

Extracting the neutron skin thickness of ^{124}Sn from the $^{12}\text{C} + ^{124}\text{Sn}$ elastic scattering angular distribution*

Zhi-Cheng Zhang (张智程)¹ Zhi-Hong Li (李志宏)^{1,2,3†} Ge-Xing Li (李歌星)¹ Chen Chen (陈晨)¹
Na Song (宋娜)¹ Chao Dong (董超)¹ Jun-Wen Tian (田俊文)¹ Jia-Ying-Hao Li (李家英豪)¹

¹China Institute of Atomic Energy, Beijing 102413, China

²School of Nuclear Science and Technology, University of Chinese Academy of Science, Beijing 101408, China

³Jinping Deep Underground Frontier Science and Dark Matter Key Laboratory of Sichuan Province, Liangshan 615000, China

Abstract: The angular distribution of elastic scattering is highly sensitive to the surface region of the nucleus, making it a powerful tool for measuring the neutron skin thickness. Utilizing the CDM3Y6 double-folding potential, we extracted the neutron skin thickness of ^{124}Sn from the $^{12}\text{C} + ^{124}\text{Sn}$ elastic scattering angular distribution, obtaining values of $0.168^{+0.025}_{-0.019}$ fm (SLy4) and 0.177 ± 0.022 fm (SLy7). These results are consistent with measurements from various other methods. Furthermore, through correlation analysis between the neutron skin thickness and nuclear symmetry energy slope parameter L , we determined the symmetry energy slope coefficients to be $L = 39.0^{+20.8}_{-16.5}$ MeV (SLy4) and $L = 46.1 \pm 18.7$ MeV (SLy7) based on the scattering data. These findings validate existing theoretical models and provide valuable insights for further studies on neutron stars and nuclear matter properties.

Keywords: Neutron skin thickness, nuclear symmetry energy, double-folding potential

DOI: 10.1088/1674-1137/adcc8f **CSTR:** 32044.14.ChinesePhysicsC.49071002

I. INTRODUCTION

Neutron stars, the remnants of massive stellar explosions, are among the most extreme and fascinating objects in the universe. Their interiors are composed of ultra-dense matter, with densities exceeding those of atomic nuclei, making them natural laboratories for studying the properties of matter under extreme conditions. Understanding the equation of state (EoS) of dense matter is crucial for interpreting neutron star observations, such as their masses, radii, and tidal deformabilities [1–3].

The EoS of neutron star matter is intimately connected to the nuclear symmetry energy, which characterizes the energy cost of deviating from equal numbers of protons and neutrons in nuclear systems. In neutron-rich nuclei, the symmetry energy plays a critical role in determining the distribution of protons and neutrons, which is governed by the isospin asymmetry $\delta = (N - Z)/A$ of the system. The symmetry energy not only influences the structure of neutron-rich nuclei but also determines the density dependence of the EoS in neutron stars, making it a key quantity for both nuclear and astrophysical studies [4, 5].

The nuclear symmetry energy $E_{\text{sym}}(\rho)$ is a key quant-

ity describing the energy difference between nuclear matter composed of protons and neutrons at equal density. It can be expanded as a function of nuclear density ρ around the saturation density ρ_0 :

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3} \left(\frac{\rho - \rho_0}{\rho_0} \right) + \dots, \quad (1)$$

where L is the slope parameter of the symmetry energy at ρ_0 :

$$L = 3\rho_0 \left. \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \right|_{\rho=\rho_0}. \quad (2)$$

Although the slope parameter is typically extracted around the saturation density, this information is essential for extrapolating to both high and low densities.

Compared to the symmetry energy at saturation density, the uncertainties in the slope parameter L are even larger, and extensive research has been conducted. One approach involves determining the neutron skin thickness of atomic nuclei to extract the slope parameter of the symmetry energy. In 2009, Centelles *et al.* determined the symmetry energy slope parameter to be $L = 55 \pm 25$ MeV

Received 20 February 2025; Accepted 15 April 2025; Published online 16 April 2025

* Supported by the National Natural Science Foundation of China (12475151, 12405164) and the Continuous-Support Basic Scientific Research Project (BJ010261223284)

