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Effective Quark-Quark Interaction Created by Quark-Antiquark Pair Excitation

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By means of the quark-antiquark pair creation model via one-gluon exchange, the effective interaction among quarks, which is equivalent to the meson exchange intervening between two hadrons, is discussed. It is shown that this process includes interactions not only between two quarks, but also among three quarks. Obviously, such effective interaction is more general than one gluon exchange potential (OGEP).

1. INTRODUCTION

In recent years, significant developments on the investigation of the nuclear force have been made by employing the quark potential model (QPM). The short-range repulsive feature of the nuclear force has been obtained in terms of the one-gluon-exchange potential (OGEP) of the Breit-Fermi type and the effect of the quark-exchange intervening between two nucleon clusters [1]. The interaction which is equivalent to the meson exchange potential in the meson exchange theory of the nuclear force can be given by exciting a pair of quark-antiquark via one-gluon exchange [2,3]. Thus, physicists are encouraged to study the light-quark systems further in the framework of QPM. Although QPM has successfully been applied to the description of the hadronic spectrum and nuclear scattering in some extend, lots of serious problems have not been solved yet. For instance, what is the unified form of the spin-orbital forces required in the hadronic spectrum and nuclear scattering? Does the color screening effect exist in the color confinement? How is it described? Are there any ways to reveal the medium-range character of the nuclear force in the quark degree of freedom? These important problems should have been intensively studied by now.

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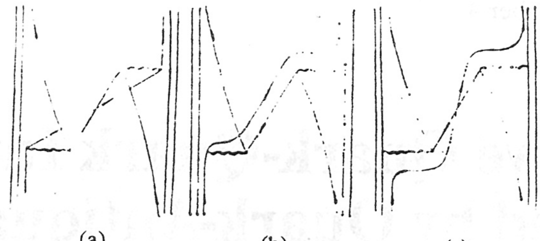


Fig. 1
Sea-quark effect of meson exchange between nucleon-nucleon interactions.

In ordinary QPM calculations, the employed V_{ij}^{OGEP} is $\lambda_i \cdot \lambda_j$, namely the color degree of freedom of interaction quarks, dependent and has typical spin, flavor and orbital structures. This is because that in V_{ij}^{OGEP}, one only considers one-gluon exchange between two quarks. However, if one further accounts for the multi-gluon exchange process, the expression of the interaction must show more complicated color, spin and orbital structures. Calculating high-order gluon exchange diagrams is rather difficult. We propose that the two gluon process where sea quarks are excited is an important mechanism because the meson exchange effect provides the medium and long-range parts of the nuclear force. In order to obtain the effect in the quark degree of freedom, which is equivalent to the effect provided by the meson exchange, one has to include the sea quark effect. In a previous paper, we obtained the effective meson exchange potential in terms of the sea quark pair creation and annihilation via two transition potential [2], indicating the importance of creating and annihilating sea quark pairs by applying the transition potential twice. In order to simplify the calculation and compare the result with that obtained from the meson exchange theory, we also derived the hadron-meson vertex function to describe the process in which a quark in the hadron excites a pair of quark-antiquark via the transition potential. Then, by using this vertex function, we approximately wrote the above-mentioned two-gluon process into the form of the effective meson exchange interaction intervening between two hadrons. Unfortunately, because of this approach, the details of the two-gluon exchange process were concealed, and some effects were lost - for instance, the effects from the hidden color states, which are considered as intermediate states, the possibly formed six-quark cluster, the quark exchange between two hadron clusters and etc. It is necessary to express the sea quark excitation process in the form of the interaction intervening between quarks. This two-gluon exchange process may give diversified color, flavor, spin and orbital structures. It would further help us to improve QPM.

In [2], we presented three diagrams (see Fig. 1) in which the sea quark effects contribute to the effective one-meson exchange potential of the nucleon-nucleon interaction. From the quark point of view, there are two kinds of quark-quark interactions in these diagrams. The one shown in Fig. 2a is the two-body quark interaction between two quarks, and the other, shown in Fig. 2b, is the three-body quark interaction among three quarks. According to the result in [2], the contribution from Fig. 2a dominates, meanwhile the effect from Fig. 2b is not negligible. In this investigation, we discuss the forms of the two body and three-body quark interactions.

