

Neutrino transition magnetic moment in the $U(1)_X$ SSM*

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Abstract: This paper investigates the neutrino transition magnetic moment in the $U(1)_X$ SSM. $U(1)_X$ SSM is the $U(1)$ extension of Minimal Supersymmetric Standard Model (MSSM) and its local gauge group is extended to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. To obtain this model, three singlet new Higgs superfields and right-handed neutrinos are added to the MSSM, which can explain the results of neutrino oscillation experiments. The neutrino transition magnetic moment is induced by electroweak radiative corrections. By applying effective Lagrangian method and on-shell scheme, we study the associated Feynman diagrams and the transition magnetic moment of neutrinos in the model. We fit experimental data for neutrino mass variances and mixing angles. Based on the range of data selection, the influences of different sensitive parameters on the results are analysed. The numerical analysis shows that many parameters have an effect on the neutrino transition magnetic moment, such as g_X , M_2 , μ , λ_H and g_{YX} . For our numerical results, the order of magnitude of μ_{ij}^M/μ_B is around $10^{-20} \sim 10^{-19}$.

Keywords: transition magnetic moment, $U(1)_X$ SSM, neutrino

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I. INTRODUCTION

The Standard Model (SM) comes under the umbrella of quantum field theory, which describes the three main forces, the strong force, the weak force, and the electromagnetic force[1–4]. Besides it predicts the existence of the Higgs. Although the SM has been a great success, its flaws are obvious. It doesn't explain the mass problem of neutrinos, the related issue of dark matter, and can't describe gravity[5–7]. Therefore, it must be extended. Scientists have made many extensions to the SM, among which the Minimal Supersymmetric Standard Model(MSSM) is a popular one. However, there are problems in MSSM such as the μ -problem[8] and massless neutrinos[9]. To break through these problems, we use the $U(1)_X$ SSM. Under this model, we study neutrino transition magnetic moment. The study of it may indirectly lead to a new understanding of the neutrino properties and the mechanism of neutrino mass generation. Be-

sides it may verify the correctness of $U(1)_X$ SSM to some extent. It is also important in the long distance propagation of neutrinos in the magnetic fields of matter and vacuum[10]. Previous research on neutrino transition magnetic moment includes analyses of Majorana neutrino effects on supernova neutrino oscillations[11] and explanations of electron recoil anomalies[12]. However, our work explores this phenomenon within a distinct model, aiming to contribute novel findings.

$U(1)_X$ SSM is the extension of the MSSM with the $U(1)_X$ gauge group, and the symmetry group is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ [13]. This extension adds three Higgs singlet superfields and right-handed neutrino superfields to the MSSM. Consequently, there are five neutral CP-even Higgs component fields (H_u^0 , H_d^0 , ϕ_η^0 , ϕ_η^0 , ϕ_S^0) in the model, and mix together, forming a 5×5 mass-squared matrix. Consequently, the mass of the lightest CP-even Higgs particle can be improved at the tree level. In the $U(1)_X$ SSM, the small hierarchy problem

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in the MSSM is alleviated through the added right-handed neutrinos, sneutrinos, and extra Higgs singlets. The μ -problem existing in the MSSM is relieved after the spontaneous symmetry breaking of the S field in vacuum through $\lambda_H \hat{S} \hat{H}_u \hat{H}_d$. Through the term $Y_\nu \hat{\nu} \hat{H}_u$, the right-handed neutrinos and left-handed neutrinos mix together, which makes light neutrinos to obtain tiny masses through the seesaw mechanism. The existence of supersymmetry provides a very natural candidate for dark matter: neutralino. While, $S U(3)_C \times S U(2)_L \times U(1)_Y \times U(1)_X$ can provide several dark matter candidates: neutralino, sneutrino(CP-even, CP-odd) etc., and it protects the Higgs mass from radiative correction by the massive particles, which solves the gauge hierarchy. Under $U(1)_X$ SSM, the transition magnetic moment of neutrino is induced by electroweak radiative corrections.

The previous studies[10] have investigated the neutrino transition magnetic moment using the effective Lagrangian method and the mass-shell scheme, yielding reasonable numerical results. In this paper, a more comprehensive study of the neutrino transition magnetic moment at the $U(1)_X$ SSM is presented. Using the effective Lagrangian method and the mass-shell scheme we obtain the expression for the neutrino transition magnetic moment. We derive the relevant Feynman diagrams and calculate the neutrino transition moment by combining the operators. In numerical calculations, we perform neutrino mixing within experimentally constrained parameter ranges to determine viable parameter values. Additionally, we compare the effects of different reasonable parameters on transition magnetic moment and get the numerical results.