† E-mail: zhliciae@163.com

©2025 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

using neutron skin thickness data from 26 stable nuclei measured in antiprotonic atom experiments, combined with liquid-drop model calculations and multiple nuclear interaction models [6]. In 2010, Chen *et al.* derived $L = 58 \pm 18$ MeV by analyzing the neutron skin thickness of ^{208}Pb through Skyrme-Hartree-Fock (SHF) calculations and constraints from heavy-ion collision experiments [7]. The following year, Chen reported $L = 52.5 \pm 20$ MeV based on neutron skin measurements of tin isotopes [8]. In 2012, Agrawal *et al.* obtained $L = 64 \pm 5$ MeV using ^{208}Pb neutron skin data with relativistic mean-field models [9]. In 2013, Zhang *et al.* found $L = 45.2 \pm 10$ MeV via SHF calculations on tin isotopes [10]. The following year, Danielewicz and Lee used the neutron skin thickness of ^{208}Pb , combined with isobaric analog state data and the Hohenberg-Kohn functional method, to constrain L to a range of 35–70 MeV [11]. In 2020, Xu *et al.* employed Bayesian analysis of Sn isotope data to estimate $L = 53.4^{+18.6}_{-29.5}$ MeV [12]. Although these data do not fully encompass all current theoretical predictions, it can be observed that the average value of L remains consistent across these analyses. Furthermore, by analyzing recent experimental data on the neutron skin thickness of ^{208}Pb published by the PREX-II collaboration [13], L predicted by the relativistic mean-field theory is 106 ± 37 MeV [14]. This value differs significantly from previous results obtained based on terrestrial laboratory experiments and astrophysical observations, and it also falls outside the range of symmetry energy slope parameters inferred from GW170817 observations [15]. Building upon the PREX measurement results, subsequent studies have focused on reconciling these discrepancies. Lattimer derived $L = 53 \pm 13$ MeV by combining PREX and CREX neutron skin data with nuclear mass systematics [16]. Essick *et al.* obtained $L = 53^{+14}_{-15}$ MeV using nonparametric EoS methods with PREX-II ^{208}Pb data and chiral effective field theory [17]. Hu *et al.* constrained L to 38.3–68.5 MeV via ab initio quantum many-body calculations of ^{208}Pb neutron skin [18], while Lim *et al.* reported 54 ± 8 MeV through Bayesian analysis of ^{208}Pb measurements [19].

Long isotopic chains provide an excellent opportunity to study the impact of symmetry energy on nucleon distributions at different δ . Tin (Sn) isotopes, with their magic number ($Z = 50$) shell structure and an extended chain of stable isotopes ($^{112-124}\text{Sn}$), are particularly advantageous for such investigations. Several methods, including proton scattering [20–22], inelastic α -scattering [23, 24], spin-dipole resonance [25], and antiprotonic atom spectroscopy [26], have been employed to study the neutron skin.

In the present work, we first quantitatively analyzed the correlation between L and the neutron skin thickness of $^{112-124}\text{Sn}$. Then, the neutron skin thickness of ^{124}Sn was extracted from the elastic scattering angular distribu-

tion of $^{12}\text{C}+^{124}\text{Sn}$. Finally, the extracted neutron skin thickness was utilized to place constraints on the symmetry energy in the high-density region. The results provide valuable references for further studies on neutron stars and nuclear matter properties.

II. CORRELATION BETWEEN L AND Δ_{np}

To study the correlation between the slope coefficient of the symmetry energy and neutron skin of Sn isotopes with different δ , we employed the SHF model, which has been widely used in nuclear physics. The SHF model, as described in Refs. [27–32], employs an interaction characterized by zero range, along with dependencies on both density and momentum. The parameters of the Skyrme interaction are calibrated to reproduce the binding energies and charge radii of numerous nuclei across the periodic table. For infinite nuclear matter, the nuclear symmetry energy from the Skyrme interaction can be expressed as follows [31, 32]:

$$E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left(\frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} - \frac{1}{8} t_0 (2x_0 + 1) \rho - \frac{1}{48} t_3 (2x_3 + 1) \rho^{\sigma+1} + \frac{1}{24} \left(\frac{3\pi^2}{2} \right)^{2/3} \times [-3t_1 x_1 + (4 + 5x_2) t_2] \rho^{5/3}, \quad (3)$$

where σ , t_0 – t_3 , and x_0 – x_3 are Skyrme interaction parameters.

The neutron skin thickness of a nucleus is defined as the difference between the root-mean-square radii of the neutron and proton distributions:

$$\Delta_{\text{np}} = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}, \quad (4)$$

The SHF model calculates neutron skin thickness by self-consistently solving mean-field equations with selected Skyrme interactions, computing neutron and proton density distributions, and evaluating the neutron-proton root-mean-square radius difference. The neutron skin thickness is sensitive to the density dependence of the nuclear symmetry energy, particularly the slope parameter L at normal nuclear matter density [27, 33–38]. Using 184 sets of Skyrme interaction parameters, we evaluated the correlation between Δ_{np} and L of $^{116,117,118,119,120,122,124}\text{Sn}$, as shown in Fig. 1. The 184 sets of Skyrme interaction parameters are BSk1, BSk2, BSk2', BSk3, BSk4, BSk5, BSk6, BSk7, BSk8, BSk9, BSk10, BSk11, BSk12, BSk13, BSk14, BSk15, BSk16, BSk17, FPLyon, KDE, KDE0v, KDE0v1, LNS, MSk1, MSk2, MSk3, MSk4, MSk5, MSk5*, MSk6, MSk7, MSk8, MSk9, MSkA, MSL0, NRAPR, Sefm074, Sefm081, Sefm09, Sefm1, SGI, SGII, SGOI, SI, SII, SIII, SIII*, SIV, Skyrme1',