It should be noted that in the sea quark effect, the contribution of fluctuating two pairs of quark-antiquark is also an important part in the hadron-hadron interaction, because they are in the same order. The effective interaction intervening between two quarks due to the fluctuation of two pairs of quark-antiquark is presented in Section 2. Finally, a discussion of the foregoing is given in Section 3.

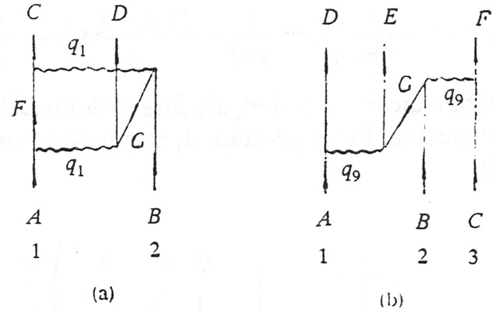
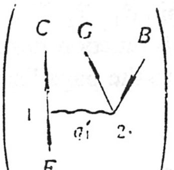


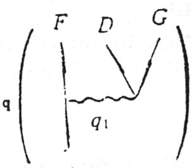
Fig. 2
Interaction between quarks via sea quark excitation.

2. SEA-QUARK-EXCITATION CAUSED INTERACTION BETWEEN TWO QUARKS AND AMONG THREE QUARKS

2.1 The Effective Interaction Between Two Quarks

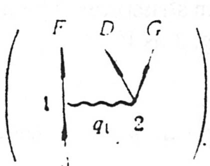
The diagrams of the two-gluon exchange due to the sea quark excitation are very complicated. Figure 2a is a diagram folded by two transition potential $V_{q\to qq\bar{q}}$ and $V_{qq\bar{q}\to q}$. It is an important diagram which can produce the meson exchange effect. Here, we only discuss the interaction between two quarks, which is folded by two transition potentials and whose intermediate state is a four-quark state. There are eight interaction diagrams with initial state $A(1)$ and $B(2)$ and final states $C(1)$ and $D(2)$. Let us take the diagram in Fig. 2a called V_1 as an example to demonstrate the method of the calculation. According to the calculating rule of the off-shell S-matrix, we can write V_1, the interaction between two quarks in the momentum space, as

$$V_1 = -\sum_{q_1}\left[V_{q\to qq\bar{q}}\left(\right.\right.$$


$$\left.\left.\right)\right]^{+}\frac{1}{E_A - E_F - E_D - E_G + i\varepsilon}\,V_{q\to qq\bar{q}}\left(\right.$$


$$\left.\right) \tag{1}$$

where the transition potential given in [2] can be expressed as

$$V_{q\to qq\bar{q}}\left(\right.$$


$$\left.\right) = \pi g_1 g_2 \lambda_1 \cdot \lambda_2[\vec{f}(m_1 m_2 \vec{q}_1 \vec{k}_A)\cdot\vec{\sigma}_2 + \vec{g}(m_1 m_2 \vec{q}_1 \vec{k}_A) \cdot (i\vec{\sigma}_1 \times \vec{\sigma}_2)]. \tag{2}$$

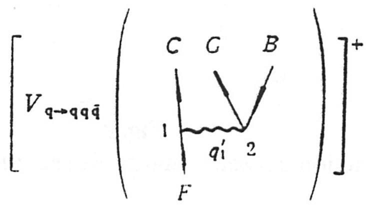
with

$$\vec{f}(m_1 m_2 \vec{q}_1 \vec{k}_A) = \frac{1}{4}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\frac{\vec{q}_1}{q_1^2} - \frac{1}{2m_1}\frac{\vec{k}_A}{q_1^2} + \frac{1}{4}\left(\frac{1}{m_1^2} + \frac{1}{m_1 m_2}\right)\frac{\vec{q}_1(\vec{q}_1\cdot\vec{k}_A)}{q_1^3}$$
$$- \frac{1}{2m_1^2}\cdot\frac{(\vec{q}_1\cdot\vec{k}_A)\vec{k}_A}{q_1^3} - \frac{1}{8}\left(\frac{1}{m_1^2} - \frac{1}{m_2^2}\right)\frac{\vec{q}_1}{q_1} + \frac{1}{4}\left(\frac{1}{m_1^2} - \frac{1}{m_1 m_2}\right)\frac{\vec{k}_A}{q_1}. \tag{3}$$