The paper is organized according to the following structure. In Sec.II, we mainly introduce the content of the $U(1)_X$ SSM including its superpotential, the general soft breaking terms, the mass matrices and couplings. In Sec.III, we give the analytical expressions of the transition magnetic moment about neutrino. In Sec.IV, we give the relevant parameters and numerical analysis. In Sec.V, we present a summary of this article. Some formulae are collected in the Appendix.

II. THE ESSENTIAL CONTENT OF $U(1)_X$ SSM

$U(1)_X$ SSM is the extension of MSSM and the local gauge group is $S U(3)_C \times S U(2)_L \times U(1)_Y \times U(1)_X$. $U(1)_X$ SSM has new superfields, which include three Higgs singlets $\hat{\eta}$, $\hat{\bar{\eta}}$, $\hat{S}$, and right-handed neutrinos $\hat{\nu}_i$. The corresponding superpotential of the $U(1)_X$ SSM is given by:

$$W = l_W \hat{S} + \mu \hat{H}_u \hat{H}_d + M_S \hat{S} \hat{S} - Y_d \hat{d} \hat{q} \hat{H}_d - Y_e \hat{e} \hat{l} \hat{H}_d + \lambda_H \hat{S} \hat{H}_u \hat{H}_d + \lambda_C \hat{S} \hat{\eta} \hat{\bar{\eta}} + \frac{\kappa}{3} \hat{S} \hat{S} \hat{S} + Y_u \hat{u} \hat{q} \hat{H}_u + Y_X \hat{\nu} \hat{\eta} \hat{\bar{\eta}} + Y_\nu \hat{\nu} \hat{l} \hat{H}_u. \quad (1)$$

The two Higgs doublets and three Higgs singlets can be listed as follows:

$$H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}(v_u + H_u^0 + iP_u^0) \end{pmatrix}, \quad H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + H_d^0 + iP_d^0) \\ H_d^- \end{pmatrix},$$

$$\eta = \frac{1}{\sqrt{2}}(v_\eta + \phi_\eta^0 + iP_\eta^0), \quad \bar{\eta} = \frac{1}{\sqrt{2}}(v_{\bar{\eta}} + \phi_{\bar{\eta}}^0 + iP_{\bar{\eta}}^0),$$

$$S = \frac{1}{\sqrt{2}}(v_S + \phi_S^0 + iP_S^0). \quad (2)$$

And the vacuum expectation values of the states $H_u, H_d, \eta, \bar{\eta}, S$ are respectively $v_u, v_d, v_\eta, v_{\bar{\eta}}$ and v_S . H_u^+ is the charged part of the doublet H_u . While, $H_u^0(P_u^0)$ is the neutral CP-even(CP-odd) part of H_u . The similar condition is for the doublet H_d . $\phi_\eta^0(P_\eta^0)$ is the CP-even(CP-odd) part of singlet η . $\phi_{\bar{\eta}}^0(P_{\bar{\eta}}^0)$ is the CP-even(CP-odd) part of singlet $\bar{\eta}$. $\phi_S^0(P_S^0)$ is the CP-even(CP-odd) part of singlet S .

There are two angles defined as $\tan\beta = v_u/v_d$ and $\tan\beta_\eta = v_{\bar{\eta}}/v_\eta$. The soft SUSY breaking terms of $U(1)_X$ SSM are shown as:

$$\begin{aligned} \mathcal{L}_{soft} = & \mathcal{L}_{soft}^{MSSM} - B_S S^2 - L_S S - \frac{T_\kappa}{3} S^3 - T_{\lambda_C} S \eta \bar{\eta} \\ & + \epsilon_{ij} T_{\lambda_H} S H_d^i H_u^j - T_X^{IJ} \bar{\eta} \tilde{\nu}_R^{*I} \tilde{\nu}_R^{*J} + \epsilon_{ij} T_\nu^{IJ} H_u^i \tilde{\nu}_R^{*I} \tilde{\nu}_R^{*J} \\ & - m_\eta^2 |\eta|^2 - m_{\bar{\eta}}^2 |\bar{\eta}|^2 - m_S^2 S^2 \\ & - (m_{\tilde{\nu}_R}^2)^{IJ} \tilde{\nu}_R^{*I} \tilde{\nu}_R^{*J} - \frac{1}{2} (M_S \lambda_X^2 + 2M_{BB'} \lambda_B \lambda_{\bar{X}}) + h.c. \quad (3) \end{aligned}$$

$\mathcal{L}_{soft}^{MSSM}$ represent the soft breaking terms of MSSM. And $\lambda_{\bar{B}}$ is the $U(1)_Y$ gaugino, which is the supersymmetric partner of the $U(1)_Y$ gauge boson B^μ . The boson of the added gauge group $U(1)_X$ is X^μ , whose supersymmetric partner is $\lambda_{\bar{X}}$.