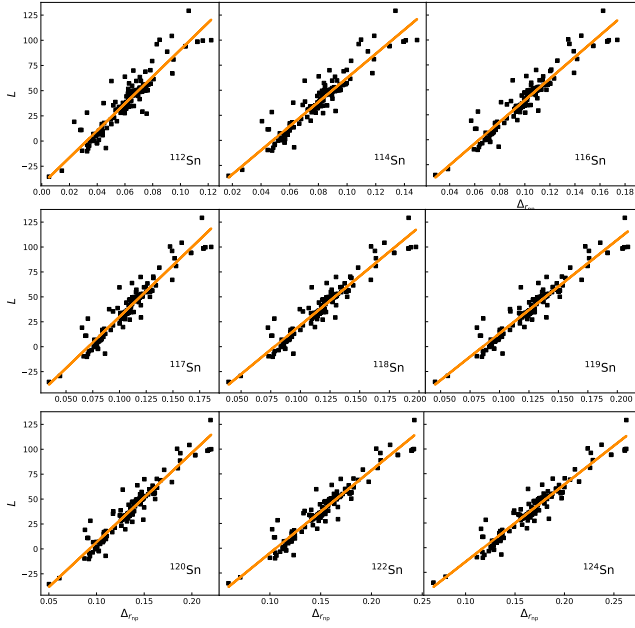


Fig. 1. (color online) L as a function of neutron skin thickness Δr_{np} for 184 sets of Skyrme interaction parameters and for stable Sn isotopes. The line is a linear fit to the data.

SkI2, SkI3, SkI5, SkM, SkM*, SkM1, SkMP, SkO', SkP, SKRA, SkS1, SkS2, SkS3, SkS4, SkSC1, SkSC2, SkSC3, SkSC4, SkSC4o, SkSC5, SkSC6, SkSC10, SkSC11, SkSC14, SkSC15, SkT, SkT1, SkT2, SkT3, SkT4, SkT5, SkT6, SkT7, SkT8, SkT9, SkT1*, SkT3*, SkT1a, SkT2a, SkT3a, SkT4a, SkT5a, SkT6a, SkT7a, SkT8a, SkT9a, SkTK, SKX, SKXce, SKXm, Skxs15, Skxs20, Skxs25, SKz1, SKz2, SKz3, SKz4, SLy0, SLy1, SLy2, SLy230a, SLy230b, SLy3, SLy4, SLy5, SLy6, SLy7, SLy8, SLy9, SLy10, SSK, SV, SV-bas, SV-min, SVI, SVII, SV-K218, SV-K226, SV-K241, SV-kap00, SV-kap02, SV-kap06, SV-mas07, SV-mas08, SV-mas10, SV-sym28, SV-sym32, SV-sym34, SV-tls, T, T11, T12, T13, T14, T15, T16, T21, T22, T23, T24, T25, T26, T31, T32, T33, T34, T35, T36, T41, T42, T43, T44, T45, T46, T51, T52, T53, T54, T55, T56, T61, T62, T63, T64, T65, T66, v070, v075, v080, v090, v100, v105, v110, Zs, and Zs*. The parameters of these Skyrme interactions can be found in Ref. [39]. The solid lines in the figures represent the linear fit of the correlation between Δr_{np} and L .

It can be observed from Fig. 1 that Sn isotopes exhibit an approximate linear correlation between L and Δr_{np} . To give a quantitative estimate of the above correlations, we define the following linear correlation coefficient R^2 :

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}, \quad (5)$$

where SS_{res} and SS_{tot} are the residual sum of squares (RSS) and total sum of squares, respectively:

$$SS_{\text{res}} = \sum_{i=1}^n [L_i - (a + b\Delta r_{np})]^2, \quad (6)$$

$$SS_{\text{tot}} = \sum_{i=1}^n (L_i - \bar{L})^2, \quad \bar{L} = \frac{1}{n} \sum_{i=1}^n L_i, \quad (7)$$

where a and b are linear regression coefficients. The linear correlation coefficient R^2 measures the degree of linear correlation, and $R^2 = 1$ corresponds to an ideal linear correlation. Table 1 presents the linear correlation coefficient R^2 for the correlations between L and Δr_{np} of Tin isotopes under different Skyrme interactions, as shown in Fig. 1, as well as the slope and intercept of the linear fit. From the figure, it also can be seen that as the isospin asymmetry increases, the linear relationship between L and Δr_{np} becomes stronger, with the strongest linearity between L and Δr_{np} observed in the ^{124}Sn nucleus. Therefore, within the stable isotopic chain of Sn, the neutron skin thickness of ^{124}Sn will provide the most effective constraint on L .

The fitted line for ^{124}Sn is given by

$$L = (-92.58 \pm 2.83) + (783.39 \pm 16.54) \times \Delta r_{np}(^{124}\text{Sn}). \quad (8)$$

where L and Δr_{np} are measured in units of MeV and fm, respectively. The errors are mainly due to statistical fluctuations arising from different interactions.

