambda^*_{i+2}}(z_2)$$
$$= \frac{\sigma_0(z_1 - z_2 + w)}{\sigma_0(w)}\,\bar{\varphi}^+_{\Lambda^*_{i+1},\Lambda^*_{i+2}}(z_1)\otimes\bar{\varphi}^+_{\Lambda^*_i,\Lambda^*_{i+1}}(z_2). \tag{24}$$

为了在第四节构造仿射椭圆变形代数,运用[27,28]文中的方法,将 $\bar{\varphi}^+$通过角转移矩阵转到 Λ 空间上,即:

$$\bar{\varphi}^+_{\Lambda^*_i,\Lambda^*_{i+1}}(z) \to \bar{\varphi}_{\Lambda_{i+1},\Lambda_i}(z).$$

这样,可以得到 $\bar{\varphi}$ 的交换关系,

$$R^{V^*,V^*}(z_1 - z_2)\bar{\varphi}_{\Lambda_{i+1},\Lambda_i}(z_1)\otimes\bar{\varphi}_{\Lambda_{i+2},\Lambda_{i+1}}(z_2)$$
$$= \frac{\sigma_0(z_1 - z_2 + w)}{\sigma_0(w)}\,\bar{\varphi}_{\Lambda_{i+2},\Lambda_{i+1}}(z_1)\otimes\bar{\varphi}_{\Lambda_{i+1},\Lambda_i}(z_2). \tag{25}$$

3 顶 角 算 子

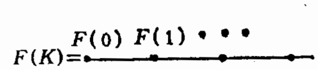
我们考虑的面模型的组态取值于水平为某一给定值 l 的权格 $P(n,l)$ 上,西东,北南相邻点为允许对,当仅考虑一条南北的半无穷链上的组态时,可以引入面型的 path $F(K)$, K 为此半无穷格点的坐标,从南到北为 $0,1,\cdots,M,\cdots\infty$, $F(K)$ 取值于 $P(n,l)$,(本文暂限于考察 $l=1$ 的基本表示.)且相邻北南点为允许对. 在主 regime 下,低温极限时,面模型的组态冻结在基态上. 由于在低温极限时, W^+ 的主要项为 $W^+\begin{pmatrix} \Lambda^*_{i+2} & \Lambda^*_{i+1} \\ \Lambda^*_{i+1} & \Lambda^*_i \end{pmatrix}$, 因此基态 path 为:

$$F_\mu(K) = \Lambda^*_{\mu+K},\ K = 0,1,\cdots,$$

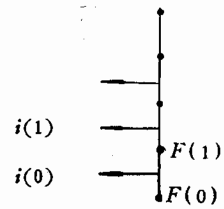
不同的 $\mu, \mu = 0,\cdots,n-1$, 对应于不同的 μ sector 的基态. 同一 μ sector 内的其它态(激发态)在无穷远的边界与 F_μ 一样.

$$F(K) = \Lambda^*_{\mu+K} = F_\mu(K),\ 当\ K \gg 0.$$

则称之为 μ 型的面 path,如果对某一面型 path

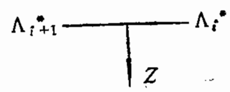


在它的边 $(i, i+1)$ 上放置一个矢量 $\varphi^+_{F(K),F(K+1)}(\xi) \in C^n$,就得到了顶角型 path. 与每一面型 path 空间 $F(K)$ 对应的为 $(C^n)^{\otimes\infty}$ 的顶角型矢量 $P(K)$,用

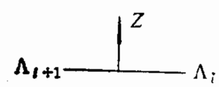


来表示 $P(K)$ 中的 path 态。μ 型面 path 对应于 μ 型顶角型 path。这里，用下图表示第二节的 $\varphi^{+}(z)$ 以及 $\bar{\varphi}(z)$。

$$\varphi^{+}_{\Lambda^*_{i+1},\Lambda^*_i}(z) =$$

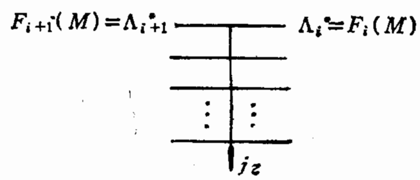


$$\bar{\varphi}_{\Lambda_{i+1},\Lambda_i}(z) =$$

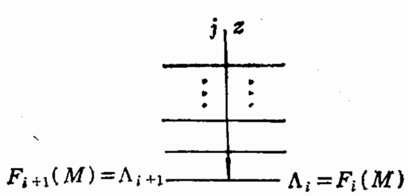


引入作用在 $P(\Lambda^*)$ 空间的算子 $\phi^{+j}_{P(\Lambda^*_{i+1}),P(\Lambda_i)}(z)$ 以及作用在对偶空间 $P(\Lambda)$ 空间上的算子 $\bar{\Phi}_{P(\Lambda_{i+1}),P(\Lambda_i)}(z)$，它们分别用下图表示，

