One texture zero for Dirac neutrinos in a diagonal charged lepton basis*

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Abstract: An analytical and numerical systematic study of the neutrino mass matrix with one texture zero is presented in a basis where the charged leptons are diagonal. Under the assumption that neutrinos are Dirac particles, the analysis is conducted in detail for the normal and inverted hierarchy mass spectra. Our study is performed without any approximations, first analytically and then numerically, using current neutrino oscillation data. The analysis constrains the parameter space in such a way that, among the six possible one-texture-zero patterns, only four are favored in the normal hierarchy and one in the inverted hierarchy by current oscillation data at the 3σ level. Phenomenological implications for the lepton CP-violating phase and neutrino masses are also explored.

Keywords: Dirac neutrinos, texture, UPMNS

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I. INTRODUCTION

Although the gauge boson sector of the standard model (SM) with the $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$ local gauge symmetry has been successfully understood so far (with $SU(3)_c$ confined and $SU(2)_L$ spontaneously broken via the Higgs mechanism [1]), its Yukawa sector is still poorly understood. Questions related to this sector, such as the number of families in nature, the hierarchy of the charged fermion mass spectrum, the smallness of the neutrino masses, quark mixing angles, neutrino oscillation parameters, and the origin of CP violation, remain open questions. Moreover, in the context of the SM, there is a lack of explanation for the dark matter and dark energy observed at present in the universe.

In the context of the SM, a neutrino flavor created by the weak interaction and associated with a charged lepton will maintain its flavor, which indicates that lepton flavor is conserved and neutrinos are massless. Moreover, from oscillation experiments, we know that neutrinos are massive particles and that they oscillate from one flavor to another, with the results sensitive only to the squared mass difference. We still do not know the mass of any of the light neutrinos. However, from cosmology, we know the upper limit for the sum of the three light neutrino masses: $\sum m_\nu < 0.12$ eV (95%CL) [2–4], whereas from tritium beta decay, there is room for an effective mass of the electron neutrino $\langle m_\beta\rangle < 0.8$ eV (90%CL) [5–7].

Current neutrino experiments are measuring the neutrino mixing parameters with unprecedented accuracy. The next generation of neutrino experiments will be sensitive to subdominant neutrino oscillation effects that can, in principle, provide information on the yet unknown neutrino parameters: the Dirac CP-violating phase in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix $U_{\rm PMNS}$, neutrino mass ordering, and the octant of the mixing angles.

To date, the solar and atmospheric neutrino oscillations have established the following values to 3 sigma of the deviation to normal ordering [8]:

$$\Delta m^2_{\rm Atm} = (2.46-2.61)\times 10^{-3}\ {\rm eV}^2 = \Delta m^2_{32},$$

$$\Delta m^2_{\rm Sol} = (6.92-8.05)\times 10^{-5}\ {\rm eV}^2 = \Delta m^2_{21},$$

$$\sin^2\theta_{\rm Atm} = (4.30-6.96)\times 10^{-1} = \sin^2\theta_{23},$$

$$\sin^2\theta_{\rm Sol} = (2.75-3.45)\times 10^{-1} = \sin^2\theta_{21},$$

$$\sin^2\theta_{\rm Reac} = (2.02-2.38)\times 10^{-2} = \sin^2\theta_{13}.$$

The numbers are obtained under the assumption that, for the charged lepton sector, the weak basis and flavor basis are the same.

In this paper, we present a systematic study of neutrino mass matrix M_ν with one texture zero, in a basis

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where the three charged leptons are diagonal, as an extension of the already presented study of the case with two texture zeros [9, 10].

In our analysis, for each of the six different one texture zeros in M_ν, we perform first an analytical and then a statistical fit of the oscillation angles, to limit the parameter space, and thus obtain neat predictions for the neutrino masses.

II. MODEL

The model used herein is a simple extension of the SM, with the following three new aspects added:

1. We extend the electroweak sector of the SM with three right handed neutrinos, $(\nu_{\alpha R};\ \alpha = e,\mu,\tau)$.
2. The charged lepton mass matrix is diagonal in the weak basis.
3. Majorana masses are forbidden.