and

$$\vec{g}(m_1 m_2 \vec{q}_1 \vec{k}_A) = -\frac{1}{4m_1}\frac{\vec{q}_1}{q_1^2} - \frac{1}{4m_1^2}\cdot\frac{\vec{q}_1(\vec{q}_1\cdot\vec{k}_A)}{q_1^3} + \frac{1}{8}\left(\frac{1}{m_1^2} - \frac{1}{m_1 m_2}\right)\frac{\vec{q}_1}{q_1}. \quad (4)$$

where 1 and 2 are quark indices; m_1 and m_2 are masses for quarks 1 and 2, respectively; E_A and $\vec{k}_A$ are the energy and momentum for quark state A_1 respectively; and $\vec{q}_1 = \vec{k}_A - \vec{k}_F$ is the momentum of the gluon. Similarly



$$= \pi g_1 g_2 \lambda_1 \cdot \lambda_2[\vec{f}(m_1 m_2 \vec{q}_1' \vec{k}_c) \cdot \vec{\sigma}_2 - \vec{g}(m_1 m_2 \vec{q}_1' \vec{k}_c) \cdot (i\vec{\sigma}_1 \times \vec{\sigma}_2)]. \quad (5)$$

where $\vec{q}_1' = \vec{k}_C - \vec{k}_F$. Due to the conservation of the momentum of initial and final states, $\vec{q}_1$ and $\vec{q}_1'$ are not independent variables, and the summation in Eq. (1) only runs over $\vec{q}_1$. Substituting Eqs. (2) and (5) into Eq. (1), we can obtain the interaction between two quarks. After simplification procedure, one can always write V_j as

$$V_j = -\pi^2 g_1^{n_1} g_2^{n_2} g_3^{n_3} V_j^{\lambda} \sum_{q_j} G_j[V_j^C + V_j^{\sigma_1\cdot\sigma_2}\vec{\sigma}_1 \cdot \vec{\sigma}_2 + i\vec{V}_j^{\sigma_1} \cdot \vec{\sigma}_1$$
$$+ i\vec{V}_j^{\sigma_2} \cdot \vec{\sigma}_2 + \vec{V}_j^{\sigma_1\times\sigma_2} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) + [(V_j^s)_2(\sigma_1\sigma_2)_2]_{00}]. \quad (6)$$

where $n_1 + n_2 + n_3 = 4$; V_j^{λ} denotes the simplified from of the color part of V_j; V_j^c represents the central part of V_j; $V_j^{\sigma_1\sigma_2}$, $V_j^{\sigma_1}$ and $V_j^{\sigma_2}$ stand for spin $\vec{\sigma}_1 \cdot \vec{\sigma}_2$, $\vec{\sigma}_1$ and $\vec{\sigma}_2$ related terms, respectively; V_j^S depicts the tensor part of V_j and G_j is the Green function describing the energy propagation. We tabulate n_i, V_j^{λ} and G_j for all V_j in Table 1. Due to the page limitation, we do not show V_j is in this paper.

2.2 The Effective Potential Among Three Quarks

The sea-quark-excitation caused interaction among three quarks is even more complicated. As an example, we only illustrate the forms of the color and spin structures. The diagram in Fig. 3 is one which can cause the meson exchange effect. We denote Fig. 3 as V_9.