$$\phi^{+j}_{P(\Lambda^*_{i+1}),P(\Lambda^*_i)}(z) =$$



$$\bar{\Phi}^{j}_{P(\Lambda_{i+1}),P(\Lambda_i)}(z) =$$



其中 $M = nZ$。可以看出顶角算子 $\phi^{+}_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_\mu)}$ 作用在 μ 型 path 上给出 $\mu+1$ 型 path；顶角算子 $\bar{\Phi}_{P(\Lambda_{i+1}),P(\Lambda_i)}$ 作用在对偶的 i 型 path 上给出 $i+1$ 型 path，即：

$$P(\Lambda^*_\mu) \xrightarrow{\phi^{+}_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_\mu)}} P(\Lambda^*_{\mu+1}) \otimes V,$$

$$P(\Lambda_i) \xrightarrow{\bar{\Phi}_{P(\Lambda_{i+1}),P(\Lambda_i)}} P(\Lambda_{i+1}) \otimes V^*.$$

利用(23)，(25)式及杨-Baxter 关系，可以得到 ϕ^{+}，$\bar{\Phi}$ 满足以下的交换关系。

$$\sum_{i,j} R_{ij,kl}(z_1 - z_2)\phi^{+i}_{P(\Lambda^*_{\mu+2}),P(\Lambda^*_{\mu+1})}(z_1)\phi^{+j}_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_\mu)}(z_2)$$

$$= \frac{\sigma_0(z_1 - z_2 + w)}{\sigma_0(w)} \phi^{+l}_{P(\Lambda^*_{\mu+2}),P(\Lambda^*_{\mu+1})}(z_2)\phi^{+k}_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_\mu)}(z_1). \tag{26}$$

$$\sum_{l,k} R_{ij,kl}(z_1 - z_2)\bar{\Phi}^l_{P(\Lambda_{\mu+2}),P(\Lambda_{\mu+1})}(z_2)\bar{\Phi}^k_{P(\Lambda_{\mu+1}),P(\Lambda_\mu)}(z_1)$$
$$= \frac{\sigma_0(z_1 - z_2 + w)}{\sigma_0(w)} \bar{\Phi}^i_{P(\Lambda_{\mu+2}),P(\Lambda_{\mu+1})}(z_1)\bar{\Phi}^j_{P(\Lambda_{\mu+1}),P(\Lambda_\mu)}(z_2). \tag{27}$$

(26),(27)式是在有限晶格上得到的，如果取热力学极限（$M \to \infty$）将会出现发散的问题，所以这里需考虑重整化的问题。对于顶角算子 $\bar{\Phi}(z)$，$\phi^+(z)$，取

$$\bar{\Phi}^i_{re}(z_1)\bar{\Phi}^j_{re}(z_2) = \bar{f}(z_1 - z_2)\bar{\Phi}^i(z_1)\bar{\Phi}^j(z_2),$$
$$\phi^{+i}_{re}(z_1)\phi^{+j}_{re}(z_2) = f(z_1 - z_2)\phi^{+i}(z_1)\phi^{+j}(z_2). \tag{28}$$

为方便，后文中出现的 $\bar{\Phi}_{re}$，ϕ^+_{re} 都用 $\bar{\Phi}$，ϕ^+ 来标记。利用两点函数，同时注意到参考文献[27]的结果，可以得到 $\bar{f}(z)$，$f(z)$ 满足差分方程：

$$\frac{\bar{f}(z_1 - z_2 + nw)}{\bar{f}(z_1 - z_2)} = \frac{\sigma_0(z_1 - z_2 + w)}{K(z_1 - z_2)\sigma_0(w)},$$
$$\frac{f(z_1 - z_2)}{f(z_1 - z_2 + nw)} = \frac{\sigma_0(z_1 - z_2 + w)}{K(z_1 - z_2)\sigma_0(w)},$$
$$\bar{f}(z_2 - z_1) = \bar{f}(z_1 - z_2 + nw),$$
$$f(z_2 - z_1) = f(z_1 - z_2 + nw). \tag{29}$$