III. NEUTRON SKIN THICKNESS OF ^{124}Sn

The angular distributions of elastic scattering for $^{12}\text{C} + ^{124}\text{Sn}$ at 66 MeV were measured using the high-precision Q3D magnetic spectrograph at the Beijing HI-13 tandem accelerator of the China Institute of Atomic Energy [40]. The isotopic abundances and thicknesses of the tin targets, as well as the experimental setup, are detailed in Ref. [41]. The measured experimental differential cross-section to Rutherford cross-section ratio and uncer-

Table 1. Linear correlation coefficient R^2 , intercept a , and slope b for the correlation between L and Δr_{np} of Sn isotopes under different Skyrme interactions.

nuclei	a	b	R^2 (%)
^{116}Sn	-68.82 ± 2.66	1084.08 ± 26.11	90.45
^{117}Sn	-73.12 ± 2.64	1031.54 ± 23.72	91.22
^{118}Sn	-76.79 ± 2.62	983.36 ± 21.80	91.79
^{119}Sn	-80.64 ± 2.65	939.13 ± 20.41	92.09
^{120}Sn	-83.08 ± 2.66	897.35 ± 19.13	92.36
^{122}Sn	-88.39 ± 2.74	835.08 ± 17.64	92.49
^{124}Sn	-92.58 ± 2.83	783.39 ± 16.54	92.50

tainties are shown as black dots and error bars in the lower panel of Fig. 2, respectively.

We used the FRESKO code [42] to analyze elastic scattering data, and the differential cross-section to Rutherford cross-section ratio was calculated. The total interaction potential can be written as the sum of three terms:

$$V_T^\ell(r) = V_C(r) + V_l(r) + V_n(r), \quad (9)$$

where r is the separation distance between the interacting nuclei. $V_C(r)$ and $V_l(r)$ are the Coulomb potential and centrifugal potential, respectively. The last term in Eq. (9) denotes the short-range and attractive nuclear potential and consists of real and imaginary potential parts, generated using the fully microscopic (FM) method. The FM method calculates the real and imaginary parts of nuclear potential using the double-folded optical model based on the CDM3Y6 effective NN interactions, *i.e.*, $V_n(r) = N_R V_n^{\text{DF}}(r) + i N_I V_n^{\text{DF}}(r)$, where $V_n^{\text{DF}}(r)$ is the double-folded potential generated by the BiFold code [43], and N_R and N_I are the normalization factors of the real and imaginary parts, respectively, which are adjusted to obtain the best fit to experimental data. This method was successfully applied in Refs. [44–46] to study of neutron skin and neutron halo.

The CDM3Y6 double-folded potential has two parts, the direct part V_D and exchange part V_{Ex} , and can be written as [47, 48]

$$V_n^{\text{DF}}(r) = V_D(r) + V_{\text{Ex}}(r) = \int dr_1 \int dr_2 \rho_p \rho_t V_D(s) + \int dr_1 \int dr_2 \rho_p \rho_t V_{\text{Ex}}(s), \quad (10)$$

where V_D and V_{Ex} represent the direct and exchange effective NN interactions, respectively. Using M3Y-Paris, V_D and V_{Ex} are expressed as [47, 48]

$$V_D(s) = 11062 \frac{e^{-4s}}{4s} - 2538 \frac{e^{-2.5s}}{2.5s} \text{ MeV}, \quad (11)$$

and

$$V_{\text{Ex}}(s) = -1524 \frac{e^{-4s}}{4s} - 518.8 \frac{e^{-2.5s}}{2.5s} - 7.847 \frac{e^{0.7072s}}{0.7072s} \text{ MeV}, \quad (12)$$

Both V_D and V_{Ex} are scaled by a weak energy-dependent function $g(E)$ and density-dependent function $F(\rho)$ [47, 48]

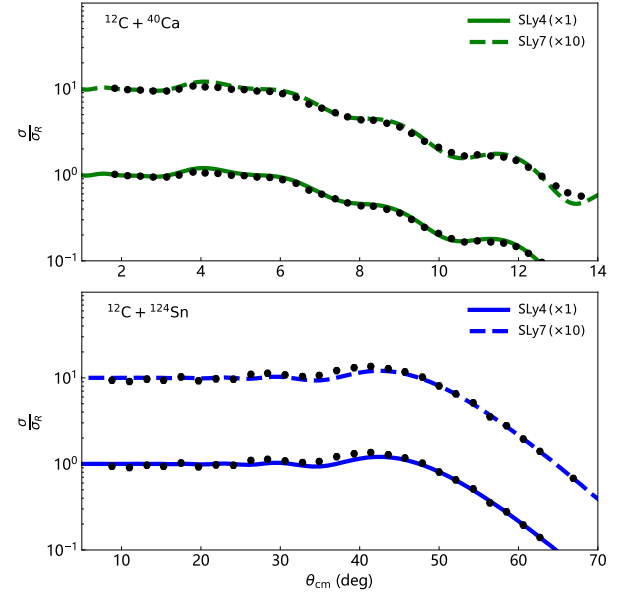


Fig. 2. (color online) Differential cross-section to Rutherford cross-section ratio for $^{12}\text{C}+^{40}\text{Ca}$ at $E_{\text{c.m.}} \approx 180$ MeV (upper panel) and $^{12}\text{C}+^{124}\text{Sn}$ at $E_{\text{c.m.}} \approx 66$ MeV (lower panel). The dots are the experimental values, while the solid line corresponds to the DF calculations.