The particle content and charge assignments for $U(1)_X$ SSM are mentioned in the Table 1. In our previous work, we have proven that $U(1)_X$ SSM is anomaly free [13].

The covariant derivatives of $U(1)_X$ SSM can be written as

$$D_\mu = \partial_\mu - i \begin{pmatrix} Y & X \end{pmatrix} \begin{pmatrix} g_1 & g_{YX} \\ 0 & g_X \end{pmatrix} \begin{pmatrix} A_\mu^Y \\ A_\mu^X \end{pmatrix}. \quad (4)$$

Compared with MSSM, $U(1)_X$ SSM has a new effect called the gauge kinetic mixing, which is produced by Abelian groups $U(1)_Y$ and $U(1)_X$. The basis conversion occurs when we use the rotation matrix R ($R^T R = 1$),

Table 1. The superfields in $U(1)_X$ SSM

Superfields	$\hat{q}_i$	$\hat{u}_i^c$	$\hat{d}_i^c$	$\hat{l}_i$	$\hat{e}_i^c$	$\hat{\nu}_i$	$\hat{H}_u$	$\hat{H}_d$	$\hat{\eta}$	$\hat{\bar{\eta}}$	$\hat{S}$
$SU(3)_C$	3	$\bar{3}$	$\bar{3}$	1	1	1	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	1	2	2	1	1	1
$U(1)_Y$	1/6	-2/3	1/3	-1/2	1	0	1/2	-1/2	0	0	0
$U(1)_X$	0	-1/2	1/2	0	1/2	-1/2	1/2	-1/2	-1	1	0

which is due to the fact that the two Abelian gauge groups are uninterrupted. The basis conversion can be described by [14–17]

$$D_\mu = \partial_\mu - i \begin{pmatrix} Y^Y & Y^X \end{pmatrix} \begin{pmatrix} g_Y & g'_{YX} \\ g'_{XY} & g'_X \end{pmatrix} R^T R \begin{pmatrix} A_\mu^Y \\ A_\mu^X \end{pmatrix} \quad (5)$$

with A_μ^Y and A_μ^X respectively representing the gauge fields of $U(1)_Y$ and $U(1)_X$. Eq. (5) can be reduced to [14, 16, 17]

$$\begin{pmatrix} g_Y & g'_{YX} \\ g'_{XY} & g'_X \end{pmatrix} R^T = \begin{pmatrix} g_1 & g_{YX} \\ 0 & g_X \end{pmatrix}, \quad R \begin{pmatrix} A_\mu^Y \\ A_\mu^X \end{pmatrix} = \begin{pmatrix} A_\mu^Y \\ A_\mu^X \end{pmatrix}. \quad (6)$$

Here g_X expresses the gauge coupling constant of the $U(1)_X$ group and g_{YX} expresses the mixing gauge coupling constant of the $U(1)_X$ and $U(1)_Y$ groups.

Some useful mass matrices and needed couplings in this model can be found in appendix A.

III. FORMULATION

The magnetic dipole moment (MDM) and electric dipole moment (EDM) of the neutrino can actually be written as the operators

$$\begin{aligned} \mathcal{L}_{MDM} &= \frac{1}{2} \mu_{ij} \bar{\psi}_i \sigma^{\mu\nu} \psi_j F_{\mu\nu}, \\ \mathcal{L}_{EDM} &= \frac{i}{2} \epsilon_{ij} \bar{\psi}_i \sigma^{\mu\nu} \gamma_5 \psi_j F_{\mu\nu}, \end{aligned} \quad (7)$$

where $F_{\mu\nu}$ is the electromagnetic field strength, $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$, $\psi_{i,j}$ denote the four-component Dirac fermions, μ_{ij} and ϵ_{ij} are respectively Dirac diagonal ($i = j$) or transition ($i \neq j$) MDM and EDM between states ψ_i and ψ_j .