其中 $K(z)$ 可见参考文献[29]中，经过求解以上差分方程组可得，

$$\bar{f}(z) = \prod_{p=0}^{\infty} \frac{\sigma_0(z + w - pnw)}{K(z - pnw)\sigma_0(w)},$$
$$f(z) = \prod_{p=0}^{\infty} \frac{K(z - pnw)\sigma_0(w)}{\sigma_0(z + w - pnw)}. \tag{30}$$

经过如此重整化后的顶角算子 $\phi^+(z)$，$\bar{\Phi}(z)$ 满足的交换关系为：

$$\sum_{i,j} \bar{R}_{ij,kl}(z_1 - z_2)\phi^{+i}_{P(\Lambda^*_{\mu+2}),P(\Lambda^*_{\mu+1})}(z_1)\ \phi^{+j}_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_\mu)}(z_2)$$
$$= \phi^{+l}_{P(\Lambda^*_{\mu+2}),P(\Lambda^*_{\mu+1})}(z_2)\phi^{+k}_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_\mu)}(z_1), \tag{31}$$

$$\sum_{k,l} \bar{R}_{ij,kl}(z_1 - z_2)\bar{\Phi}^l_{P(\Lambda_{\mu+2}),P(\Lambda_{\mu+1})}(z_2)\bar{\Phi}^k_{P(\Lambda_{\mu+1}),P(\Lambda_\mu)}(z_1)$$
$$= \bar{\Phi}^i_{P(\Lambda_{\mu+2}),P(\Lambda_{\mu+1})}(z_1)\bar{\Phi}^j_{P(\Lambda_{\mu+1}),P(\Lambda_\mu)}(z_2). \tag{32}$$

其中 $\bar{R}(z_1 - z_2) = R(z_1 - z_2)K^{-1}(z_1 - z_2)$，它是重整化后的 Boltzmann 权。

4　椭圆 q 变形仿射代数

从第三节定义的顶角算子 ϕ^+ 可以看出，

$$P(\Lambda^*_i)\xrightarrow{\phi^+} P(\Lambda^*_{i+1})\otimes V \xrightarrow{\phi^+\otimes id} P(\Lambda^*_{i+2})\otimes V^{\otimes 2} \to \cdots \to P(\Lambda^*_i)\otimes V^{\otimes n}$$

经过几个 ϕ^+ 的作用后回到原来的 path sector $P(\Lambda^*_i)$。利用这一结果，可以定义 $P(\Lambda^*_\mu)$ 上的自同态算子 $L^\pm$。如果运用[27,28]文中的方法，可以将$(n-1)$个 ϕ^+ 算子反对称聚合后，通过角转移矩阵顺时针和逆时针转到 $P(\Lambda)$ 空间，这样可以得到作用在 $P(\Lambda^*)\otimes P(\Lambda)$ 空间上的 $L^\pm$ 算子。用角转移矩阵顺时针和逆时针转过来的 $\bar{\Phi}$ 算子，其谱相差

nw，这是由物理模型及它所对应的仿射代数中心值所决定的。这样构造的 $L^{\pm}$ 如下:

$$L_{ij}^{+}(z)=\phi^{+i}\left(z+\frac{nw}{2}\right)\hat{\otimes}\bar{\phi}^{j}\left(z-\frac{nw}{2}\right),$$

$$L_{ij}^{-}(z)=\phi^{+i}\left(z-\frac{nw}{2}\right)\hat{\otimes}\bar{\phi}^{j}\left(z+\frac{nw}{2}\right). \tag{33}$$

其中 $\hat{\otimes}$ 表示量子空间的张量积. 可以得到以下的算子交换关系:

$$\bar{R}_{12}(z_1-z_2)L_1^{\pm}(z_1)L_2^{\pm}(z_2)=L_2^{\pm}(z_2)L_1^{\pm}(z_1)\bar{R}_{12}(z_1-z_2),$$

$$\bar{R}_{12}(z_1-z_2+nw)L_1^{+}(z_1)L_2^{-}(z_2)=L_2^{-}(z_2)L_1^{+}(z_1)\bar{R}_{12}(z_1-z_2-nw). \tag{34}$$