$$V_{D(\text{Ex})}(E, \rho, s) = g(E)F(\rho)V_{D(\text{Ex})}(s), \quad (13)$$

The energy-dependent function is expressed as [47, 48]

$$g(E) = 1 - 0.003 \frac{E_{\text{lab}}}{A_p}, \quad (14)$$

where E_{lab} is the incident energy in MeV, and A_p is the mass of the projectile nucleus. Furthermore, the density-dependent function $F(\rho)$ can be written as [47, 48]

$$F(\rho) = 0.2658(1 + 3.8033 \exp(1.4099\rho) - 4.0\rho), \quad (15)$$

where ρ is the overlapping density of the interacting nuclei.

The matter density distribution of ^{12}C projectile nucleus is expressed by the two-parameter Fermi (2pF) model $\rho(r) = \rho_0 / \left[1 + \exp\left(\frac{r-R}{a}\right) \right]$ with $\rho_0 = 0.207 \text{ fm}^{-3}$, $R = 2.1545 \text{ fm}$, and $a = 0.425 \text{ fm}$ [45]. For an ^{124}Sn target nucleus, we consider proton and neutron densities calculated with the SHF with the widely used SLy4 parameter set and its refined variant SLy7 as the original density $\rho(r)$. The scaled density $\rho_{\text{scaling}}(r)$ is determined from the original density $\rho(r)$ as

$$\rho_{\text{scaling}}(r) = \frac{1}{\alpha^3} \rho(r/\alpha), \quad (16)$$

with a scaling factor

$$\alpha = \sqrt{\frac{\langle r^2 \rangle_{\text{scaling}}}{\langle r^2 \rangle}}. \quad (17)$$

The actual procedure to determine α (of p and n) for each case is as follows. First, we scale the proton density according to $R_p(\text{scaling}) = R_p(\text{exp})$ [21, 49]. Although this is a minute adjustment, secondly, we scale the neutron density such that the theoretical angular distribution reproduces the experimental data well [24].

To verify the reliability of the method, we tested the results of ^{40}Ca neutron skin thickness extracted from elastic scattering of $^{12}\text{C} + ^{40}\text{Ca}$. The experimental angular distribution data was digitally extracted from Fig. 3 of Ref. [50], shown as black dots and error bars in the upper panel of Fig. 2. The values of N_R and N_I that provide the best fit to the data are 0.85 and 0.6, and the scaling factors α_p and α_n are listed in Table 2. The results are shown and compared with the experimental data in the upper panel of Fig. 2. The errors of radii were extracted by adjusting α_n such that the chi-squared χ^2 value $\chi^2_{\min} + 1$. We determined the neutron skin thickness of ^{40}Ca as $-0.018^{+0.012}_{-0.015}$ fm (SLy4) and -0.013 ± 0.013 fm (SLy7). These values together with RMS radius of neutron and mass are listed in Table 3. The neutron skin thicknesses are consistent with $-0.010^{+0.022}_{-0.024}$ fm by J. Zenihiro *et al.* [51] and -0.011 ± 0.011 fm by T. Wakasa *et al.* [52].

Next, we applied the same methodology to ^{124}Sn . The values of N_R and N_I that provide the best fit to the data are 0.7 and 0.4, respectively, and the scaling factors α_p and α_n are listed in Table 2. The results are shown and compared with the experimental data in the lower panel

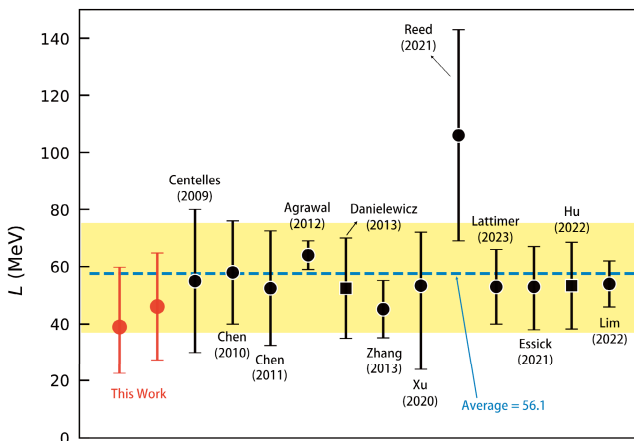


Fig. 3. (color online) Constraints on L from this work and 12 other studies. The blue dashed line represents the mean of all 14 results, and the yellow band indicates the 1σ uncertainty of the mean. Square markers denote data with no error information.

of Fig. 2. We determined the neutron skin thickness of ^{124}Sn as $0.168^{+0.025}_{-0.019}$ fm (SLy4) and 0.177 ± 0.022 fm (SLy7). These values are also listed in Table 3.