Since $p = m_f \ll m_\nu$ for on-shell fermions and $k \rightarrow 0 \ll m_\nu$ for photons, we can conveniently obtain the contribution of the loop diagram to the fermionic diagonal MDM and EDM using the effective Lagrangian method. Then we can expand the amplitude of corresponding

triangle diagrams based on the external momenta of fermion and photon. After matching the effective theory with the holonomic theory, we obtain all high-dimension operators along with their coefficients. We only need to keep those dimension 6 operators for later calculations.

$$\begin{aligned} O_1^{L,R} &= e \bar{\psi}_i (i \not{D})^3 P_{L,R} \psi_j, \\ O_2^{L,R} &= e (i \not{D}_\mu \psi_i) \gamma^\mu F \cdot \sigma P_{L,R} \psi_j, \\ O_3^{L,R} &= e \bar{\psi}_i F \cdot \sigma \gamma^\mu P_{L,R} (i \not{D}_\mu \psi_j), \\ O_4^{L,R} &= e \bar{\psi}_i (\partial^\mu F_{\mu\nu}) \gamma^\nu P_{L,R} \psi_j, \\ O_5^{L,R} &= e m_{\psi_i} \bar{\psi}_i (i \not{D})^2 P_{L,R} \psi_j, \\ O_6^{L,R} &= e m_{\psi_i} \bar{\psi}_i F \cdot \sigma P_{L,R} \psi_j, \end{aligned} \quad (8)$$

where $P_L = \frac{1}{2}(1 - \gamma_5)$, $P_R = \frac{1}{2}(1 + \gamma_5)$, $\mathcal{D}_\mu = \partial^\mu + ieA_\mu$, and m_{ψ_i} is the mass of fermion ψ_i . The effective vertices with one external photon are written as

$$\begin{aligned} O_1^{L,R} &= ie \{ [(p+k)^2 + p^2] \gamma_\rho + (\not{p} + \not{k}) \gamma_\rho \not{p} \} P_{L,R}, \\ O_2^{L,R} &= ie (\not{p} + \not{k}) [k, \gamma_\rho] P_{L,R}, \\ O_3^{L,R} &= ie [k, \gamma_\rho] \not{p} P_{L,R}, \\ O_4^{L,R} &= ie (k^2 \gamma_\rho - k k_\rho) P_{L,R}, \\ O_5^{L,R} &= i e m_{\psi_i} \{ (\not{p} + \not{k}) \gamma_\rho + \gamma_\rho \not{p} \} P_{L,R}, \\ O_6^{L,R} &= i e m_{\psi_i} [k, \gamma_\rho] P_{L,R}. \end{aligned} \quad (9)$$

By applying the equations of motion to the outer fermions, we obtain the relations in the effective Lagrangian [18]:

$$\begin{aligned} &C_2^R O_2^R + C_2^L O_2^L + C_2^{L*} O_3^R + C_2^{R*} O_3^L + C_6^R O_6^R + C_6^{R*} O_6^L \\ &\Rightarrow (C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) O_6^R + (C_2^{R*} + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^L + C_6^{R*}) O_6^L \\ &= e m_{\psi_i} \Re(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) \bar{\psi}_i \sigma^{\mu\nu} \psi_j F_{\mu\nu} \\ &+ i e m_{\psi_i} \Im(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) \bar{\psi}_i \sigma^{\mu\nu} \gamma_5 \psi_j F_{\mu\nu}, \end{aligned} \quad (10)$$

Comparing with Eq. (7) and Eq. (10), we can obtain

$$\begin{aligned}\mu_{ij} &= 4m_e m_{\psi_i} \Re(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) \mu_B, \\ \epsilon_{ij} &= 4m_e m_{\psi_i} \Im(C_2^R + \frac{m_{\psi_j}}{m_{\psi_i}} C_2^{L*} + C_6^R) \mu_B,\end{aligned}\quad (11)$$

where $\Re(\dots)$ and $\Im(\dots)$ are the real and imaginary parts of the complex numbers respectively, $\mu_B = e/(2m_e)$, and m_e is the electron mass. The Wilson coefficients ($C_2^R, C_2^L, C_6^R, C_6^L$) related to our study in this paper have been included in the Appendix B.