V_9 can be written as

$$V_9 = -\pi^2 g_1 g_2^2 g_3 V_9^{\lambda} \cdot \sum_{q_9} G_9[V_9^C + V_9^{\sigma} + V_9^{\sigma\cdot\sigma} + V_9^{\sigma\times\sigma} + V_9^{\sigma\sigma\sigma} + V_9^{s}], \quad (7)$$

with

$$V_9^{\lambda} = \frac{2}{3}\lambda_1 \cdot \lambda_3 + \sum_{a,b,c}(d_{abc} + if_{abc})\lambda_{1a}\lambda_{2c}\lambda_{3b}, \quad (8)$$

Table 1
n_i, V^{λ}_j, and G_i in V_j.

j	n_1	n_2	n_3	V^{λ}_f	G_j
1	2	2	0	$32/9 + 14/3\, \lambda_1 \cdot \lambda_2$	$(E_A - E_F - E_D - E_G + i\epsilon)^{-1}$
2	2	2	0	$32/9 + 14/3\, \lambda_1 \cdot \lambda_2$	$(E_B - E_F - E_C - E_G + i\epsilon)^{-1}$
3	1	3	0	$-2/3\, \Pi_1 \cdot \lambda_2$	$(E_A - E_F - E_C - E_G + i\epsilon)^{-1}$
4	3	1	0	$-2/3\, \lambda_1 \cdot \lambda_2$	$(E_B - E_F - E_D - E_D + i\epsilon)^{-1}$
5	3	1	0	$-2/3\, \lambda_1 \cdot \lambda_2$	$(E_A - E_F - E_G - E_C + i\epsilon)^{-1}$
6	1	3	0	$-2/3\, \lambda_1 \cdot \lambda_2$	$(E_B - E_D - E_G - E_F + i\epsilon)^{-1}$
7	1	1	2	$2\, \lambda_1 \cdot \lambda_2$	$(E_A - E_C - E_F - E_G + i\epsilon)^{-1}$
8	1	1	2	$2\, \lambda_1 \cdot \lambda_2$	$(E_B - E_D - E_G - E_F + i\epsilon)^{-1}$

where d_{abe} and f_{abe} are coefficients in following two equations

$$[\lambda_a, \lambda_b] = 2if_{abe}\lambda_e \tag{9}$$

and

$$\{\lambda_a, \lambda_b\} = 2d_{abe}\lambda_e + \frac{4}{3}\delta_{ab}, \tag{10}$$

and

$$G_9 = (E_A - E_D - E_E - E_G + i\varepsilon)^{-1}. \tag{11}$$

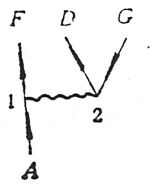
Due to the space limitations, the expressions of $V^c_9, V^{\sigma}_9, \cdots, V^s_9$ are ignored.

2.3 The Two-Quark Effective Potential in the Coordinate Space.

In principle, we can obtain the potential in the coordinate space by taking Fourier transformation of the potential in the momentum space. However, because this potential function presents great complexity, we cannot give an analytical expression which can be used to make actual calculations. Taking V_1 and V_2 as examples, we give the potential by folding two transition potentials in the coordinate space and making the closure approximation to the energy propagator of the intermediate state. We give special consideration to these two diagrams because they represent the mechanisms of effective meson exchange. Parts of these effective potentials are color independent, while the rest are color dependent. It is noteworthy that the color-independent terms would give some new results.

The transition potential in the coordinate space in [2] is given by

$$V_{q\to qq\bar{q}}\left(\quad \right)_{r=r_1-r_2}$$



$$= i\frac{1}{4}g_1g_2\lambda_1 \cdot \lambda_2 \left[-\frac{1}{4}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\frac{1}{r^3}\vec{r}\cdot\vec{\sigma}_2 + i\frac{1}{2m_1 r}\vec{k}_A\cdot\vec{\sigma}_2 \right.$$

$$\left. + \frac{1}{4m_1 r^3}\, i(\vec{\sigma}_1 \times \vec{\sigma}_2)\cdot\vec{r} \right] \tag{12}$$

Similar to the calculation in the momentum space, we can easily obtain the effective potential between two quarks under the local approximation. If the masses of two interacting quarks are the same, by keeping the terms to the order of $1/m$, we can express the effective potential in the coordinate space as:

$$V_1(1,2) + V_2(1,2) = -\frac{1}{16} g^4 G V^\lambda [V^C + V^{\sigma_1\cdot\sigma_2}\vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_S^{L\cdot S}\vec{L}\cdot(\vec{\sigma}_1 + \vec{\sigma}_2) + V_{AS}^{L\cdot S}(\vec{r}\times\vec{K})\cdot(\vec{\sigma}_1 - \vec{\sigma}_2) + i\vec{V}^{\sigma_1}\cdot\vec{\sigma}_1 + i\vec{V}^{\sigma_2}\cdot\vec{\sigma}_2 + \vec{V}^{\sigma_1\times\sigma_2}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2) + ((V^S)_2(\sigma_1\sigma_2)_2)_{00}]. \tag{13}$$