Table 4 lists the neutron skin thickness Δr_{np} of ^{124}Sn from this work and other studies. It can be seen that the neutron skin thickness extracted in the present work is consistent with the values obtained by other methods. Substituting the neutron skin thickness into Eq. (8), we obtained a constraint on the slope parameter of the symmetry energy: $L = 39.0^{+20.8}_{-16.5}$ MeV (SLy4) and $L = 46.1 \pm 18.7$ MeV (SLy4). Figure 3 presents the results of this work and other studies mentioned in the introduction. As evident from the figure, our data align with other data within the error margins.

The average of all 14 results shown in Fig. 3 yields $L = 56.1 \pm 18.9$ MeV. Using the results from this work (SLy7) and the average, we derive a constraint on the symmetry energy described by $E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) \times (\rho/\rho_0)^{L/3E_{\text{sym}}(\rho_0)}$ [53], as indicated by the red-shaded and blue-shaded regions in Fig. 4, respectively. Among the 184 sets of Skyrme interaction parameters mentioned earlier, only 62 sets satisfy the constraints of this work

Table 2. Scaling factors α_p and α_n for neutrons and protons.

Isotope	SLy4		SLy7	
	α_p	α_n	α_p	α_n
^{40}Ca	0.9832	0.9917	0.9893	0.9969
^{124}Sn	0.9925	0.9924	0.9939	0.9960

Table 3. Our results for R_n , R_m , and Δr_{np} . The radii are in units of fm.

Isotope	Skyrme set	R_n	R_m	Δr_{np}
^{40}Ca	SLy4	$3.366^{+0.012}_{-0.015}$	3.375 ± 0.007	$-0.018^{+0.012}_{-0.015}$
^{40}Ca	SLy7	3.371 ± 0.013	3.378 ± 0.007	-0.013 ± 0.013
^{124}Sn	SLy4	$4.767^{+0.025}_{-0.019}$	$4.700^{+0.014}_{-0.012}$	$0.168^{+0.025}_{-0.019}$
^{124}Sn	SLy7	4.775 ± 0.022	4.705 ± 0.014	0.177 ± 0.022

Table 4. Neutron skin thickness Δr_{np} of ^{124}Sn from different research.

Method	$\Delta r_{\text{np}}/\text{fm}$	Ref.
^{12}C Scattering	$0.168^{+0.025}_{-0.019}$	This work
	0.177 ± 0.022	This work
Proton Scattering	0.25 ± 0.05	[20]
Inelastic α -Scattering	0.21 ± 0.11	[23]
Spin-Dipole Resonance	0.19 ± 0.07	[25]
Proton Scattering	0.185 ± 0.017	[21]
^4He Reaction Cross Sections	0.180 ± 0.142	[24]
Proton Scattering	0.156 ± 0.022	[22]

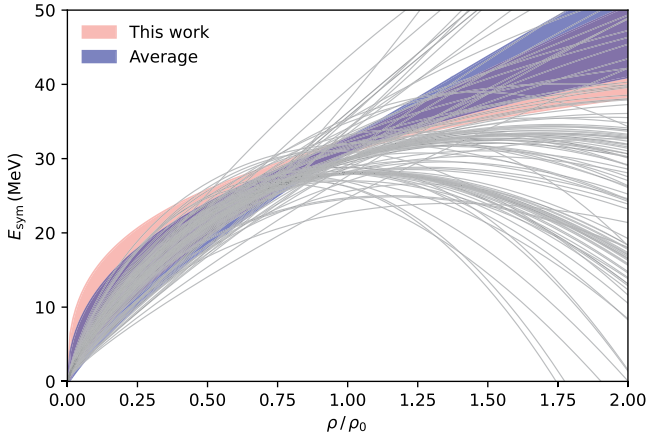


Fig. 4. (color online) Boundary of the constrained region for the symmetry energy as a function of density. The gray lines depict the density dependence of the symmetry energy with $\rho_0 = 0.16 \text{ fm}^{-3}$ for 122 sets of Skyrme interaction parameters that do not satisfy the constraints of the average.

(FPLyon, KDE, LNS, SkM, SKRA, SkT1, SkT2, SkT3, SkT1*, SkT3*, SkT1a, SkT2a, SkT3a, SLy0, SLy1, SLy2, SLy230a, SLy230b, SLy3, SLy4, SLy5, SLy6, SLy7, SLy8, SLy10, SV-sym32, T11, T12, T13, T14, T15, T16, T21, T22, T23, T24, T25, T26, T31, T32, T33, T34, T35, T36, T41, T42, T43, T44, T45, T46, T51, T52, T53, T54, T55, T56, T61, T62, T63, T64, T65, and T66). When considering average constraints, this number remains 62, but with exclusions of FPLyon, KDE, SkM, and SLy10, while incorporating MSkA, NRAPR, SIV, and SkO' sets.

