

Searching for doubly charmed baryon from $\bar{B}_c$ meson decays*

Ye Xing (邢晔)^{1,2,3†} Ji Xu (徐吉)^{1,2,3‡}

¹School of Physics, China University of Mining and Technology, Xuzhou 221000, China

²School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China

³School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China

Abstract: In this paper, we study the production of doubly charmed baryon from anti-bottom charmed meson. Using the effective Lagrangian approach, we discuss triangle diagrams in hadronic level to get access to the branching ratios of $\bar{B}_c \rightarrow \mathcal{B}_{ccq} + \mathcal{B}_{\bar{c}q\bar{q}}$. It seems that the specific process $\bar{B}_c \rightarrow \Xi_{cc}^+ \Xi_{\bar{c}}^0$ occupies the largest possibility in the order of 9.1×10^{-5} . In addition, although the production of undiscovered Ω_{cc}^+ is Cabibbo suppressed in $\bar{B}_c \rightarrow \Omega_{cc}^+ \Xi_{\bar{c}}^0$, it's branching ratio can still reach 10^{-7} level. These results are expected to be fairly valuable supports for future experiments.

Keywords: Doubly charmed baryon, Bc meson, Effective Lagrangian approach

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I. PHENOMENOLOGICAL ANALYSIS

The LHCb collaboration claimed the experimental evidence of doubly charmed baryon candidate Ξ_{cc}^{++} in the $\Lambda_c^+ K^- \pi^+ \pi^+$ final state in 2017 [1]. Its mass was determined to be 3621.55 MeV [2], and its lifetime measured by the weakly decays, was 0.256 ps [3]. Prior to this, the doubly charmed baryon Ξ_{cc}^+ with a mass of 3518.7 MeV was first reported by the SELEX collaboration in decay modes $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$ and $\Xi_{cc}^+ \rightarrow p D^+ K^-$ [4, 5], its lifetime was found to be less than 33 fs. However, subsequent experiments FOCUS [6], BaBar [7], Belle [8] and LHCb [9] have not confirmed this state yet. Likewise, for Ω_{cc}^+ baryon, the most recent searches from LHCb collaboration still have not reported significant detection signal. Therefore more theoretical and experimental efforts propelling the study of doubly heavy baryons are urgently called for.

It is of key importance for completing the hadron spectrum and revealing the nature of perturbative and non-perturbative QCD dynamics about doubly heavy baryons. Presently various theoretical studies, such as discussions carried by lattice QCD [10–14], quark model [15–22], QCD sum rules [23–28], heavy baryon chiral perturbation theory [29–32] have been applied into the research area of doubly heavy system. For ground states

of Ξ_{cc} , the majority theoretical predictions concentrate on masses and lifetimes, i.e., the masses are declared in the range from 3.5-3.7 GeV, being close to current experimental measurements. Due to effect of the destructive Pauli interference of the c quark decay products and the valence u quark in the initial state, the lifetime of Ξ_{cc}^{++} is expected to be around 2-4 times larger than the one of Ξ_{cc}^+ , consequently most of theoretical predictions show that the lifetime of Ξ_{cc}^+ is in the range of 40-160 fs. For ground state Ω_{cc}^+ , there are also plentiful theoretical predictions, for instance the mass is predicted to be around 3.6-3.9 GeV, and its lifetime is 75-180 fs. We arrange some typical results into Table 1.

The aim of our work is to present a new study about production of doubly charmed baryon from $\bar{B}_c$ meson, it may be achieved by LHCb and future b-factory. We derive the production processes in terms of effective Lagrangian method [33, 34]. The complete transition is divided into weakly decay part associated with transition $\mathcal{O}^i: \bar{b} \rightarrow \bar{c} d \bar{s}$, and strongly couple part related to two strong coupled vertices, i.e., $\mathcal{B}_{cc}\mathcal{B}_c D$ and $\mathcal{B}_c \mathcal{B}_c J/\psi$ ($\mathcal{B}$ is the general baryon), the production can then be described by some triangle diagrams formally in hadronic level. Under the SU(3) light quark flavor symmetry [35–41], it is convenient to relate different production channels, wherein the final states can be one doubly charmed bary-

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† E-mail: xingye_guang@cumt.edu.cn

‡ E-mail: xuji_phy@zzu.edu.cn



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Table 1. The masses and lifetimes of doubly charmed baryon predicted from several theoretical methods or experiments.

	LQCD[11]	QM[17]	[1]	QCDSR[24]	NR model [42]	OPE [43, 44]	Exp(mass & lifetime)	
Ξ_{cc}^{++}	3.610 GeV	3.676 GeV	3.627 GeV	3.72 GeV	0.67 ps	0.52 ps	3.622 GeV [2]	0.256 ps [3]
Ξ_{cc}^+	3.610 GeV	3.676 GeV	3.627 GeV	3.72 GeV	0.25 ps	0.19 ps	3.519 GeV [5]	< 33 fs [5]
Ω_{cc}^+	3.738 GeV	3.832 GeV	—	3.73 GeV	0.21 ps	0.22 ps	—	—

on $\mathcal{B}_{cc}$ (Ξ_{cc}^{++} , Ξ_{cc}^+ , Ω_{cc}^+), and anti-charmed triplet baryon $\mathcal{B}_{\bar{c}3}$ or anti-sextet baryon $\mathcal{B}_{\bar{c}\bar{6}}$,

$$\begin{aligned}
\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Xi}_{\bar{c}}^-) &= \Gamma(\bar{B}_c \rightarrow \Xi_{cc}^+ \bar{\Xi}_{\bar{c}}^0), \\
\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Lambda}_{\bar{c}}^-) &= \Gamma(\bar{B}_c \rightarrow \Omega_{cc}^+ \bar{\Xi}_{\bar{c}}^0), \\
\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Xi}_{\bar{c}}^{\prime -}) &= \Gamma(\bar{B}_c \rightarrow \Xi_{cc}^+ \bar{\Xi}_{\bar{c}}^{\prime 0}), \\
\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Sigma}_{\bar{c}}^-) &= 2\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^+ \bar{\Sigma}_{\bar{c}}^0).
\end{aligned} \quad (1)$$

Furthermore, the rates of decay widths are related to the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{cs} and V_{cd} ,

$$\frac{\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Xi}_{\bar{c}}^-)}{\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Lambda}_{\bar{c}}^-)} = \frac{\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Xi}_{\bar{c}}^{\prime -})}{\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \bar{\Sigma}_{\bar{c}}^-)} = \frac{|V_{cs}|^2}{|V_{cd}|^2}, \quad (2)$$

Consistently, we calculate the branching ratio from effective Lagrangian method, comparing with the predictions from SU(3) analysis in the hope of providing assistance to understand and search for doubly heavy baryons.