A. Neutrino mass matrix

According to the previous hypothesis, for the charged lepton sector, we have

$$M_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{1}$$

which indicates that the most general 3×3 neutrino mass matrix is complex; however, a complex matrix can always be decomposed as the product of a Hermitian and unitary matrix, according to the polar decomposition theorem $(M = HU)$, where the unitary factor can be absorbed into the $SU(2)_L$ singlet gauge structure [11−13]. Then, we can assume it as a Hermitian matrix without loss of generality, which indicates

$$M_\nu = \begin{pmatrix} m_{\nu_e\nu_e} & m_{\nu_e\nu_\mu} & m_{\nu_e\nu_\tau} \\ m_{\nu_\mu\nu_e} & m_{\nu_\mu\nu_\mu} & m_{\nu_\mu\nu_\tau} \\ m_{\nu_\tau\nu_e} & m_{\nu_\tau\nu_\mu} & m_{\nu_\tau\nu_\tau} \end{pmatrix} = V_{\rm PMNS}\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^\dagger_{\rm PMNS} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix}\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} U^*_{e1} & U^*_{\mu1} & U^*_{\tau1} \\ U^*_{e2} & U^*_{\mu2} & U^*_{\tau2} \\ U^*_{e3} & U^*_{\mu3} & U^*_{\tau3} \end{bmatrix}$$
$$= \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix}\begin{bmatrix} m_1U^*_{e1} & m_1U^*_{\mu1} & m_1U^*_{\tau1} \\ m_2U^*_{e2} & m_2U^*_{\mu2} & m_2U^*_{\tau2} \\ m_3U^*_{e3} & m_3U^*_{\mu3} & m_3U^*_{\tau3} \end{bmatrix} \tag{2}$$

where mixing matrix $V_{\rm PMNS}$ for Dirac neutrinos is parametrized in the usual way as follows:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}\begin{bmatrix} c_{13} & 0 & s_{13}{\rm e}^{-{\rm i}\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}{\rm e}^{{\rm i}\delta_{\rm CP}} & 0 & c_{13} \end{bmatrix}\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}{\rm e}^{-{\rm i}\delta_{\rm CP}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}{\rm e}^{{\rm i}\delta_{\rm CP}} & c_{12}c_{23}-s_{12}s_{23}s_{13}{\rm e}^{{\rm i}\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}{\rm e}^{{\rm i}\delta_{\rm CP}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}{\rm e}^{{\rm i}\delta_{\rm CP}} & c_{23}c_{13} \end{bmatrix} \tag{3}$$

where ${\rm Diag}(m_1,m_2,m_3)$ represents the neutrino mass eigenvalues, and $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ are the cosine and sine of oscillation angles θ_{ij}, $i<j = 1,2,3$, respectively, in the standard parametrization.

Now, owing to the hermiticity constraint, the elements of M_ν satisfy: $m_{\nu_e\nu_e} = m^*_{\nu_e\nu_e}$, $m_{\nu_\mu\nu_\mu} = m^*_{\nu_\mu\nu_\mu}$, $m_{\nu_\tau\nu_\tau} = m^*_{\nu_\tau\nu_\tau}$, $m_{\nu_\mu\nu_e} = m^*_{\nu_e\nu_\mu}$, $m_{\nu_\tau\nu_e} = m^*_{\nu_e\nu_\tau}$, and $m_{\nu_\mu\nu_\tau} = m^*_{\nu_\tau\nu_\mu}$.

In our analysis, we use the numerical values for the entries of $U_{\rm PMNS}$ measured at the 3σ ranges presented in the literature [8] and quoted above.

B. Counting parameters

When the mass matrices for the lepton sector are given by equations (1) and (2), in the weak basis, the most general Hermitian mass matrix M_ν has six real parameters and three phases that we can use to explain seven physical parameters: three real oscillating angles θ_{12}, θ_{13}, and θ_{23}, three real neutrino masses m_1, m_2, and m_3, and one CP violating phase δ. Hence, in principle, we have a redundant number of parameters (two more phases).

Now, contrary to the quark sector, we cannot introduce texture zeros via weak basis transformations [14−18] in mass matrix M_ν, as it will change the charged lepton diagonal mass matrix. However, as shown elsewhere [9, 10, 19], the "weak basis transformations" can be used to eliminate the two redundant phases, and consequently, our study considers only seven parameters, six real and one phase, which are sufficient to accommodate, in principle, the seven physical parameters.

Now, as the number of analytical parameters is equal to the number of physical ones, one (or more) texture zero(s) in mass matrix M_ν will imply relationships between the physical parameters; in particular, for one texture zero, we may expect a relationship between the neutrino masses and the oscillating angles in the U_{PMNS} matrix (as is the case, for example, in the quark sector [17,18]).

The origin of texture zeros has been extensively discussed in the literature (see, for instance, [20−24]). Such vanishing entries can indeed arise from the imposition of discrete flavor symmetries or specific group structures acting on the lepton fields.

III. ONE TEXTURE ZERO

The introduction of texture zeros in a general mass matrix has been an outstanding hypothesis that may provide relationships in the lepton sector between the oscillation angles and mass eigenvalues.