Then we investigate the $\nu_j \rightarrow \nu_i \gamma$ processes about the transition magnetic moment of neutrino under the $U(1)_X$ SSM. The amplitude of $\nu_j \rightarrow \nu_i \gamma$ can be obtained from the following Feynman diagrams shown in Fig. 1. After calculating the left one in Fig.(1) and connecting with Eq.(9), we can get

$$\begin{aligned}\mathcal{M} &= -i \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - m_F^2)(k^2 - m_S^2)^2} \\ &\times \left\{ -\frac{1}{4} \left(\frac{k^2}{k^2 - m_S^2} - \frac{k^4}{(k^2 - m_S^2)^2} \right) \right. \\ &\times (O_2^L + O_3^L) A_R B_L - \frac{1}{2} \left(1 - \frac{k^2}{k^2 - m_S^2} \right) O_6^L A_L B_L \left. \right\},\end{aligned}\quad (12)$$

where k is the photon momentum, m_F corresponds to the chargino mass, m_S corresponds to the scalar lepton mass. A_L, A_R, B_R and B_L are

$$\begin{aligned}A_L &= -g_2 U_{j1}^* \sum_{a=1}^3 U_{ia}^{V*} Z_{ka}^E + U_{j2}^* \sum_{a=1}^3 U_{ia}^{V*} Y_{e,a} Z_{k,(3+a)}^E, \\ A_R &= \sum_{a=1}^3 Y_{\nu,a}^* U_{i,(3+a)}^V Z_{ka}^E V_{j2}, \\ B_R &= A_L^*, \quad B_L = A_R^*.\end{aligned}\quad (13)$$

Through the general description of the electromagnetic form factors of Dirac and Majorana neutrinos, we can get the MDM and EDM for Majorana neutrinos

$$\mu_{ij}^M = \mu_{ij}^D - \mu_{ji}^D, \quad \epsilon_{ij}^M = \epsilon_{ij}^D - \epsilon_{ji}^D. \quad (14)$$

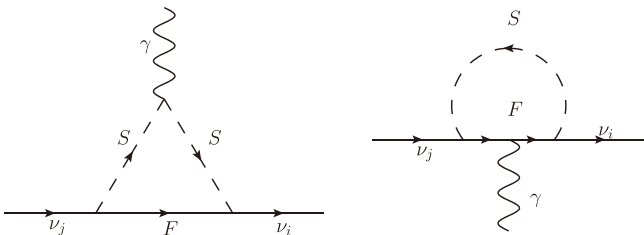


Fig. 1. Feynman diagrams for the $\nu_j \rightarrow \nu_i \gamma$ processes in the $U(1)_X$ SSM

Finally, we simplify Eq.(12) and use numerical calculation software(Mathematica) to get the numerical results.

IV. NUMERICAL ANALYSIS

In this section of the numerical results, we consider some constraints from experiments, including:

1. The lightest CP-even Higgs h^0 mass is around 125.1 GeV. The Higgs h^0 decays ($h^0 \rightarrow \gamma + \gamma, Z + Z, W + W, b + \bar{b}, \tau + \bar{\tau}$) can well meet the latest experimental constraints[19–21].

The mass of the lightest CP-even Higgs boson should consider the stop quark contributions at loop level[22, 23]

$$m_h^0 = \sqrt{(m_{h_1}^0)^2 + \Delta m_h^2}, \quad (15)$$

with $m_{h_1}^0$ representing the lightest tree-level Higgs boson mass. The concrete form of Δm_h^2 is

$$\begin{aligned}\Delta m_h^2 &= \frac{3m_t^4}{4\pi^2 v^2} \left[\left(\tilde{t} + \frac{1}{2} \tilde{X}_t \right) + \frac{1}{16\pi^2} \left(\frac{3m_t^2}{2v^2} - 32\pi\alpha_3 \right) (\tilde{t}^2 + \tilde{X}_t \tilde{t}) \right], \\ \tilde{t} &= \log \frac{M_{\tilde{T}}^2}{m_t^2}, \quad \tilde{X}_t = \frac{2\tilde{A}_t^2}{M_{\tilde{T}}^2} \left(1 - \frac{\tilde{A}_t^2}{12M_{\tilde{T}}^2} \right).\end{aligned}\quad (16)$$

α_3 is the strong coupling constant. $M_{\tilde{T}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ and $m_{\tilde{t}_{1,2}}$ are the stop masses. $\tilde{A}_t = A_t - \mu \cot \beta$ and A_t is the trilinear Higgs stop coupling. We use the parameter values to fix $m_h^0 \sim 125.1$ GeV.