The rest of this paper is organized as follows. In Sec. II, we simply introduce the calculation framework about the production of doubly charmed baryons from $\bar{B}_c$ decay, several amplitudes of triangle diagrams according to the effective Lagrangian method are achieved. Then we give numerical analysis and branching ratios of different processes in Sec. III. The last section contains a brief summary.

II. THE BRANCHING RATIOS

The production of doubly charmed baryons $\mathcal{B}_{cc}$ from

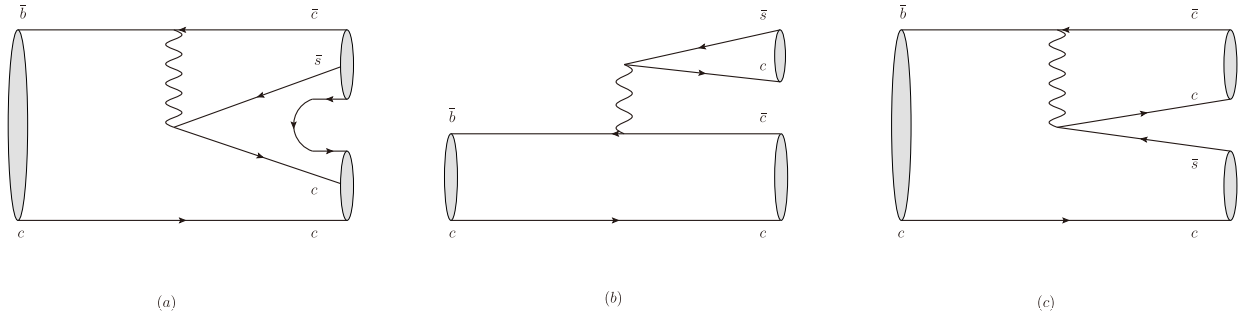


Fig. 1. (a). The production processes of $\bar{B}_c \rightarrow \mathcal{B}_{cc} + \bar{\mathcal{B}}_{\bar{c}}$, which are non-factorizable and color suppressed. (b,c). The weak decays of $\bar{B}_c \rightarrow D(D^*) + J/\psi(\eta_c)$, which possess the factorizable external and internal W -emission diagrams respectively.

$\bar{B}_c$ occurs through the weak decay of bottom quark $\bar{b} \rightarrow \bar{c}d\bar{s}$, whose topological diagram in quark level is shown in Fig. 1(a). The non-factorizable color suppressed diagram with two baryons in final states are unusual, it is hard to handle directly by common factorization schemes. In this paper, we shall consider the process within the framework of the effective Lagrangian approach—a convenient method to study hadrons in hadronic level. The total production matrix element can be further divided into weakly transition matrix as well as strongly coupled matrix by inserting some complete basis,

$$\langle \mathcal{B}_{cc} \bar{\mathcal{B}}_{\bar{c}} | \mathcal{H}_{eff} | \bar{B}_c \rangle = \sum_{\lambda} \langle \mathcal{B}_{cc} \bar{\mathcal{B}}_{\bar{c}} | \mathcal{H}_{\lambda} | \lambda \rangle \langle \lambda | \mathcal{H}_{eff} | \bar{B}_c \rangle. \quad (3)$$

Here H_{λ} represents the strong interaction term written in the interaction picture, which will be given in the following. It is not hard to see that the dominant contribution respecting with weakly transition matrix corresponds to $\langle D(D^*) \eta_c | \mathcal{H}_{eff} | \bar{B}_c \rangle$ or $\langle D(D^*) J/\psi | \mathcal{H}_{eff} | \bar{B}_c \rangle$ (here, we only consider the major contribution $\lambda = D(D^*) J/\psi(\eta_c)$), which is depicted with an accessible emission diagram [45] in Fig. 1(b,c). Meanwhile, the remaining one can be decoded by several effective Lagrangians which will be given below. Combining them, the productions of spin- $\frac{1}{2}$ doubly charmed baryons $\mathcal{B}_{cc}$ (Ξ_{cc}^{++} , Ξ_{cc}^+ , Ω_{cc}^+) stemming from the triangle diagrams can be attained. We proceed with processes under quark transition $\bar{b} \rightarrow \bar{c}c\bar{s}$:

$$\bar{B}_c \rightarrow \Xi_{cc}^{++} + \bar{\Xi}_{\bar{c}}^- (/ \bar{\Xi}_{\bar{c}}^{\prime -}), \quad \bar{B}_c \rightarrow \Xi_{cc}^+ + \bar{\Xi}_{\bar{c}}^0 (/ \bar{\Xi}_{\bar{c}}^{\prime 0}), \quad (4)$$

and the transition $\bar{b} \rightarrow \bar{c}d\bar{d}$:

$$\bar{B}_c \rightarrow \Xi_{cc}^{++} + \bar{\Lambda}_c^- (\bar{\Sigma}_c^-), \quad \bar{B}_c \rightarrow \Xi_{cc}^+ + \bar{\Sigma}_c^0, \quad \bar{B}_c \rightarrow \Omega_{cc}^+ + \bar{\Xi}_c^0. \quad (5)$$

The corresponding triangle diagrams are drawn in Fig. 2. Here, we have excluded some unallowable processes in phase space, as well as the processes with decoupling vertex.

A. Effective Lagrangians

We adopt the effective Lagrangian to study the production of doubly charmed baryons. The process can be regarded as $\bar{B}_c$ meson firstly decays into $D(D^*)$ and $J/\psi(\eta_c)$ mesons, then exchanging with charm baryon $\mathcal{B}_c$ to produce anti-charmed baryon $\bar{\mathcal{B}}_c$ and doubly charmed baryon $\mathcal{B}_{cc}$, $\bar{B}_c \rightarrow M_1 M_2 \xrightarrow{\mathcal{B}_c} \bar{\mathcal{B}}_c \mathcal{B}_{cc}$ ($M_1 M_2$ can be $J/\psi D, \eta_c D, J/\psi D^*$ or ηD^*). The weak decay $\bar{B}_c \rightarrow J/\psi(\eta_c) + D(D^*)$ can occur via the factorizable W -emission process which is shown in Fig. 1(b), it usually offers the largest contribution according to the topological classification of weak decays. The effective Hamiltonian is

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} (C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)) + h.c., \quad (6)$$

here, G_F is the Fermi constant, $C_{1,2}(\mu)$ s are the Wilson coefficients, and $O_{1,2}(\mu)$ s are the tree level fermion operators, with $O_1 = (\bar{b}_\alpha c_\beta)_{V-A} (\bar{c}_\beta s_\alpha)_{V-A}$ and $O_2 = (\bar{b}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta s_\beta)_{V-A}$. The combinations of the effective Wilson coefficient $a_{1,2}$ appears as $a_1 = C_1 + C_2/N_c$, $a_2 = C_2 + C_1/N_c$ with N_c the number of colors.