As discussed earlier, the six real mathematical parameters of the most general Hermitian mass matrix for Dirac neutrinos provide sufficient room to accommodate the five real experimental values with no prediction at all. One texture zero, although not conducive to any prediction either, limits the parameter space sufficiently to accommodate the experimentally measured numbers.

In the following, for the case of "normal ordering" (NO) and "inverted ordering" (IO), we will study six different cases of one texture zero in Hermitian Dirac mass matrix M_ν, particularly for the lightest Dirac neutrino mass, which consequently implies the knowledge of the neutrino mass spectrum.

Our aim in this study is to perform first an analytical and then a statistical analysis of the parameter space, when one texture zero is introduced in M_ν.

For the implications of texture zeros in M_ν, two cases must be analyzed: texture zeros in the diagonal, and texture zeros outside the diagonal.

A. Diagonal texture zeros

Let us first assume that $m_{\nu_e\nu_e}=0$ and consider its implications:

From equation (2), we have

$$m_{\nu_e\nu_e}=m_1|U_{e1}|^2+m_2|U_{e2}|^2+m_3|U_{e3}|^2=0, \tag{4}$$

and dividing this by m_3 and using the unitary constraint of matrix *U*, that is, $|U_{e1}|^2+|U_{e2}|^2+|U_{e3}|^2=1$, we can write equation (4) as

$$\frac{m_1}{m_3}|U_{e1}|^2+\frac{m_2}{m_3}|U_{e2}|^2+1-|U_{e1}|^2-|U_{e2}|^2=0;$$

which we can rearrange as

$$|U_{e2}|^2=\frac{m_3}{m_3-m_2}-\frac{m_3-m_1}{m_3-m_2}|U_{e1}|^2. \tag{5}$$

By rewriting equation (5) in terms of the physical parameters, we obtain

$$s_{12}^2c_{13}^2=\frac{m_3}{(m_3-m_2)}-\frac{(m_3-m_1)}{(m_3-m_2)}c_{12}^2c_{13}^2, \tag{6}$$

As anticipated above, we obtain the relationship between the neutrino masses and oscillation parameters.

Similarly, for $m_{\nu_\mu\nu_\mu}=0$, we have

$$|U_{\mu 2}|^2=\frac{m_3}{m_3-m_2}-\frac{m_3-m_1}{m_3-m_2}|U_{\mu 1}|^2, \tag{7}$$

and for $m_{\nu_\tau\nu_\tau}=0$, we have

$$|U_{\tau 2}|^2=\frac{m_3}{m_3-m_2}-\frac{m_3-m_1}{m_3-m_2}|U_{\tau 1}|^2. \tag{8}$$

The former three cases can be summarized as follows:

$$|U_{\alpha 2}|^2=\frac{m_3}{m_3-m_2}-\frac{m_3-m_1}{m_3-m_2}|U_{\alpha 1}|^2, \tag{9}$$

for $\alpha=e$ if $m_{\nu_e\nu_e}=0$; $\alpha=\mu$ if $m_{\nu_\mu\nu_\mu}=0$, and $\alpha=\tau$ if $m_{\nu_\tau\nu_\tau}=0$.

B. Texture zeros outside the diagonal

Let us now consider a texture zero outside the diagonal. Let us start with $m_{\nu_e\nu_\mu}=0$ (notice that $m_{\nu_\mu\nu_e}=m^*_{\nu_e\nu_\mu}=0$).

Here, equation (2) implies

$$m_{\nu_e\nu_\mu}=m_1U_{e1}U^*_{\mu 1}+m_2U_{e2}U^*_{\mu 2}+m_3U_{e3}U^*_{\mu 3}=0, \tag{10}$$

and dividing this by m_3 and using the orthogonality condition $U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2}+U_{e3}U^*_{\mu 3}=0$, we can write

$$\left(\frac{m_1}{m_3}-1\right)U_{e1}U^*_{\mu 1}+\left(\frac{m_2}{m_3}-1\right)U_{e2}U^*_{\mu 2}=0, \tag{11}$$

which is multiplied by $U^*_{e2}U_{\mu 2}$ and rearranged to obtain

$$\left(\frac{m_1}{m_3}-1\right)U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}+\left(\frac{m_2}{m_3}-1\right)|U_{e2}|^2|U_{\mu 2}|^2=0, \tag{12}$$

which we can finally write as

$$U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}+\left(\frac{m_3-m_2}{m_3-m_1}\right)|U_{e2}|^2|U_{\mu 2}|^2=0, \tag{13}$$

which, together with its complex conjugate, can be separated into two parts: a real part equal to zero and an imaginary part also equal to zero (notice that, for a Hermitian matrix, its eigenvalues must be real but not necessarily positive).