here $G = 1/(E_i - E_m + i\epsilon)$ and E_m is the energy of the intermediate state. Under the closure approximation, we replace G by an average value. $\vec{L} = \vec{l}_1 + \vec{l}_2$ is the sum of the angular momenta of two quarks $\vec{K} = \vec{k}_1 + \vec{k}_2$ is the total angular momentum of two quarks. All other symbols are:

$$V^\lambda = \frac{32}{9} + \frac{14}{3}\lambda_1\cdot\lambda_2. \tag{14}$$

$$V^C = \frac{3}{4m^2r^4} - i\frac{1}{4m^2r^4}\vec{r}\cdot(\vec{k}_A - \vec{k}_B - \vec{k}_C + \vec{k}_D) + \frac{1}{4m^2r^2}(\vec{k}_A\cdot\vec{k}_C + \vec{k}_B\cdot\vec{k}_D). \tag{15}$$

$$V^{\sigma_1\cdot\sigma_2} = \frac{5}{12m^2r^4} - i\frac{1}{12m^2r^4}\vec{r}\cdot(\vec{k}_A - \vec{k}_C - \vec{k}_B + \vec{k}_D). \tag{16}$$

$$V_S^{L\cdot S} = -\frac{3}{4m^2r^4}. \tag{17}$$

$$V_A^{L\cdot S} = \frac{1}{8m^2r^4}. \tag{18}$$

$$\vec{V}^{\sigma_1} = \frac{1}{4m^2r^2}(\vec{k}_B\times\vec{k}_D). \tag{19}$$

$$\vec{V}^{\sigma_2} = \frac{1}{4m^2r^2}(\vec{k}_A\times\vec{k}_C). \tag{20}$$

$$\vec{V}^{\sigma_1\times\sigma_2} = -i\frac{1}{16m^2r^4}\vec{r}\times(\vec{k}_A + \vec{k}_B + \vec{k}_C + \vec{k}_D) = 0. \tag{21}$$

and

$$(V^S)_2 = -\frac{\sqrt{5}}{4m^2r^6}(\vec{r}\vec{r})_2 + i\frac{\sqrt{5}}{8m^2r^4}[(\vec{r}\vec{k}_A)_2 - (\vec{r}\vec{k}_B)_2 - (\vec{r}\vec{k}_C)_2 + (\vec{r}\vec{k}_D)_2]. \tag{22}$$

From Eqs. (13) and (17), we see that because the value of $\langle G\rangle$ is always a negative number, in the calculation of the hadronic spectrum, the contribution from the color independent terms of the two-gluon exchange potential via two transition potentials and the contribution from the spin-orbital term of the one-gluon exchange potential are proved to be contrary.

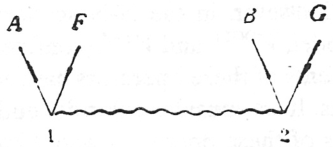


Fig. 3
Two $q\bar{q}$ excitation from quark sea.

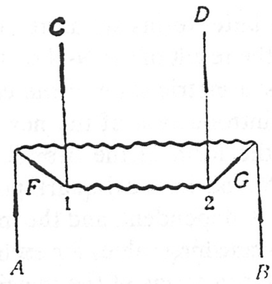


Fig. 4
Interaction between q-q via direct excitation of two $q\bar{q}$.