The amplitude of the weak process can be expressed as the products of two current hadronic matrix elements companied by form factor and decay constant,

$$\begin{aligned} \mathcal{M}(\bar{B}_c \rightarrow J/\psi D(D^*)) &= \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_1 \langle J/\psi | (\bar{b}c)_{V-A} | \bar{B}_c \rangle \langle D(D^*) | (\bar{c}s)_{V-A} | 0 \rangle \\ &+ \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_2 \langle J/\psi | (\bar{b}c)_{V-A} | \bar{B}_c \rangle \langle D(D^*) | (\bar{c}s)_{V-A} | 0 \rangle, \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{M}(\bar{B}_c \rightarrow \eta_c D(D^*)) &= \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_1 \langle \eta_c | (\bar{b}c)_{V-A} | \bar{B}_c \rangle \langle D(D^*) | (\bar{c}s)_{V-A} | 0 \rangle \\ &+ \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_2 \langle \eta_c | (\bar{b}c)_{V-A} | \bar{B}_c \rangle \langle D(D^*) | (\bar{c}s)_{V-A} | 0 \rangle. \end{aligned} \quad (8)$$

The matrix elements between pseudoscalar or vector

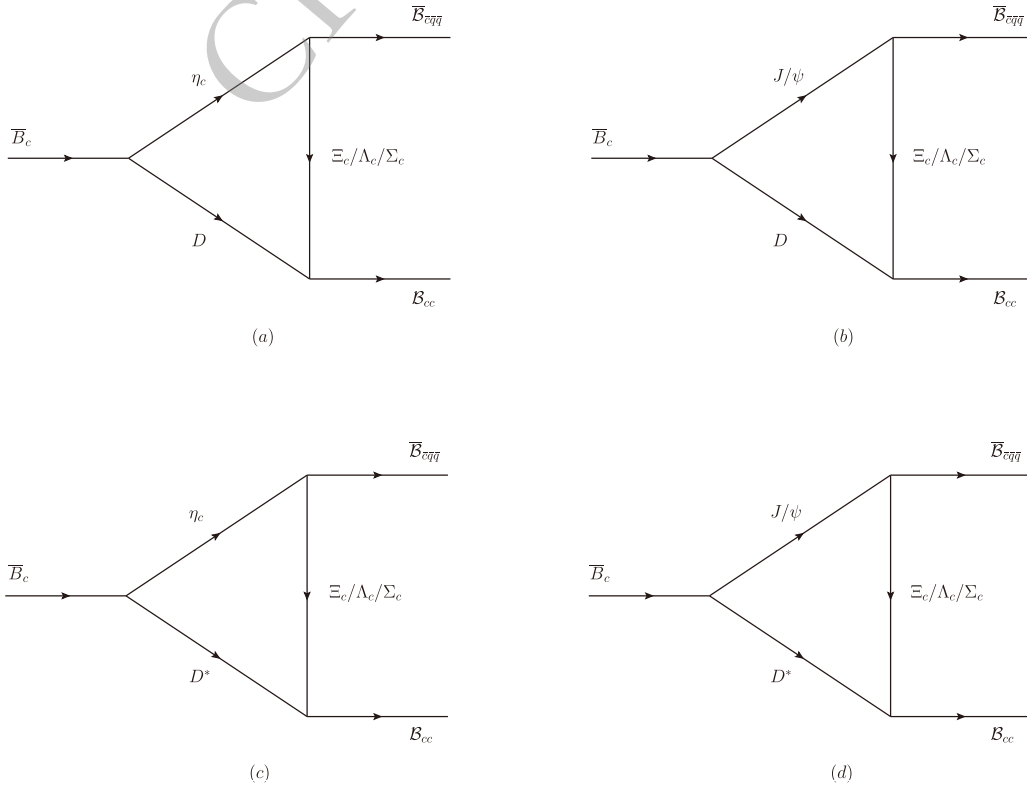


Fig. 2. The triangle diagrams attached to the production of doubly charmed baryons, brought by exchanging charm baryons $\Xi_c, \Lambda_c, \Sigma_c$.

meson and vacuum have the following formations [46],

$$\langle P | (\bar{c}s)_{V-A} | 0 \rangle = i f_P p_\mu, \quad \langle V | (\bar{c}s)_{V-A} | 0 \rangle = m f_V \varepsilon_\mu^*. \quad (9)$$

where P, V stand for pseudoscalar and vector mesons; f_P and f_V are the pseudoscalar and vector meson decay constants; ε_μ^* represents the polarization vector of vector meson.

We follow the parameterized form of the transition $\bar{B}_c \rightarrow J/\psi D(D^*)$ [47],

$$\begin{aligned} & \langle J/\psi(P_V) | (\bar{b}c)_{V-A} | \bar{B}_c(P_{B_c}) \rangle \\ &= \frac{2}{m_{B_c} + m_V} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} P_{B_c}^\rho P_V^\sigma V(q^2) + i \varepsilon_\mu^* (m_{B_c} + m_V) A_1(q^2) \\ & \quad - i \frac{\varepsilon^* \cdot q}{m_{B_c} + m_V} (P_{B_c} + P_V)_\mu A_2(q^2) - i \frac{\varepsilon^* \cdot q}{q^2} 2 m_V q_\mu A_3(q^2) \\ & \quad + i \frac{\varepsilon^* \cdot q}{q^2} 2 m_V q_\mu A_0(q^2), \end{aligned} \quad (10)$$

$$\begin{aligned} & \langle \eta(P_P) | (\bar{b}c)_{V-A} | \bar{B}_c(P_{B_c}) \rangle \\ &= (P_{B_c} + P_P - \frac{m_{B_c}^2 - m_P^2}{q^2} q)_\mu F_1(q^2) + \frac{m_{B_c}^2 - m_P^2}{q^2} q_\mu F_0(q^2). \end{aligned} \quad (11)$$

here ε_μ denotes the polarization vector of J/ψ , transition

momentum $q_\mu \equiv (P_{B_c} - P_P)_\mu$. In order to cancel the poles at $q^2 = 0$, invariant weak form factors $F_0(q^2), F_1(q^2), A_0(q^2)$ and $A_3(q^2)$ satisfy the following conditions:

$$\begin{aligned} F_1(0) &= F_0(0), \quad A_3(0) = A_0(0), \\ A_3(q^2) &= \frac{m_{B_c} + m_V}{2m_V} A_1(q^2) - \frac{m_{B_c} - m_V}{2m_V} A_2(q^2). \end{aligned} \quad (12)$$