As $m_{\nu_\mu\nu_e}$ must also be equal to zero, the two relations must be equivalent to make the real and imaginary parts in equation (13) equal to zero. As the second term in equation (13) is real, considering the imaginary part equal to zero produces

$$\mathrm{Im}(U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2}) = J = 0; \tag{14}$$

which indicates that this texture zero is associated with a Jarlskog invariant equal to zero and no CP violation is present for this texture zero.

Similarly, for $m_{\nu_e\nu_\tau} = 0$, we have

$$m_{\nu_e\nu_\tau} = m_1U_{e1}U_{\tau 1}^{*} + m_2U_{e2}U_{\tau 2}^{*} + m_3U_{e3}U_{\tau 3}^{*} = 0. \tag{15}$$

Dividing by m_3 and using the orthogonality relationship $U_{e1}U_{\tau 1}^{*} + U_{e2}U_{\tau 2}^{*} + U_{e3}U_{\tau 3}^{*} = 0$, we can write equation (15) as

$$\left(\frac{m_1}{m_3} - 1\right)U_{e1}U_{\tau 1}^{*} + \left(\frac{m_2}{m_3} - 1\right)U_{e2}U_{\tau 2}^{*} = 0, \tag{16}$$

which is multiplied by $U_{e2}^{*}U_{\tau 2}$ and rearranged to obtain

$$\left(\frac{m_1}{m_3} - 1\right)U_{e1}U_{\tau 1}^{*}U_{e2}^{*}U_{\tau 2} + \left(\frac{m_2}{m_3} - 1\right)|U_{e2}|^2|U_{\tau 2}|^2 = 0, \tag{17}$$

which in turns implies

$$U_{e1}U_{\tau 1}^{*}U_{e2}^{*}U_{\tau 2} + \left(\frac{m_3 - m_2}{m_3 - m_1}\right)|U_{e2}|^2|U_{\tau 2}|^2 = 0, \tag{18}$$

which again yields

$$Im(U_{e1}U_{\tau 1}^{*}U_{e2}^{*}U_{\tau 2}) = J = 0. \tag{19}$$

The former indicates that this texture zero outside the diagonal, and also the former case, are associated with a Jarlskog invariant equal to zero and again, there is no CP violation in this case.

Similarly, for $m_{\nu_\mu\nu_\tau} = 0$, we have

$$m_{\nu_\mu\nu_\tau} = m_1U_{\mu 1}U_{\tau 1}^{*} + m_2U_{\mu 2}U_{\tau 2}^{*} + m_3U_{\mu 3}U_{\tau 3}^{*} = 0, \tag{20}$$

and dividing this by m_3 and using the appropriate orthogonality relationship, we have

$$\left(\frac{m_1}{m_3} - 1\right)U_{\mu 1}U_{\tau 1}^{*} + \left(\frac{m_2}{m_3} - 1\right)U_{\mu 2}U_{\tau 2}^{*} = 0, \tag{21}$$

which is multiplied by $U_{\mu 2}^{*}U_{\tau 2}$ to obtain

$$U_{\mu 1}U_{\tau 1}^{*}U_{\mu 2}^{*}U_{\tau 2} + \left(\frac{m_3 - m_2}{m_3 - m_1}\right)|U_{\mu 2}|^2|U_{\tau 2}|^2 = 0, \tag{22}$$

which again yields

$$\mathrm{Im}(U_{\mu 1}U_{\tau 1}^{*}U_{\mu 2}^{*}U_{\tau 2}) = J = 0; \tag{23}$$

IV. NUMERICAL RESULTS

After the spontaneous symmetry breaking of the local gauge symmetry, the Lagrangian mass term for the lepton sector is given by

$$-\mathcal{L} = \bar{\nu}_L M_\nu \nu_R + \bar{l}_L M_l l_R + \text{h.c.}, \tag{24}$$

where M_ν and M_l are, in general, complex matrices. We will analyze each of the texture forms with one texture zero in the neutral sector and choose the charged lepton sector in the diagonal form.