IV. SUMMARY

The study of nuclear symmetry energy is of paramount importance for understanding the properties of neutron stars, as it directly influences the EoS of dense matter, the structure of neutron stars, and their maximum mass and radius. In particular, the slope parameter L at saturation density, which characterizes the density dependence of the symmetry energy, plays a crucial role in determining the pressure of neutron-rich matter and thus the thickness of the neutron star crust and the neutron

skin of heavy nuclei. However, the relatively large uncertainty in the slope parameter L remains a significant challenge in nuclear physics, making it a focal point of symmetry energy research. Using the SHF method, we investigated the relationship between L and Δr_{np} in stable Sn isotopes. Our results revealed an approximately linear correlation between Δr_{np} and L for Sn isotopes. This linear correlation strengthens with increasing isospin asymmetry, with the strongest linear relationship observed in the ^{124}Sn nucleus. This indicates that the neutron skin thickness of ^{124}Sn provides the most effective constraint on L . We can imagine that uranium isotopes with even larger isospin asymmetry would be a better choice for constraining L . Additionally, the nucleon distribution in neutron stars, which have an isospin asymmetry close to 1, would provide the best data for constraining L .

We extracted the neutron skin thickness of ^{124}Sn from the elastic scattering angular distribution of $^{12}\text{C} + ^{124}\text{Sn}$ using the CDM3Y6 double-folding potential. The extracted neutron skin thicknesses were $0.168^{+0.025}_{-0.019} \text{ fm}$ (SLy4) and $0.177 \pm 0.022 \text{ fm}$ (SLy7), which is consistent with the results obtained by other methods. Additionally, we constrained the slope parameter L of the nuclear symmetry energy to $L = 39.0^{+20.8}_{-16.5} \text{ MeV}$ (SLy4) and $L = 46.1 \pm 18.7 \text{ MeV}$ (SLy4), which is in agreement with the average values predicted by other studies and eliminates 122 sets of Skyrme interaction parameters out of the original 184. Furthermore, the average constraint derived from 13 works gives $L = 56.1 \pm 18.9 \text{ MeV}$, significantly reducing the theoretical uncertainty in the behavior of high-density nuclear matter, thereby providing a theoretical foundation for understanding the properties of high-density neutron stars.

Despite good agreement, future investigations are needed to address potential systematic uncertainties. In this study, for the sake of simplicity, we considered only the elastic scattering channel in the folding potential analysis. In reality, nucleus–nucleus interactions can involve various inelastic processes and coupling effects [54]. In future studies, a coupled-channel analysis could be employed to reduce the deviations caused by inelastic scattering.

References

- [1] J. Lattimer and M. Prakash, *Astrophys. J.* **550**(1), 426 (2001)
- [2] D. Lunney, (2020), arXiv: 2011.08645
- [3] A. Li, *Nucl. Phys. Rev.* **41**(01), 308 (2024)
- [4] B. A. Li, L. W. Chen, and C. M. Ko, *Phys. Rep.* **464**(4), 113 (2008)
- [5] M. Baldo and G. Burgio, *Prog. Part. Nucl. Phys.* **91**, 203 (2016)
- [6] M. Centelles, X. Roca-Maza, X. Vinas *et al.*, *Phys. Rev. Lett.* **102**(12), 122502 (2009)
- [7] L. W. Chen, C. M. Ko, B. A. Li *et al.*, *Phys. Rev. C* **82**(2), 024321 (2010)
- [8] L. W. Chen, *Phys. Rev. C* **83**(4), 044308 (2011)
- [9] B. Agrawal, J. De, and S. Samaddar, *Phys. Rev. Lett.* **109**(26), 262501 (2012)
- [10] Z. Zhang and L. W. Chen, *Phys. Lett. B* **726**(1), 234 (2013)
- [11] P. Danielewicz and J. Lee, *Nucl. Phys. A* **922**, 1 (2014)
- [12] J. Xu, W. J. Xie, and B. A. Li, *Phys. Rev. C* **102**(4), 044316 (2020)
- [13] D. Adhikari, H. Albatineh, D. Androic *et al.*, *Phys. Rev. Lett.* **126**(17), 172502 (2021)
- [14] B. T. Reed, F. J. Fattoyev, C. J. Horowitz *et al.*, *Phys. Rev.*