The mass matrix of chargino includes the parameters v_u, v_d, λ_H, v_S , and the mass squared matrix of scalar lepton includes $v_u, v_d, \lambda_H, v_S, v_\eta, v_{\bar{\eta}}, g_X, g_{YX}$. The CP-even Higgs mass squared matrix at tree level also has these parameters. As we fit CP-even Higgs h^0 mass, these parameters will be restricted, which affects the values of these parameters. In the whole, these parameters affect the mass matrix of chargino and the mass squared matrix of scalar lepton. Therefore, they have effects on the neutrino transition magnetic moment.

2. The constraints from neutrino experiment data including mixing angles and mass variances are considered[24–26].

3. The Z' boson mass is larger than 5.1 TeV. The gauge boson masses are[13]

$$\begin{aligned}M_\gamma^2 &= 0, \\ M_{Z'}^2 &= \frac{1}{8} \left((g_1^2 + g_2^2 + g_{YX}^2) v^2 + 4g_X^2 \xi^2 \right. \\ &\quad \left. \mp \sqrt{(g_1^2 + g_2^2 + g_{YX}^2)^2 v^4 + 8(g_{YX}^2 - g_1^2 - g_2^2) g_X^2 v^2 \xi^2 + 16g_X^4 \xi^4} \right).\end{aligned}\quad (17)$$

For $M_{Z'}$, it can be much simplified with the supposi-

tion $\xi^2 \gg v^2$. It is shown as

$$\begin{aligned} M_{Z'}^2 &= \frac{1}{8} \left((g_1^2 + g_2^2 + g_{YX}^2) v^2 + 4g_X^2 \xi^2 \right. \\ &\quad \left. + \sqrt{(g_1^2 + g_2^2 + g_{YX}^2)^2 v^4 + 8(g_{YX}^2 - g_1^2 - g_2^2)g_X^2 v^2 \xi^2 + 16g_X^4 \xi^4} \right) \\ &\approx \frac{1}{8} \left(4g_X^2 \xi^2 + \sqrt{16g_X^4 \xi^4} \right) \\ &= g_X^2 \xi^2. \end{aligned} \quad (18)$$

$M_{Z'}/g_X > 6$ TeV is the results of the particle[27].

4. The neutralino mass is limited to more than 116 GeV, and the chargino mass is limited to more than 1000 GeV. The slepton mass is limited to more than 600 GeV [19].

With the above experimental requirements, we generally take the values of new mass parameters ($M_{BB'}, M_{BL}, M_S$) around the energy scale of new physics (10^3 GeV). $M_{\tilde{L}}, M_{\tilde{E}}$ are all of mass square dimension, and can be up to the order of 10^6 GeV². Non-diagonal elements of scalar lepton mass matrix affect T_e , which can reach 10^{-1} GeV. $\tan\beta$ and v_S affect the mass matrix of chargino. $v_{\tilde{\eta}}$ and v_{η} affect slepton mass. The loop diagram is produced by chargino and scalar lepton. We adopt the following parameters that can affect the neutrino transition magnetic moment in the numerical calculation:

$$\begin{aligned} \tan\beta &= 23, \quad v_S = 4.3 \text{ TeV}, \quad \tan\beta_{\eta} = \frac{v_{\tilde{\eta}}}{v_{\eta}} = 0.8, \\ v_{\tilde{\eta}} &= 17 \sin(\beta_{\eta}) \text{ TeV}, \quad v_{\eta} = 17 \cos(\beta_{\eta}) \text{ TeV}, \\ T_{e11} &= T_{e22} = T_{e33} = 0.5 \text{ GeV}, \\ M_{L11} &= M_{L22} = M_{L33} = 3 \text{ TeV}^2, \\ M_{\tilde{E}11} &= M_{\tilde{E}22} = M_{\tilde{E}33} = 8 \text{ TeV}^2. \end{aligned} \quad (19)$$

And the parameters we selected are of good universality. In the following numerical analysis, the parameters to be studied include:

$$g_X, \lambda_H, M_2, \mu, g_{YX}. \quad (20)$$

Without special statement, the non-diagonal elements of the parameters are supposed as zero.

A. Neutrino Mixing

In the neutrino mass matrix, elements such as Y_{ν} are relevant to neutrino mixing. Transition magnetic moment is closely related to the mass matrix including Y_{ν} . In this subsection, using the top-down approach we can derive the formulae for the neutrino mass and mixing angle from the effective neutrino mass matrix. Here, we adopt the

normal ordering spectrum to calculate the neutrino observables ($\sin^2(\theta_{ij})$ etc.). The detailed procedure is outlined in Appendix C.