A. Textures forms

There are six different one texture zero patterns in M_ν, for which we use the following notation:

$$T_1 = \begin{pmatrix} 0 & a & b \\ a^* & c & d \\ b^* & d^* & e \end{pmatrix}, T_2 = \begin{pmatrix} a & b & c \\ b^* & 0 & d \\ c^* & d^* & e \end{pmatrix},$$

$$T_3 = \begin{pmatrix} a & b & c \\ b^* & d & e \\ c^* & e^* & 0 \end{pmatrix}, T_4 = \begin{pmatrix} a & 0 & b \\ 0 & c & d \\ b^* & d^* & e \end{pmatrix},$$

$$T_5 = \begin{pmatrix} a & b & 0 \\ b^* & c & d \\ 0 & d^* & e \end{pmatrix}, T_6 = \begin{pmatrix} a & b & c \\ b^* & d & 0 \\ c^* & 0 & e \end{pmatrix}.$$

In the following, we will perform a numerical analysis for each of the textures.

B. Texture T_1

Texture T_1, featuring a vanishing $(1,1)$ element, provides a good fit to the observed neutrino oscillation data. In this case, parameters a, b, and d are complex, whereas c and e are real. Mass matrix M_ν must satisfy the diagonalization condition:

$$U^\dagger M_\nu U = \mathrm{diag}(m_1, m_2, m_3),$$

where U is the same $U_{\rm PMNS}$ matrix known experimentally. To quantify the deviation between the observables predicted by mass matrix M_ν and the experimentally measured values, we construct a chi-squared error function:

$$\chi^2 = \sum_i \left(\frac{x_i^{\rm pred} - x_i^{\rm obs}}{\sigma_i} \right)^2,$$

where x_i represents the relevant physical observables (such as oscillation angles and mass-squared differences), $x_i^{\rm pred}$ represents the theoretical predictions obtained from the diagonalization of M_ν, $x_i^{\rm obs}$ represents the experimentally measured values, and σ_i represents the corresponding experimental uncertainties, as noted in the introduction.

The goal is to find the set of parameters in M_ν that minimizes χ^2, ensuring that the matrix reproduces the observed neutrino oscillation data within the allowed confidence level. The same procedure is applied to the other texture types.

1. *Mass matrix parameters*

Our numerical analysis shows that the mass matrix elements are

$$M_\nu = \begin{pmatrix} 0 & 0.00516+0.0057i & -0.00411+0.00461i \\ 0.00516-0.0057i & 0.0281 & 0.02225-0.00005i \\ -0.00411-0.00461i & 0.02225+0.00005i & 0.02507 \end{pmatrix} \mathrm{eV}$$

with the resulting eigenvalues given by

$$m_1 = -0.00542\ \mathrm{eV},\ m_2 = 0.00860\ \mathrm{eV},$$
$$m_3 = 0.04999\ \mathrm{eV},\ \sum_{i=1}^{3} |m_i| = 0.06402\ \mathrm{eV}.$$

This spectrum is consistent with a normal mass ordering and complies with cosmological bounds on the sum of neutrino masses [2], and

$$\Delta m_{21}^2 = 4.46\times 10^{-5}\ \mathrm{eV}^2,\ \Delta m_{31}^2 = 2.50\times 10^{-3}\ \mathrm{eV}^2,$$

which are in excellent agreement with experimental constraints from solar and atmospheric neutrino data [25]. These values are evaluated with the best fit point for the minimum $\chi^2 = 1.43\times 10^{-3}$.

The above values allow us to obtain the following mixing matrix:

$$U_{\rm PMNS} = \begin{pmatrix} -0.818602+0.0905867i & -0.537971-0.101074i & -0.0162952-0.147616i \\ 0.374339-0.153715i & -0.541351-0.128917i & -0.725065-0.0290143i \\ -0.3974 & 0.625053 & -0.67185 \end{pmatrix}.$$

This matrix is unitary and consistent with the standard parameterization of the lepton mixing matrix. The oscillation angles extracted from this matrix are

$$\theta_{13} = \arcsin(|U_{13}|),\ \theta_{12} = \arctan\left(\frac{|U_{12}|}{|U_{11}|}\right),$$
$$\theta_{23} = \arctan\left(\frac{|U_{23}|}{|U_{33}|}\right).$$

To extract Dirac CP-violating phase δ, one can use the Jarlskog invariant J, defined as

$$J = \mathrm{Im}\left(U_{11}U_{22}U_{12}^*U_{21}^*\right)$$
$$= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta.$$
$$\theta_{12} = 33.61^\circ,\ \theta_{13} = 8.54^\circ,\ \theta_{23} = 47.21^\circ,\ \delta = 270.02^\circ.$$

These values are in good agreement with the experimentally measured values at the 3σ level reported by the NuFIT collaboration.

Texture T_1 is highly successful in accommodating the current experimental data on neutrino mixing and mass-squared differences, with a physically acceptable prediction for the CP-violating phase. The choice of a zero value at the $(1,1)$ position can be motivated by underlying flavor symmetries, such as $L_e - L_\mu - L_\tau$, and encourage us to further study texture zeros in neutrino physics [25–28].