In the SU(4) symmetry, the interaction Lagrangians related to the processes are given as [48]

$$\mathcal{L}_{\mathcal{B}\mathcal{B}P} = \frac{g_{\mathcal{B}\mathcal{B}P}}{m_P} \langle \bar{\mathcal{B}} \gamma_5 \gamma_\mu \partial^\mu P \mathcal{B} \rangle, \quad (13)$$

$$\mathcal{L}_{\mathcal{B}\mathcal{B}V} = g_{\mathcal{B}\mathcal{B}V} \langle \bar{\mathcal{B}} \gamma_\mu V^\mu \mathcal{B} \rangle + \frac{f_{\mathcal{B}\mathcal{B}V}}{2m_V} \langle \bar{\mathcal{B}} \sigma_{\mu\nu} \partial_\mu V^\nu \mathcal{B} \rangle. \quad (14)$$

Where $g_{\mathcal{B}\mathcal{B}P}$ and $g_{\mathcal{B}\mathcal{B}V}$ are the couplings of pseudoscalar meson or vector meson and two baryons $\mathcal{B}\mathcal{B}$. Since there is no experimental data to extract values of the needed couplings, we deduce them from the generic SU(4) symmetry, i.e., the empirical values $g_{NN\pi} = 13.5(5)$, $g_{\rho NN} = 3.25(90)$, $f_{\rho NN} = \kappa_\rho g_{\rho NN}$, $\kappa_\rho = 6.1(1)$ [49, 50], are used to reach the couplings between doubly charmed baryon and charmed baryon. One can resort to Appendix A for more details. Within SU(4) symmetry, pseudoscalar and vector mesons reclassified as 15-plet, whose expressions are provided as [51]

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3\eta_c}{\sqrt{12}} \end{pmatrix}. \quad (15)$$

and

$$V_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega + \frac{J/\psi}{\sqrt{12}} & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & -\frac{3J/\psi}{\sqrt{12}} \end{pmatrix}. \quad (16)$$

Baryons respresented as $\mathcal{B}$ belong to a 20-plet,

$$\begin{aligned}
\mathcal{B}^{121} &= p, \quad \mathcal{B}^{122} = n, \quad \mathcal{B}^{132} = \frac{1}{\sqrt{2}}\Sigma^0 - \frac{1}{\sqrt{6}}\Lambda, \quad \mathcal{B}^{213} = \sqrt{\frac{2}{3}}\Lambda, \quad \mathcal{B}^{231} = \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda, \quad \mathcal{B}^{232} = \Sigma^-, \quad \mathcal{B}^{233} = \Xi^-, \\
\mathcal{B}^{311} &= \Sigma^+, \quad \mathcal{B}^{313} = \Xi^0, \quad \mathcal{B}^{141} = -\Sigma_c^{++}, \quad \mathcal{B}^{142} = \frac{1}{\sqrt{2}}\Sigma_c^+ + \frac{1}{\sqrt{6}}\Lambda_c, \quad \mathcal{B}^{143} = \frac{1}{\sqrt{2}}\Xi_c'^+ - \frac{1}{\sqrt{6}}\Xi_c^+, \quad \mathcal{B}^{241} = \frac{1}{\sqrt{2}}\Sigma_c^+ - \frac{1}{\sqrt{6}}\Lambda_c, \\
\mathcal{B}^{242} &= \Sigma_c^0, \quad \mathcal{B}^{243} = \frac{1}{\sqrt{2}}\Xi_c'^0 + \frac{1}{\sqrt{6}}\Xi_c^0, \quad \mathcal{B}^{341} = \frac{1}{\sqrt{2}}\Xi_c'^+ + \frac{1}{\sqrt{6}}\Xi_c^+, \quad \mathcal{B}^{342} = \frac{1}{\sqrt{2}}\Xi_c'^0 - \frac{1}{\sqrt{6}}\Xi_c^0, \quad \mathcal{B}^{343} = \Omega_c^0, \\
\mathcal{B}^{124} &= \sqrt{\frac{2}{3}}\Lambda_c, \quad \mathcal{B}^{234} = \sqrt{\frac{2}{3}}\Xi_c^0, \quad \mathcal{B}^{314} = \sqrt{\frac{2}{3}}\Xi_c^+, \quad \mathcal{B}^{144} = \Xi_{cc}^{++}, \quad \mathcal{B}^{244} = -\Xi_{cc}^+, \quad \mathcal{B}^{344} = \Omega_{cc}^+.
\end{aligned} \tag{17}$$

These states are expressed by tensors $\mathcal{B}^{ijk}$, where the first two indices is anti-symmetric.

B. Amplitudes and Decay Rates

Having the effective Lagrangians at hand, we can obtain the decay amplitudes of Fig. 2. For instance, the amplitude of $\bar{B}_c \rightarrow \Xi_{cc}^{++}\Xi_c^-$ is