这里只给出 $L^{+}(z)$ 与 $L^{-}(z)$ 交换关系的证明,其它交换关系的证明是类似的.

$$\begin{aligned}
&\bar{R}(z_1-z_2+nw)L_1^{+}(z_1)L_2^{-}(z_2)\\
&=\bar{R}_{k'l',kl}(z_1-z_2+nw)\phi^{+k'}\left(z_1+\frac{nw}{2}\right)\phi^{+l'}\left(z_2-\frac{nw}{2}\right)\\
&\quad\hat{\otimes}\bar{\phi}^{i}\left(z_2-\frac{nw}{2}\right)\bar{\phi}^{j}\left(z_2+\frac{nw}{2}\right)\\
&=\phi^{+l}\left(z_2-\frac{nw}{2}\right)\phi^{+k}\left(z_1-\frac{nw}{2}\right)\hat{\otimes}\phi^{i}\left(z_2-\frac{nw}{2}\right)\bar{\phi}^{j}\left(z_2+\frac{nw}{2}\right)
\end{aligned}$$

这里用到了(32)式。同样应用(31)式,

$$\begin{aligned}
&\phi^{+l}\left(z_2-\frac{nw}{2}\right)\phi^{+k}\left(z_1-\frac{nw}{2}\right)\hat{\otimes}\bar{\phi}^{l'}\left(z_2+\frac{nw}{2}\right)\bar{\phi}^{k'}\left(z_1-\frac{nw}{2}\right)\bar{R}_{ij,k'l'}(z_1-z_2-nw)\\
&=\phi^{+l}\left(z_2-\frac{nw}{2}\right)\phi^{+k}\left(z_1-\frac{nw}{2}\right)\hat{\otimes}\bar{\phi}^{i}\left(z_1-\frac{nw}{2}\right)\bar{\phi}^{j}\left(z_2+\frac{nw}{2}\right).
\end{aligned}$$

这样就证明了 L^{+} 与 L^{-} 之间的交换关系. 因此给出了椭圆 q 变形仿射代数的实现.

下面将给出作为椭圆 q 变形仿射代数的顶角算子 ϕ^{+},ϕ^{-} 与 $L^{\pm}$ 之间的交换关系. 利用(31)(32)式可以得到以下的交换关系.

$$\begin{aligned}
&\sum_{i,j}\bar{R}\left(z_1-z_2\mp\frac{nw}{2}\right)_{ij,kl}(\phi^{+i}_{P(\Lambda^*_{\mu+2}),P(\Lambda^*_{\mu+1})}(z_1)P_{P(\Lambda_{\mu+2}),P(\Lambda_{\mu+1})})L^{\pm il'}(z_2)\\
&\quad=L^{\pm ll'}(z_2)(\phi^{+k}_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_{\mu})}\hat{\otimes}P_{P(\Lambda_{\mu+1}),P(\Lambda_{\mu})}),\\
&\sum_{k,l}\bar{R}\left(z_1-z_2\mp\frac{nw}{2}\right)_{ij,kl}(P_{P(\Lambda^*_{\mu+2}),P(\Lambda^*_{\mu+1})}\hat{\otimes}\bar{\phi}^{k}_{P(\Lambda_{\mu+2}),P(\Lambda_{\mu+1})}(z_2))L^{\pm li}(z_1)\\
&\quad=L^{\pm ji}(z_1)(P_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_{\mu})}\hat{\otimes}\bar{\phi}^{i}_{P(\Lambda_{\mu+1}),P(\Lambda_{\mu})}(z_2)).
\end{aligned} \tag{35}$$

其中 $P_{P(\Lambda^*_{\mu+1}),P(\Lambda^*_{\mu})}=\prod_{k=1}^{\infty}\delta_{P_{\mu+1}(K+1),P_{\mu}(K)}$.

即为 path 空间上东南方向到西北方向的对角矩阵.

附录: 1.