C. Texture T_2

Texture T_2, featuring a vanishing $(2,2)$ element, provides a partial fit to the observed neutrino oscillation data. The atmospheric mass-squared difference Δm_{31}^2 and two of the mixing angles are within the 3σ experimental ranges.

$$M_\nu \approx \begin{pmatrix} -0.02006 & -0.00319+0.00454i & -0.00883+0.00648i \\ -0.00319-0.00454i & 0 & 0.03037+0.01680i \\ -0.00883-0.00648i & 0.03037-0.01680i & 0.02616 \end{pmatrix}$$

The texture produced the following observables:

$$m_1 = -0.020929\ \mathrm{eV},\ m_2 = -0.025179\ \mathrm{eV},$$
$$m_3 = 0.052208\ \mathrm{eV},\ \sum_{i=1}^{3} |m_i| = 0.0983\ \mathrm{eV}.$$

$$\Delta m^2_{21} = 1.96\times10^{-4}\ \text{eV}^2,\ \Delta m^2_{31} = 2.29\times10^{-3}\ \text{eV}^2$$

$$U_{\text{PMNS}} = \begin{pmatrix} 0.840 & 0.517 & 0.166 \\ 0.036+0.503\text{i} & 0.025-0.659\text{i} & -0.259-0.495\text{i} \\ -0.045-0.196\text{i} & 0.290+0.463\text{i} & -0.677-0.451\text{i} \end{pmatrix}$$

$$\theta_{12} = 31.61^\circ,\ \theta_{13} = 9.53^\circ,\ \theta_{23} = 34.46^\circ,\ \ \delta = 223.4^\circ$$

These values are evaluated with the best fit point for the minimum $\chi^2 = 0.39$. A vanishing (2,2) element partially accommodates the current neutrino oscillation data. While Δm^2_{31} and two mixing angles lie within the 3σ experimental ranges, solar mass-squared difference Δm^2_{21} and atmospheric angle θ_{23} are significantly outside the preferred bounds. Thus, although textures with zeros are theoretically suitable for reducing parameter space [20, 29], the T_2 structure is disfavored by present data [8, 30–31].

D. Texture T_3

For texture T_3, featuring an input zero in (3,3), the form of the mass matrix that we obtain is

$$M_\nu \approx \begin{pmatrix} -0.03041 & -0.00996+0.00705\text{i} & -0.00871-0.00019\text{i} \\ -0.00996-0.00705\text{i} & 0.02277 & 0.03326+0.02773\text{i} \\ -0.00871+0.00019\text{i} & 0.03326-0.02773\text{i} & 0 \end{pmatrix}$$

The observables are given by

$$m_1 = -0.032442\ \text{eV},\ m_2 = -0.033881\ \text{eV},$$
$$m_3 = \ 0.058679\ \text{eV},\ \sum_{i=1}^{3}|m_i| = 0.1250\ \text{eV}.$$

$$\Delta m^2_{21} = 9.54\times10^{-5}\ \text{eV}^2,\ \Delta m^2_{31} = 2.39\times10^{-3}\ \text{eV}^2,$$

$$U_{\text{PMNS}} = \begin{pmatrix} 0.802 & -0.574 & -0.166 \\ -0.269+0.064\text{i} & -0.557-0.044\text{i} & 0.629+0.464\text{i} \\ 0.436-0.301\text{i} & 0.436-0.410\text{i} & 0.600-0.035\text{i} \end{pmatrix}$$

$$\theta_{12} = 35.63^\circ,\ \theta_{13} = 9.54^\circ,\ \theta_{23} = 52.42^\circ,\ \ \delta = 328.7^\circ.$$

These values are evaluated with the best fit point for the minimum $\chi^2 = 2.10$. This texture is close to experimental expectations, with mixing angles and mass-squared differences lying within the 3σ ranges.