$$\begin{aligned}
M_a &= \int \frac{d^4 k_1}{(2\pi)^4} g_{\Xi_{cc}\Xi_c D} g_{\Xi_c\Xi_c\eta_c} \frac{\bar{u}_{\Xi_{cc}}(p_2)\gamma_\nu(k_1 - m_{\Xi_c})\gamma_\mu v_{\Xi_c}(p_1)k_3^\mu k_2^\nu}{(k_1^2 - m_{\Xi_c}^2)(k_2^2 - m_D^2)(k_3^2 - m_{\eta_c}^2)} \mathcal{F}^2(k_1^2)(if_D)((p \cdot k_2 + k_3 \cdot k_2 - m_{B_c}^2 + m_\eta^2)F_1(k_2^2) + (m_{B_c}^2 - m_\eta^2)F_0(k_2^2)), \\
M_b &= i \int \frac{d^4 k_1}{(2\pi)^4} g_{\Xi_{cc}\Xi_c D} g_{\Xi_c\Xi_c\psi} \frac{\bar{u}_{\Xi_{cc}}(p_2)(k_1 - m_{\Xi_c})\gamma_5(\gamma_\mu - i\kappa_\psi\sigma_{\alpha\mu}k_3^\alpha/(2m_\psi))v_{\Xi_c}(p_1)}{(k_1^2 - m_{\Xi_c}^2)(k_2^2 - m_D^2)(k_3^2 - m_\psi^2)} \mathcal{F}^2(k_1^2)(if_D)ik_{2\nu} \\
&\quad (-g^{\mu\nu} + \frac{k_3^\mu k_3^\nu}{m_\psi^2})((m_{B_c} + m_\psi)A_1(k_2^2) - \frac{(p + k_3) \cdot k_2}{m_{B_c} + m_\psi}A_2(k_2^2) - 2m_\psi A_3(k_2^2) + 2m_\psi A_0(k_2^2)), \\
M_c &= i \int \frac{d^4 k_1}{(2\pi)^4} g_{\Xi_{cc}\Xi_c D^*} g_{\Xi_c\Xi_c\eta_c} \frac{\bar{u}_{\Xi_{cc}}(p_2)(\gamma_\mu - i\kappa_{D^*}\sigma_{\alpha\mu}k_2^\alpha/(2m_{D^*}))(k_1 + m_{cqq})\gamma_5(k_1 + \not{p}_1)v_{\Xi_c}(p_1)}{-(k_1^2 - m_{\Xi_c}^2)(k_2^2 - m_{D^*}^2)(k_3^2 - m_{\eta_c}^2)} \\
&\quad \mathcal{F}^2(k_1^2)(-g^{\mu\nu} + \frac{k_2^\mu k_2^\nu}{m_{D^*}^2})(m_{D^*}f_{D^*})((p + k_3 - \frac{m_{B_c}^2 - m_\eta^2}{k_2^2}k_2)^\nu F_1(k_2^2) + \frac{(m_{B_c}^2 - m_\eta^2)}{k_2^2}k_2^\nu F_0(k_2^2)), \\
M_d &= \int \frac{d^4 k_1}{(2\pi)^4} \frac{\bar{u}_{\Xi_{cc}}(p_2)(\gamma_\mu - i\kappa_{D^*}\sigma_{\alpha\mu}k_2^\alpha/(2m_{D^*}))(k_1 + m_{\Xi_c})(\gamma_\nu - i\kappa_\psi\sigma_{\beta\nu}k_3^\beta/(2m_\psi))v_{\Xi_c}(p_1)}{-(k_1^2 - m_{\Xi_c}^2)(k_2^2 - m_{D^*}^2)(k_3^2 - m_\psi^2)} \\
&\quad (g_{\Xi_{cc}\Xi_c D^*} g_{\Xi_c\Xi_c\psi})\mathcal{F}^2(k_1^2)(m_{D^*}f_{D^*})(-g^{\mu\alpha} + \frac{k_2^\mu k_2^\alpha}{m_{D^*}^2})(-g^{\nu\beta} + \frac{k_3^\nu k_3^\beta}{m_\psi^2})(\frac{2}{m_{B_c} + m_\psi}\epsilon_{\alpha\beta\rho\sigma}p^\rho k_2^\sigma V(k_2^2) \\
&\quad + ig_{\alpha\beta}(m_{B_c} + m_\psi)A_1(k_2^2) - i\frac{(p + k_3)_\alpha k_{2\beta}}{m_{B_c} + m_\psi}A_2(k_2^2) - i\frac{k_{2\alpha}k_{2\beta}}{k_2^2}2m_\psi A_3(k_2^2) + i\frac{k_{2\beta}}{k_2^2}2m_\psi k_{2\alpha}A_0(k_2^2)).
\end{aligned} \tag{18}$$

where $k_3 = p_1 + k_1$ is the momentum of $J/\psi(\eta_c)$ and $k_2 = p_2 - k_1$ is the momentum of $D(D^*)$ respectively. Hadrons have finite sizes, therefor the monopole form factor is adopted at each vertex,

$$\mathcal{F}(k_1^2) = \left(\frac{\Lambda^2 - m_i^2}{\Lambda^2 - k_1^2}\right)^2, \tag{19}$$

here Λ is the so-called cutoff mass, which governs the range of suppression. We take the value $\Lambda = m_E + \alpha$ [39], m_E is the mass of exchanged particle and $\alpha = \eta\Lambda_{QCD}$. Usually the parameter η is expected to be of order unity and it depends not only on the exchanged particle but also on the external particles involved in the strong-interac-

tion vertex. In principle, we can not determine it from first-principles calculations. However, we can fix the parameter using the measured decay rates. In this B_c meson system [52], we take naive value for α in range of 100-300 MeV. The decay rate for nonleptonic transition $\bar{B}_c \rightarrow P_1 P_2$ is expressed in terms of the decay amplitude $\mathcal{M}(\bar{B}_c \rightarrow P_1 P_2)$

$$\Gamma(\bar{B}_c \rightarrow \mathcal{B}_{cc} + \bar{\mathcal{B}}_{cqq}) = \frac{|\mathbf{P}_1|}{8\pi m_{B_c}^2} |\mathcal{M}|^2, \tag{20}$$

where $\mathbf{P}_1$ is the magnitude of three momentum of the final state meson, given as $|\mathbf{P}_1| = \frac{1}{2m_{B_c}} \sqrt{\lambda(m_{B_c}^2, m_{\mathcal{B}_{cc}}^2, m_{\bar{\mathcal{B}}_c}^2)}$; $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$.

III. NUMERICAL ANALYSIS

The calculation of branching ratios requires the decay constants of several pseudoscalar and vector mesons, which are presented as follows.

$$\begin{aligned} f_D &= 0.208(15) \text{ GeV}, f_{D^*} = 0.273(12) \text{ GeV}, \\ f_{D^*} &= 0.245(20) \text{ GeV}, f_{D_s^*} = 0.273(19) \text{ GeV}. \end{aligned} \quad (21)$$

Furthermore, the masses of mesons and baryons refer to PDG [53]. Numerous strong couplings are deduced from SU(4) symmetry shown in Appendix A.

On the physical region, each of the transition form factors can be accurately interpolated by the following function [54]:

$$f(q^2) = \alpha_1 + \alpha_2 q^2 + \frac{\alpha_3 q^4}{m_{B_c}^2 - q^2},$$

where the parameters $\alpha_{1,2,3}$ are taken from Table 2.