对于(13)式,如果取 $a=\Lambda_i,\hat{\mu}=\hat{i},\hat{\nu}=\widehat{i+1}$,并根据 $\Lambda_{i+1}=\Lambda_i+\hat{i}$, 则(13)式变为:

$$\begin{aligned}
&\bar{\varphi}_{\Lambda_{i+1},\Lambda_{i+2}}(z_1)\otimes\bar{\varphi}_{\Lambda_i,\Lambda_{i+1}}(z_2)R(z_1-z_2)\\
&=W\left(\begin{matrix}\Lambda_i & \Lambda_{i+1}\\ \Lambda_{i+1} & \Lambda_{i+2}\end{matrix}\middle|\frac{z_1-z_2}{w}\right)\bar{\varphi}_{\Lambda_i,\Lambda_{i+1}}(z_1)\otimes\bar{\varphi}_{\Lambda_{i+1},\Lambda_{i+2}}(z_2)
\end{aligned}$$

$$+W\begin{pmatrix} \Lambda_i & \Lambda_i+\widehat{i+1} \\ \Lambda_{i+1} & \Lambda_{i+2} \end{pmatrix}\left|\frac{z_1-z_2}{w}\right)\bar{\varphi}_{\Lambda_i,\Lambda_i+\widehat{i+1}}(z_1)\otimes\bar{\varphi}_{\Lambda_i+\widehat{i+1},\Lambda_{i+2}}(z_2).$$

根据定义$W\begin{pmatrix} \Lambda_i & \Lambda_i+\widehat{i+1} \\ \Lambda_{i+1} & \Lambda_{i+2} \end{pmatrix}\left|\frac{z_1-z_2}{w}\right)=0$,而 $\bar{\varphi}_{\Lambda_i+\widehat{i+1},\Lambda_{i+2}}(z_2)$ 的分母为零,同样只得到方程右边仍为两项。当取 $a=\Lambda_i,\hat{\mu}=\widehat{i+1}$, $\hat{\nu}=\hat{i}$ 得:

$$\bar{\varphi}_{\Lambda_i+\widehat{i+1},\Lambda_{i+2}}(z_1)\otimes\bar{\varphi}_{\Lambda_i,\Lambda_i+\widehat{i+1}}(z_2)R(z_1-z_2)$$

$$=W\begin{pmatrix} \Lambda_i & \Lambda_i+\widehat{i+1} \\ \Lambda_i+\widehat{i+1} & \Lambda_{i+2} \end{pmatrix}\left|\frac{z_1-z_2}{w}\right)\bar{\varphi}_{\Lambda_i,\Lambda_i+\widehat{i+1}}(z_1)\otimes\bar{\varphi}_{\Lambda_i+\widehat{i+1},\Lambda_{i+2}}(z_2)$$

$$+W\begin{pmatrix} \Lambda_i & \Lambda_{i+1} \\ \Lambda_i+\widehat{i+1} & \Lambda_{i+2} \end{pmatrix}\left|\frac{z_1-z_2}{w}\right)\bar{\varphi}_{\Lambda_i,\Lambda_{i+1}}(z_1)\otimes\bar{\varphi}_{\Lambda_{i+1},\Lambda_{i+2}}(z_2).$$

经过运算可将等式右边第二项消掉,但方程的形式已改变。

附录: 2.

经过直接的计算可以得到(22)式中的规范交换矩阵元的具体表达式:

$$F_{\mu k}=\frac{\det(a^*,z_1)\ \det(a^*+\hat{\mu}^*,z_2)}{\det(a^*+\hat{k}^*,z_1)\ \det(a^*,\ z_2)},\quad \mu\neq k$$

$$F_{\mu\mu}=\frac{\det(a^*,z_1)\ \det(a^*+\hat{\mu}^*,\ z_2)}{\det(a^*+\hat{\mu}^*,z_1)\ \det(a^*,\ z_2)}$$

$$f_{\mu k}=\frac{\det(a^*,z_1)\ \det(a^*+\hat{\mu}^*,z_2)}{\det(a^*+\hat{\mu}^*,z_1)\ \det(a^*,\ z_2)},\quad \mu\neq k$$

其中

$$\det(b^*,z)=\det(M_{ij})$$

$$M_{ij}=\phi^i_{b^*,b^*+\hat{j}}(z)$$

由(13a)式的结论,可以知道 $F_{\mu k},F_{\mu\mu},f_{\mu k}$ 不依赖于谱参数 z_1,z_2。

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q-deformed Quantum Affine Algebra in Z_n Lattice Model

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Abstract

The Z_n-Baxter model in the principal regime is studied, and it is verified that two different Boltzmann weights are conjugate to each other. Using the two different startriangular relations and antisymmetric fusion, a realization of q-deformed quantum affine algebra is obtained.

Key words affine algebra, Boltzmann weight, elliptic. function.