E. Texture T_4

We find a mass matrix with the following form (note that, for this particular case, all entries are real):

$$M_\nu = \begin{pmatrix} -0.00610 & 0 & 0.02191 \\ 0 & 0.03603 & 0.02242 \\ 0.02191 & 0.02242 & 0.02762 \end{pmatrix}\ \text{eV}$$

The observables are given by

$$m_1 = -0.02236\ \text{eV},\ m_2 = \ 0.02411\ \text{eV},$$
$$m_3 = \ 0.05581\ \text{eV},\ \sum_{i=1}^{3}|m_i| = 0.1023\ \text{eV}.$$

$$\Delta m^2_{21} = 8.12\times10^{-5}\ \text{eV}^2,\ \Delta m^2_{31} = 2.61\times10^{-3}\ \text{eV}^2.$$

$$U_{\text{PMNS}} = \begin{pmatrix} -0.8155 & 0.5594 & 0.1484 \\ -0.3027 & -0.6308 & 0.7145 \\ 0.4933 & 0.5377 & 0.6837 \end{pmatrix}$$

$$\theta_{12} = 34.45^\circ,\ \theta_{13} = 8.53^\circ,\ \theta_{23} = 46.26^\circ,$$
$$\delta = 0^\circ\ \text{or}\ \pi\quad (\text{Jarlskog invariant}\ J = 0)$$

This texture provides an excellent fit to the experimental data. All the mixing angles and mass-squared differences fall within the 3σ ranges, and the CP-violating phase corresponds to a vanishing Jarlskog invariant, consistent with the imposed matrix structure. These values are evaluated with the best fit point for the minimum $\chi^2 = 4.61\times10^{-8}$.

F. Texture T_5

The mass matrix is observed to have the following form:

$$M_\nu = \begin{pmatrix} -0.00028 & 0.00774+0.00782\text{i} & 0 \\ 0.00774-0.00782\text{i} & 0.02442 & 0.02114+0.00508\text{i} \\ 0 & 0.02114-0.00508\text{i} & 0.03048 \end{pmatrix}\ \text{eV}$$

The observables are given by

$$m_1 = -0.00681\ \text{eV},\ m_2 = \ 0.01097\ \text{eV},$$
$$m_3 = \ 0.05046\ \text{eV},\ \sum_{i=1}^{3}|m_i| = 0.0682\ \text{eV}.$$

$$\Delta m_{21}^2 = 1.53\times10^{-4}\ \text{eV}^2,\ \Delta m_{31}^2 = 2.55\times10^{-3}\ \text{eV}^2$$

$$U_{\text{PMNS}}=\begin{pmatrix} 0.8243 & 0.5472 & -0.1453 \\ -0.3440+0.3476i & 0.3933-0.3974i & -0.4709+0.4759i \\ 0.1476-0.2439i & -0.3225+0.5329i & -0.3772+0.6232i \end{pmatrix}$$

$$\theta_{12} = 33.58°,\ \theta_{13} = 8.35°,\ \theta_{23} = 42.58°,\ \delta = 0°\ \text{or}\ \pi.$$

This texture successfully reproduces the observed neutrino oscillation parameters with all the values within the 3σ experimental ranges and an effectively vanishing CP-violating phase. However, the minimum $\chi^2 = 37.65$. Although the model contains the necessary parameters to describe the primary observables, it is missing one or more parameters required to capture smaller but statistically significant variations in the data. In this form, the number of free parameters (degrees of freedom) must be increased to verify if the value of χ^2 decreases to an acceptable level.

G. Texture T_6

We find a Hermitian neutrino mass matrix of the form:

$$M_\nu = \begin{pmatrix} 0.05467 & 0.00173+0.00467i & 0.00111+0.00461i \\ 0.00173-0.00467i & 0.02970 & 0 \\ 0.00111-0.00461i & 0 & -0.00394 \end{pmatrix} \text{eV}$$

The observables obtained are

$$m_1 = 0.06352\ \text{eV},\ m_2 = -0.06633\ \text{eV},$$
$$m_3 = -0.07548\ \text{eV},\ \sum_{i=1}^{3}|m_i| = 0.2053\ \text{eV}.$$

$$\Delta m_{21}^2 = 3.68\times10^{-4}\ \text{eV}^2,\ \Delta m_{31}^2 = 1.43\times10^{-3}\ \text{eV}^2$$

$$U_{\text{PMNS}} = \begin{pmatrix} 0.9251 & 0.2856 & 0.2500 \\ -0.3713+0.0296i & 0.8077-0.0644i & 0.4511-0.0360i \\ 0.0689+0.0250i & 0.4812+0.1744i & -0.8048-0.2917i \end{pmatrix}$$

$$\theta_{12} = 17.16°,\ \theta_{13} = 14.48°, \theta_{23} = 27.86°,\ \delta = 0°\ \text{or}\ \pi.$$

This texture fails to reproduce the experimental values of the mixing angles within the 3σ range and yields an unacceptably high sum of masses. Therefore, it is disfavored by current data.

V. INVERTED ORDERING

Although the oscillation experiments have precisely measured the two mass-squared differences, Δm_{21}^2 and $|\Delta m_{21}^2|$, the ordering of the mass eigenstates remains an open question. The inverted hierarchy (IH) corresponds to the mass ordering

$$m_3 < m_1 < m_2,$$

in which two heavier mass eigenstates m_1 and m_2 form a close pair separated from the lighter state m_3. This contrasts with the normal hierarchy (NH), where m_3 is the heaviest.