Armed with these, we determine the production of doubly charmed baryon from $\bar{B}_c$ meson, acquiring the numerical number of widths and branching ratios collected into Table 3, particularly the branching ratio can reach to order 10^{-5} . In our analysis, we fix the parameter $\alpha = \Lambda_{QCD}$ in Λ with a value of 250 MeV [55]. For completeness, the branching ratios respecting to α are studied still in Fig. 3. Moreover, we further verify the dominant sub-process of productions $\bar{B}_c \rightarrow M_1 M_2 \xrightarrow{EP} \bar{B}_c \mathcal{B}_{cc}$ ($M_1 M_2$

$= J/\psi D, \eta_c D, J/\psi D^*$ or ηD^*). In regard to the results, we draw several inferences below.

- Our calculation shows that the production $\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_{\bar{c}}^{-}$ occupies the largest branching ratio among the considering processes in this work, which should be confirmed experimentally. Once verified, it could be an ideal candidate to produce doubly charmed baryon in the future. The dominant contribution of amplitude comes from sub-process $\bar{B}_c \rightarrow J/\psi D \xrightarrow{\Xi_c} \Xi_{cc}^{++} \Xi_{\bar{c}}^{-}$ (branching ratio of the subprocess is 2.1×10^{-5}), and $\bar{B}_c \rightarrow \eta_c D^* \xrightarrow{\Xi_c} \Xi_{cc}^{++} \Xi_{\bar{c}}^{-}$ (branching ratio of the subprocess is 1.1×10^{-5}). Therefore, the major contribution of branching ratio comes from these two sub-processes and their cross ones.

- It seems that the leading contributions of strong couplings are subprocesses with both $\mathcal{B}\mathcal{B}V$ or $\mathcal{B}\mathcal{B}P$ vertices in terms of the processes, i.e., for $\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_{\bar{c}}^{-}$, the isolated contribution from a pair of $\mathcal{B}\mathcal{B}P$ s or $\mathcal{B}\mathcal{B}V$ s is triviality, merely at the order of 10^{-8} .

- Different from the first four Cabibbo allowed processes, the last four processes are Cabibbo suppressed. However, the branching ratio are still considerable. Especially, the undiscovered Ω_{cc} can reach to 10^{-6} order.

- Table 3 shows the rates of decay widths $\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_{\bar{c}}^{-})/\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \Lambda_{\bar{c}}^{-}) \approx 16.4$, $\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_{\bar{c}}^{-})/\Gamma(\bar{B}_c \rightarrow \Xi_{cc}^{++} \Sigma_{\bar{c}}^{-}) \approx 15.5$, which turn out to

Table 2. The interpolation parameters of form factors in Eq.(10), from which α_1 is dimensionless and $\alpha_{2,3}$ have dimension GeV^{-1} .

	$\bar{B}_c \rightarrow \eta_c$			$\bar{B}_c \rightarrow J/\psi$			
	$F_1(0)$	$F_0(0)$	$V_0(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$A_3(0)$
α_1	0.632(13)	0.632(13)	0.836(41)	0.574(33)	0.551(25)	0.561(28)	0.574(33)
α_2	0.032(24)	0.025(11)	0.045(80)	0.030(43)	0.018(38)	0.021(35)	0.030(43)
α_3	0.061(13)	0.034(7)	0.028(110)	0.055(25)	0.036(61)	0.037(77)	0.055(25)

Table 3. Branching ratios of doubly charmed baryons from $\bar{B}_c$ ($\bar{B}_c \rightarrow M_1 M_2 \xrightarrow{EP} \bar{B}_c \mathcal{B}_{cc}$) with $\Lambda = m_{B_c} + 0.25 \text{ GeV}$. The exchanged particles (EP) can be $\Lambda_c, \Xi_c, \Sigma_c$. In addition, we show the dominant contribution from the $M_1 M_2$.

Process	EP	Decay width($\times 10^{-19}$)	Branching ratio($\times 10^{-6}$)	Dominant $M_1 M_2$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_{\bar{c}}^{-}$	Ξ_c	$4.88^{+1.07}_{-0.92}$	$3.78^{+0.83}_{-0.71}$	$\eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{+} \Xi_{\bar{c}}^0$	Ξ_c	$5.51^{+1.27}_{-1.06}$	$4.27^{+0.98}_{-0.82}$	$\eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \Xi_{\bar{c}}^{'-}$	Ξ_c'	$83.1^{+20.1}_{-17.3}$	$64.4^{+15.6}_{-13.4}$	$J/\psi D, \eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{+} \Xi_{\bar{c}}^{'0}$	Ξ_c'	$117.9^{+29.2}_{-24.5}$	$91.4^{+22.6}_{-19.0}$	$J/\psi D, \eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \Lambda_{\bar{c}}^{-}$	Λ_c	$0.30^{+0.08}_{-0.07}$	$0.23^{+0.06}_{-0.05}$	$\eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{+} \Sigma_{\bar{c}}^0$	Σ_c	$11.9^{+3.29}_{-2.72}$	$9.25^{+2.55}_{-2.11}$	$J/\psi D, \eta_c D^*$
$\bar{B}_c \rightarrow \Xi_{cc}^{++} \Sigma_{\bar{c}}^{-}$	Σ_c	$5.37^{+1.50}_{-1.23}$	$4.16^{+1.16}_{-0.95}$	$J/\psi D, \eta_c D^*$
$\bar{B}_c \rightarrow \Omega_{cc}^{+} \Xi_{\bar{c}}^0$	Ξ_c	$0.13^{+0.04}_{-0.03}$	$0.10^{+0.03}_{-0.02}$	$\eta_c D^*$