2.4 The Effective Potential Caused by Fluctuating Two Pairs of Quark-Antiquark

In the sea-quark-excitation caused effective potential between two quarks, the process of two pair quark-antiquark creation directly from the vacuum is also an important mechanism. It can provide the central and spin-spin interactions in the same order, in which the color independent terms also exist. If the masses of quarks are the same, the potential in the coordinate space for Fig. 3 is.

$$V_{qq\bar{q}\bar{q}}(r_{12}) = \frac{g^2}{4}\lambda_1 \cdot \lambda_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \left[-\frac{1}{\pi m r_{12}^2} + \frac{\pi}{m^2}\delta(\vec{r}_{12}) + 0\left(\frac{1}{m^3}\right)\right]. \tag{23}$$

The diagram in Fig. 4 describes the effective two-quark interaction folded by two $V_{qq\bar{q}\bar{q}}$. By keeping terms to the order of $1/m^2$ there exist the central and spin-spin terms in the potential only.

$$V(r_{12}) = \frac{1}{16}g^4\left(\frac{32}{9} - \frac{4}{3}\lambda_1 \cdot \lambda_2\right)(3 - 2\vec{\sigma}_1 \cdot \vec{\sigma}_2)G\frac{1}{\pi^2 m^2 r^4}. \tag{24}$$

where $G = (-E_C - E_F - E_D - E_G + i\epsilon)^{-1}$. In the calculation we also replace G by an average value.

3. DISCUSSION

In the preceding section, we derived the effective meson exchange potential caused by two transition potentials and the process in which two pairs of quark-antiquark are created and annihilated from the vacuum. These processes are actually equivalent to the interactions between two quarks and among three quarks. These interactions have more complicated color, spin and orbital structures than V^{OGEP} does. If we want to solve the problem of the hadronic system from the view point of the interaction between quarks, we should consider two-quark interactions with more complicated structures and three-quark interactions because some effects cannot be obtained by simply changing the strength of the V^{OGEP} potential and the form of the orbital function.

The calculation of the three-body force is a rather complicated problem. But from the form of the two-quark potential, we find some new features. One of them is that apart from the terms which are related to the color degree of freedom of two quarks $\lambda_i \cdot \lambda_j$, there exist color-independent terms. The former terms function like modifying the strength of the original interaction and the form of the orbital function. As a consequence, it affects the hadronic spectrum and the result of the N-N scattering. The latter terms are more interesting. The consideration of this kind of terms may cause large effect on the result of the N-N scattering calculation. It is well known that two quarks in hadron are in the antisymmetric state of the color $SU(3)$ group. The matrix element of $\lambda_i \cdot \lambda_j$ is always a constant. The introduction of the new color independent term only modifies the strength of the potential and the form of the orbital function. However, in the N-N scattering calculation, the situation is different. In usual quark potential model, V^{OGEP} and V^{conf} (confinement potential) are quark color $\lambda_i \cdot \lambda_j$ dependent, and the matrix elements of these operators have zero values for direct terms and non-vanishing values for exchange terms. If we consider color-dependent operators, both direct and exchange terms of the matrix elements of these operators would have non-zero values. Thus, it is possible to bring a large effect to the N-N scattering calculation. Furthermore, we should point out that the color-independent part of the spin-orbital force, produced by two transition potentials, has opposite sign to the corresponding part of V^{OGEP}. As a consequence, the excessive spin-orbital force in the hadronic spectrum calculation by using OGEP may partially be canceled by considering additional sea quark effects.

In this paper, we only derive the general forms of the two-quark and three-quark interactions, which are folded by two transition potentials via creating and annihilating two pairs of quark-antiquark, and simply discuss their effects. Although the obtained potentials are under the closure approximation and local approximation, one can use them in the quantitative calculation. We will publish the numerical result in a forthcoming paper.

REFERENCES

[1] K. Holinde, *Nucl. Phys.*, **A415** (1984), p. 477; Ch. Ulster and K. Holinde, *Phys. Lett.*, **136B** (1984), p. 135; Y. Suzuki and K. T. Hecht, *Phys. Rev.*, **C27** (1983), p. 299; **C28** (1983), p. 1548.

[2] Y. W. Yu and Z. Y. Zhang, *Nucl. Phys.*, **A426** (1984), p. 557; **A455** (1986), p. 737; Y. W. Yu, Z. Y. Zhang, and P. N. Shen, *Nucl. Phys.*, **A528** (1991), p. 513.

[3] Y. Fujiwara and K. T. Hecht, *Nucl. Phys.*, **A444** (1985), p. 541; **A451** (1986), p. 625; **A456** (1986), p. 669; **A462** (1987), p. 621.