- Lett. **126**(17), 172503 (2021)
- [15] L. Baiotti, *Prog. Part. Nucl. Phys.* **109**, 103714 (2019)
- [16] J. M. Lattimer, *Particles* **6**(1), 30 (2023)
- [17] R. Essick, P. Landry, A. Schwenk *et al.*, *Phys. Rev. C* **104**(6), 065804 (2021)
- [18] B. Hu, W. Jiang, T. Miyagi *et al.*, *Nat. Phys.* **18**(10), 1196 (2022)
- [19] Y. Lim and J. W. Holt, *Galaxies* **10**(5), 99 (2022)
- [20] L. Ray, *Phys. Rev. C* **19**(5), 1855 (1979)
- [21] S. Terashima, H. Sakaguchi, H. Takeda *et al.*, *Phys. Rev. C* **77**(2), 024317 (2008)
- [22] S. Tagami, T. Wakasa, and M. Yahiro, *Results Phys.* **46**, 106296 (2023)
- [23] A. Krasznahorkay, A. Balanda, J. Bordewijk *et al.*, *Nucl. Phys. A* **567**(3), 521 (1994)
- [24] M. Matsuzaki, S. Tagami, and M. Yahiro, *Phys. Rev. C* **104**(5), 054613 (2021)
- [25] A. Krasznahorkay, M. Fujiwara, P. Van Aarle *et al.*, *Phys. Rev. Lett.* **82**(16), 3216 (1999)
- [26] A. Trzcińska, J. Jastrzębski, P. Lubiński *et al.*, *Phys. Rev. Lett.* **87**(8), 082501 (2001)
- [27] B. A. Brown, *Phys. Rev. Lett.* **85**(25), 5296 (2000)
- [28] M. Brack, C. Guet, and H. B. Håkansson, *Phys. Rep.* **123**(5), 275 (1985)
- [29] J. Friedrich and P. G. Reinhard, *Phys. Rev. C* **33**(1), 335 (1986)
- [30] B. A. Brown, *Phys. Rev. C* **58**(1), 220 (1998)
- [31] L. W. Chen, C. M. Ko, and B. A. Li, *Phys. Rev. C* **72**(6), 064309 (2005)
- [32] J. R. Stone, J. Miller, R. Konciewicz *et al.*, *Phys. Rev. C* **68**(3), 034324 (2003)
- [33] A. Dieperink, Y. Dewulf, D. Van Neck *et al.*, *Phys. Rev. C* **68**(6), 064307 (2003)
- [34] C. Horowitz and J. Piekarewicz, *Phys. Rev. Lett.* **86**(25), 5647 (2001)
- [35] C. Horowitz and J. Piekarewicz, *Phys. Rev. C* **66**(5), 055803 (2002)
- [36] S. Typel and B. A. Brown, *Phys. Rev. C* **64**(2), 027302 (2001)
- [37] R. Furnstahl, *Nucl. Phys. A* **706**(1-2), 85 (2002)
- [38] S. Karataglidis, K. Amos, B. A. Brown *et al.*, *Phys. Rev. C* **65**(4), 044306 (2002)
- [39] M. Dutra, O. Lourenço, J. Sá Martins *et al.*, *Phys. Rev. C* **85**(3), 035201 (2012)
- [40] Z. Li, Y. Cheng, C. Yan *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **336**(1), 150 (1993)
- [41] L. Gan, H. Sun, Z. Li *et al.*, *Phys. Rev. C* **101**(1), 014612 (2020)
- [42] I. Thompson, *Comput. Phys. Rep* **7**, 167 (1988)
- [43] M. Karakoç, *Comput. Phys. Commun.* **284**, 108613 (2023)
- [44] M. Lewitowicz, C. Borcea, F. Carstoiu *et al.*, *Nucl. Phys. A* **562**(2), 301 (1993)
- [45] A. H. Amer, Z. M. Mahmoud, and Y. Penionzhkevich, *Nucl. Phys. A* **1020**, 122398 (2022)
- [46] D. Gupta and C. Samanta, *J. Phys. G: Nucl. Part. Phys.* **28**(1), 85 (2001)
- [47] D. T. Khoa, W. Von Oertzen, H. Bohlen *et al.*, *J. Phys. G: Nucl. Part. Phys.* **34**(3), R111 (2007)
- [48] D. T. Khoa, G. Satchler, and W. Von Oertzen, *Phys. Rev. C* **56**(2), 954 (1997)
- [49] I. Angeli and K. P. Marinova, *At. Data Nucl. Data Tables* **99**(1), 69 (2013)
- [50] C. C. Sahm, T. Murakami, J. G. Cramer *et al.*, *Phys. Rev. C* **34**, 2165 (1986)
- [51] J. Zenihiro, H. Sakaguchi, S. Terashima *et al.*, (2018), arXiv: 1810.11796
- [52] T. Wakasa, S. Tagami, J. Matsui *et al.*, *Phys. Rev. C* **107**, 024608 (2023)
- [53] L. W. Chen, C. M. Ko, and B. A. Li, *Phys. Rev. Lett.* **94**, 032701 (2005)
- [54] C. H. Rong, G. L. Zhang, L. Gan *et al.*, *Chin. Phys. C* **44**(10), 104003 (2020)