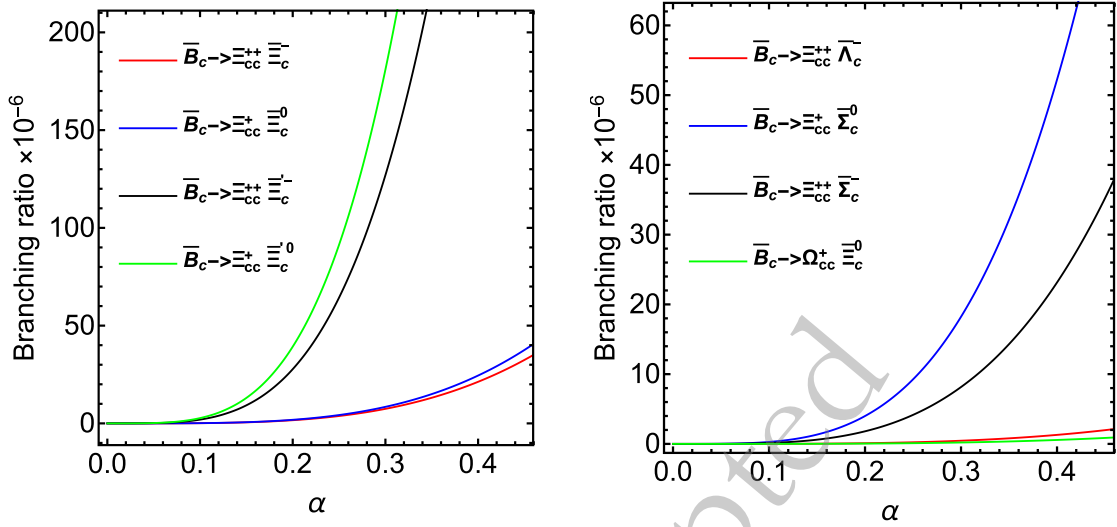


Fig. 3. Branching ratios respecting to α in Λ , α varies from 100-300 MeV. Left image contains four Cabibbo allowed processes, right one describes Cabibbo suppressed processes with transition $\bar{b} \rightarrow \bar{c} c \bar{d}$.

be consistent with straightforward symmetry analysis in Eq. (2), $|V_{cs}|^2/|V_{cd}|^2 \approx 19.9$. This result exhibits that the SU(3) symmetry is basically maintained in these processes.

- We conduct a systematic error analysis, considering the uncertainties of form factors, decay constants, and coupling constants. It is easy to see that the deviation of central value can reach 20%. Further analysis reveals that the main uncertainties come from coupling constant $\mathcal{B}\mathcal{B}V$, i.e. $g_{pNN} = 3.25(90)$, $g_{\Xi_{cc}\Sigma_c D^*} = -4.60(113)$, once excluding these, the error will be less than 10%.

- The decay channels $\bar{B}_c \rightarrow \bar{\mathcal{B}}_c \mathcal{B}_{cc}$ are computed for the first time in this work. We note that there are studies investigating the branching ratio for processes such as $\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p}$ in the framework of perturbative QCD (pQCD) [56], the effective operator involved $\bar{b} \rightarrow \bar{c} \bar{u} d$ is similar with ours $\bar{b} \rightarrow \bar{c} c \bar{s}$. However, it is well established that a complete theoretical prediction within factorization schemes necessitates non-perturbative inputs. In the present case, the wave functions of doubly charmed baryons remain inaccessible, as there are currently no theoretical or experimental studies on the LCDAs. Consequently, we adopt the effective Lagrangian approach, which circumvents the need for explicit knowledge of hadronic LCDAs.

IV. CONCLUSIONS

The observation of Ξ_{cc}^{++} by the LHCb Collaboration opens a new field of research about the nature of baryons containing two heavy quarks. The doubly charmed baryons can form an SU(3) triplet $\Xi_{cc}^{++}, \Xi_{cc}^{+}$ and Ω_{cc}^{+} , therefore, after the experimental observation of Ξ_{cc}^{++} , it is of prime

importance to pursue searches for additional production modes of this particle and the other two. It is worth noting that the charmonium pentaquarks have been discovered from the B -meson decays [57, 58], this inspired us to search for doubly charmed baryons through meson decays.

In this paper, we have presented a theoretical study on the production of doubly charmed baryon from $\bar{B}_c$ meson decays. The decay widths and branching ratios are achieved in terms of effective Lagrangian method and collected in Table 3. We hope these results may provide hints for the exploration of new decay modes in the future.

APPENDIX A. COUPLINGS

The Lagrangian of $\mathcal{B}\mathcal{B}P$ in SU(3) symmetry can be written as,

$$\mathcal{L}_{\mathcal{B}\mathcal{B}P}^{SU3} = a_1 \bar{\mathcal{B}}_j^i \gamma_5 \gamma_\mu \partial^\mu P_k^j \mathcal{B}_i^k + a_2 \bar{\mathcal{B}}_j^i \gamma_5 \gamma_\mu \partial^\mu P_i^k \mathcal{B}_k^j, \quad (\text{A1})$$

here parameters $a_1 = D + F$, $a_2 = D - F$ and $\frac{D}{D+F} = \alpha_D = 0.64$ [59]. Expanding the Lagrangian, we take the couplings of $NN\pi$ and $K\Lambda N$ and the rate is

$$\frac{g_{NN\pi}}{g_{K\Lambda N}} = \frac{\sqrt{3}}{2\alpha_D - 3}. \quad (\text{A2})$$

Furthermore, the Lagrangian of $\mathcal{B}\mathcal{B}P$ in SU(4) symmetry is

$$\begin{aligned}\mathcal{L}_{\mathcal{B}\mathcal{B}P}^{SU4} &= g_0(b_1\bar{\mathcal{B}}_{[jk]}^i\gamma_5\gamma_\mu\partial^\mu P_i^j\mathcal{B}_i^{[kl]} + b_2\bar{\mathcal{B}}_{[kl]}^i\gamma_5\gamma_\mu\partial^\mu P_i^j\mathcal{B}_j^{[kl]}) \\ &= g_0(g_1\bar{\mathcal{B}}^{(ij)k}\gamma_5\gamma_\mu\partial^\mu P_i^l\mathcal{B}_{[l]jk} + g_2\bar{\mathcal{B}}^{[ij]k}\gamma_5\gamma_\mu\partial^\mu P_i^l\mathcal{B}_{[lk]j}).\end{aligned}\quad (\text{A3})$$

Here, we show two equivalent tensor representations of baryon $\mathcal{B}_{[\alpha\beta]}^i\epsilon^{jk\alpha\beta}$ and $\mathcal{B}^{(ij)k}$, $[]$ and $\{\}$ represent symmetric and anti-symmetric indices respectively, $\epsilon^{jk\alpha\beta}$ is the total antisymmetric tensor. Expanding the Lagrangian

$$\frac{g_{NN\pi}}{g_{K\Lambda N}} = \frac{b_2}{-\frac{b_1+4b_2}{2\sqrt{3}}} = \frac{-2\sqrt{3}}{\frac{b_1}{b_2}+4}.\quad (\text{A4})$$