A. Numerical analysis

The numerical analysis for the IH follows the same procedure described in Section IV. In particular, we employ the same texture structures introduced in Section IV.A. Among the considered textures, only the T2 texture successfully reproduces the experimental data within the 1σ level. The methodology used to obtain these results is outlined below.

The T2 texture is defined as

$$T_2 = \begin{pmatrix} a & b & c \\ b^* & 0 & d \\ c^* & d^* & e \end{pmatrix}.$$

The diagonalization procedure was performed under the constraint $m_2 > m_1 > m_3$, appropriate for the inverted mass hierarchy. This approach allows us to express the UPMNS matrix in terms of parameters m_1, m_2, m_3, and b.

Applying the χ^2 minimization method described in Section IV.B yields the numerical results presented below.

$$m_1 = 0.0561248\ \text{eV},\ m_2 = 0.0567404\ \text{eV},$$
$$m_3 = 0.0256819\ \text{eV},\ \sum_{i=1}^{3}|m_i| = 0.138\ \text{eV}.$$

The calculated χ^2 statistic was 2.21×10^{-8} with three degrees of freedom ($df = 3$)

$$\Delta m_{21}^2 = 6.95\times10^{-5}\ \text{eV}^2,\ \Delta m_{31}^2 = 2.50\times10^{-3}\ \text{eV}^2.$$

$$U_{\text{PMNS}} = \begin{pmatrix} 0.821372 & 0.550543 & 0.149164 \\ -0.47604 & 0.517586 & 0.710979 \\ 0.314219 & -0.654986 & 0.687211 \end{pmatrix}$$

$$\theta_{12} = 34.54°,\ \theta_{13} = 8.54°,$$
$$\theta_{23} = 46.13°,\ \delta = 0°\ \text{or}\ \pi\ (\text{Jarlskog invariant}\ J = 0)$$

VI. CONCLUSIONS

We systematically analyzed the six possible one zero texture patterns for Hermitian neutrino mass matrices under the assumption of both neutrino normal and inverted mass hierarchies, working in the charged lepton diagonal basis. In this scenario, the lepton mixing matrix arises entirely from the neutrino sector. Mathematically, as the number of free parameters exceeds the number of physical observables, analytical solutions always exist. Specifically, the mass-squared differences, three mixing angles, and CP-violating phase can all be determined analytically for each texture. A crucial observation is that, when the texture zero appears off the diagonal, the Jarlskog invariant vanishes, implying that the Dirac CP-violating phase must be either $\delta = 0$ or $\delta = \pi$. Therefore, nontrivial CP violation ($\delta \neq 0, \pi$) is only possible when the texture zero is located in one of the diagonal entries of the mass matrix.

Our numerical analyses in the normal ordering reveal that:

- **Texture** T_1 yields an excellent fit to all current experimental data. All three mixing angles, mass-squared differences, and CP phase are reproduced within 1σ of the deviation of the global best fit values. In particular, the best fit CP phase is observed to be close to $\delta \approx 270°$, and the total sum of neutrino masses is consistent with cosmological bounds, such as those from supernova neutrino constraints and the Planck satellite.

- **Texture** T_2 fails to accommodate the experimental data, with significant deviations in the atmospheric mixing angle and solar mass-squared difference.

- **Texture** T_3 provides aviable fit to all observables within 3σ.

- In the remaining textures T_4, T_5, and T_6, the phase of CP violation is necessarily trivial ($\delta = 0$ or π), as expected owing to the texture zero being located off-diagonal. However,
  - Texture T_4, which is real, and
  - Texture T_5, which is complex,

both successfully reproduce all the observables within 3σ.

- **Texture** T_6 fails to match current experimental constraints and is thus disfavored.

Our numerical analyses in the inverted ordering reveal that:

- **Texture** T_2 fit to current experimental information is excellent. All key parameters—the three mixing angles, mass differences, and CP phase—agree with the global best-fit values at the 1σ level of deviation. The optimal CP phase is observed to be near $270°$, and the total neutrino mass is in conformance with cosmological bounds, including those from supernovae and the Planck satellite.

- The other five single texture zero configurations are found to be in disagreement with the current experimental data. Therefore, they must be ruled out (or excluded) as viable solutions within this framework.

In summary, we have shown that, although analytical solutions always exist for Hermitian neutrino mass matrices with a single texture zero, not all such patterns yield physically acceptable predictions.

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