The constraints from neutrino experiment data are[19]

$$\begin{aligned} \sin^2(\theta_{12}) &= 0.307_{-0.012}^{+0.013}, \\ \sin^2(\theta_{23}) &= 0.546 \pm 0.021, \\ \sin^2(\theta_{13}) &= 0.022 \pm 0.0007, \\ \Delta m_{\odot}^2 &= (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \\ |\Delta m_A^2| &= (2.453 \pm 0.033) \times 10^{-3} \text{ eV}^2. \end{aligned} \quad (21)$$

To fit the data of neutrino physics, we take the parameters as

$$\begin{aligned} Y_{X11} &= Y_{X22} = Y_{X33} = 0.1, \quad Y_{\nu 22} = 1.4000 \times 10^{-6}, \\ Y_{\nu 33} &= 1.352420 \times 10^{-6}, \quad Y_{\nu 12} = 7.604202 \times 10^{-8}. \end{aligned} \quad (22)$$

By fixing some matrix elements in Eq.(22) and taking others as variables, we can discuss data easier.

In Fig. 2, $\sin^2(\theta_{12})$, $\sin^2(\theta_{23})$ and $10 \sin^2(\theta_{13})$ are plotted in the plane of $Y_{\nu 23}$ versus $Y_{\nu 13}$. If the area satisfies the $10 \sin^2(\theta_{13})$ in the 30σ , it of course satisfies $\sin^2(\theta_{23})$ in the 3σ . With $Y_{\nu 11} = 1.092847 \times 10^{-6}$, the constraints of three mixing angles are satisfied (They all in the range of 3σ). The yellow, blue and green area represent $0.483 < \sin^2(\theta_{23}) < 0.609$, $0.271 < \sin^2(\theta_{12}) < 0.346$ and $0.199 < 10 \sin^2(\theta_{13}) < 0.241$ respectively. The yellow region resembles a rectangle, the blue region a fragmented ribbon, and the green region a continuous ribbon. The

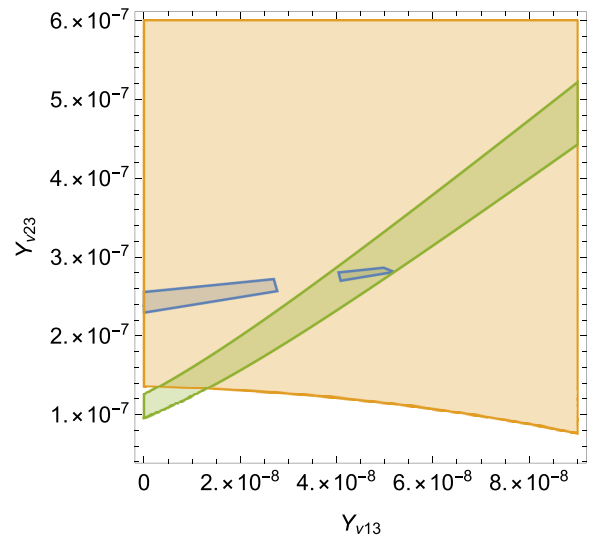


Fig. 2. (color online) $\sin^2(\theta_{12})$, $\sin^2(\theta_{23})$ and $10 \sin^2(\theta_{13})$ are plotted in the plane of $Y_{\nu 13}$ versus $Y_{\nu 23}$. The yellow, blue and green area represent $0.483 < \sin^2(\theta_{23}) < 0.609$, $0.271 < \sin^2(\theta_{12}) < 0.346$ and $0.199 < 10 \sin^2(\theta_{13}) < 0.241$ respectively.

overlapping area represents values of $Y_{\nu 23}$ and $Y_{\nu 13}$ that satisfy all three mixing angle constraints.

Similarly, in Fig. 3 the three constraints from the mixing angles are satisfied (They are all in the range of 3σ). $\Delta m_{\odot}^2$ and $|\Delta m_A^2|$ are plotted in the plane of $Y_{\nu 23}$ versus $Y_{\nu 13}$. The yellow area represents $2.353 \times 10^{-21} \text{ eV}^2 < |\Delta m_A^2| < 2.553 \times 10^{-21} \text{ eV}^2$, which looks like a rectangle. The blue area represents $6.99 \times 10^{-23} \text{ eV}^2 < \Delta m_{\odot}^2 < 8.07 \times 10^{-23} \text{ eV}^2$, which looks like a band. Overall the overlapping part is needed.