Comparing the results between (A2) and (A4),

$$\frac{b_1}{b_2} = 2 - 4\alpha_D.\quad (\text{A5})$$

It is similar for the case of $\mathcal{B}\mathcal{B}V$ couplings. Generally, the Lagrangian of $\mathcal{B}\mathcal{B}V$ in SU(3) symmetry can be written as

$$\mathcal{L}_{\mathcal{B}\mathcal{B}V}^{SU3} = c_1\bar{\mathcal{B}}_j^i\gamma_\mu(V^\mu)_k^j\mathcal{B}_i^k + c_1\bar{\mathcal{B}}_k^j\gamma_\mu(V^\mu)_i^j\mathcal{B}_j^k + c_2\bar{\mathcal{B}}_k^j\gamma_\mu(V^\mu)_i^j\mathcal{B}_i^k.\quad (\text{A6})$$

Expanding the Lagrangian and comparing the coupling ρNN with $K^*\Lambda N$,

$$g_{\rho NN} = -\sqrt{3}g_{K^*\Lambda N}.\quad (\text{A7})$$

The Lagrangian of $\mathcal{B}\mathcal{B}V$ in SU(4) symmetry is given below,

$$\begin{aligned}\mathcal{L}_{\mathcal{B}\mathcal{B}V}^{SU4} &= g'_0(g'_1\bar{\mathcal{B}}_{[jk]}^i\gamma_\mu(V^\mu)_l^j\mathcal{B}_i^{[kl]} + g'_2\bar{\mathcal{B}}_{[kl]}^i\gamma_\mu(V^\mu)_i^j\mathcal{B}_j^{[kl]} \\ &\quad + g'_3\bar{\mathcal{B}}_{[kl]}^i\gamma_\mu(V^\mu)_j^j\mathcal{B}_i^{[kl]}) \\ &= g'_0(h_1\bar{\mathcal{B}}^{(ij)k}\gamma_\mu(V^\mu)_i^l\mathcal{B}_{[l]jk} + h_2\bar{\mathcal{B}}^{[ij]k}\gamma_\mu(V^\mu)_i^l\mathcal{B}_{[lk]j}).\end{aligned}$$

We again show two equivalent formulas as before. Similarly, one can get

$$\frac{g_{\rho NN}}{g_{K^*\Lambda N}} = \frac{g'_2}{-\frac{g'_1+4g'_2}{2\sqrt{3}}} = \frac{-2\sqrt{3}}{\frac{g'_1}{g'_2}+4},\quad (\text{A9})$$

finally, we have

$$\frac{g'_1}{g'_2} = -2.\quad (\text{A10})$$

According to the couplings $g_{NN\pi} = 13.5(5)$, $g_{\rho NN} = 3.25(90)$,

$\kappa_\rho = 6.1(1)$ as well as the relations in (A5) and (A10), we

determine the required couplings,

$$\begin{aligned}g_{\Xi_c\Xi_c\eta_c} &= \frac{1}{3\sqrt{6}}(b_1 - 5b_2) = -10.2(3), & g_{\Xi_c\Xi'_c\eta_c} &= 0, \\ g_{\Lambda_c\Lambda_c\eta_c} &= \frac{1}{3\sqrt{6}}(b_1 - 5b_2) = -10.2(3), & g_{\Lambda_c\Sigma_c\eta_c} &= 0, \\ g_{\Sigma_c\Sigma_c\eta_c} &= \frac{1}{\sqrt{6}}(b_2 - b_1) = 8.6(3), \\ g_{\Xi'_c\Xi'_c\eta_c} &= \frac{1}{\sqrt{6}}(b_2 - b_1) = 8.6(3), \\ g_{\Xi_{cc}^{++}\Xi_{cc}D_s} &= -\frac{1}{\sqrt{3}}(b_1 + b_2) = -3.4(2), \\ g_{\Xi_{cc}^{++}\Xi'_cD_s} &= b_2 = 13.5(5), \\ g_{\Xi_{cc}^{++}\Xi_cD_s} &= -\frac{1}{\sqrt{3}}(b_1 + b_2) = -3.4(2), \\ g_{\Xi_{cc}^{++}\Xi'_cD_s} &= -b_2 = -13.5(5), \\ g_{\Xi_{cc}^{++}\Lambda_cD} &= \frac{1}{\sqrt{3}}(b_1 + b_2) = 3.4(2), \\ g_{\Xi_{cc}^{++}\Sigma_cD} &= b_2 = 13.5(5), \\ g_{\Xi_{cc}^{++}\Sigma_cD} &= -\sqrt{2}b_2 = -19.1(7), \\ g_{\Omega_{cc}^{+}\Xi_cD} &= -\frac{1}{\sqrt{3}}(b_1 + b_2) = -3.4(2), \\ g_{\Xi_c\Xi_c J/\psi} &= \frac{1}{3\sqrt{6}}(g_1 - 5g_2) = -3.10(75), & g_{\Xi_c\Xi'_c J/\psi} &= 0, \\ g_{\Lambda_c\Lambda_c J/\psi} &= \frac{1}{3\sqrt{6}}(g_1 - 5g_2) = -3.10(75), & g_{\Lambda_c\Sigma_c J/\psi} &= 0, \\ g_{\Sigma_c\Sigma_c J/\psi} &= \frac{g_2 - g_1}{\sqrt{6}} = 3.98(90), \\ g_{\Xi'_c\Xi'_c J/\psi} &= \frac{g_2 - g_1}{\sqrt{6}} = 3.98(90), \\ g_{\Xi_{cc}^{++}\Xi_{cc}D_s^*} &= -\frac{1}{\sqrt{3}}(g_1 + g_2) = 1.88(45), \\ g_{\Xi_{cc}^{++}\Xi'_cD_s^*} &= g_2 = 3.25(90), \\ g_{\Xi_{cc}^{++}\Xi_cD_s^*} &= -\frac{1}{\sqrt{3}}(g_1 + g_2) = 1.88(45), \\ g_{\Xi_{cc}^{++}\Xi'_cD_s^*} &= -g_2 = -3.25(90), \\ g_{\Xi_{cc}^{++}\Lambda_cD^*} &= \frac{1}{\sqrt{3}}(g_1 + g_2) = -1.88(45), \\ g_{\Xi_{cc}^{++}\Sigma_cD^*} &= g_2 = 3.25(90), \\ g_{\Xi_{cc}^{++}\Sigma_cD^*} &= -\sqrt{2}g_2 = -4.60(113), \\ g_{\Omega_{cc}^{+}\Xi_cD^*} &= -\frac{1}{\sqrt{3}}(g_1 + g_2) = 1.88(45).\end{aligned}\quad (\text{A11})$$

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