In Fig. 4, we combine Fig. 2 and Fig. 3 to find reasonable parameter space apparently. The overlapping area in Fig. 2 satisfies $0.483 < \sin^2(\theta_{23}) < 0.609$, $0.271 < \sin^2(\theta_{12}) <$

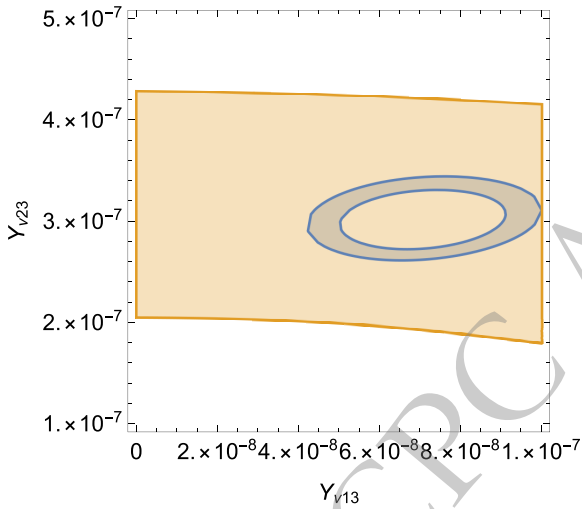


Fig. 3. (color online) $\Delta m_{\odot}^2$ and $|\Delta m_A^2|$ are plotted in the plane of $Y_{\nu 13}$ versus $Y_{\nu 23}$. The yellow area represents $1.2 \times 10^{-21} \text{ eV}^2 < |\Delta m_A^2| < 4.3 \times 10^{-21} \text{ eV}^2$ and the blue area represents $7.3 \times 10^{-23} \text{ eV}^2 < \Delta m_{\odot}^2 < 9.9 \times 10^{-23} \text{ eV}^2$.

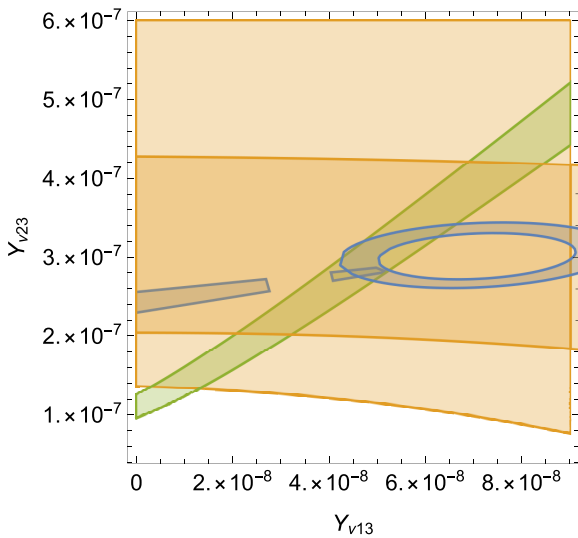


Fig. 4. (color online) Combining Fig. 2 and Fig. 3, the overlapping area satisfy all constraints.

0.346 and $0.199 < 10\sin^2(\theta_{13}) < 0.241$. In Fig. 3, the overlapping area satisfies $1.2 \times 10^{-21} \text{ eV}^2 < |\Delta m_A^2| < 4.3 \times 10^{-21} \text{ eV}^2$ and $7.3 \times 10^{-23} \text{ eV}^2 < \Delta m_{\odot}^2 < 9.9 \times 10^{-23} \text{ eV}^2$. In this figure, all shadow overlapping areas meet five constraints.

Now we discuss how the matrix element Y_{ν} such as $Y_{\nu 11}$ affects the $\sin^2(\theta_{12})$, $\sin^2(\theta_{23})$ and $10 \times \sin^2(\theta_{13})$. In Fig. 5, the constraints from two mass variances are satisfied. Then $\sin^2(\theta_{12})$, $\sin^2(\theta_{23})$ and $10 \times \sin^2(\theta_{13})$ are plotted as $Y_{\nu 11}$ changes. According to the analysis of Fig. 4, we can take $Y_{\nu 13} = 4.516926 \times 10^{-8}$ and $Y_{\nu 23} = 2.803229 \times 10^{-7}$. With the above data, the blue, yellow and green regions correspond to the values of $\sin^2(\theta_{12})$, $\sin^2(\theta_{23})$ and $10 \times \sin^2(\theta_{13})$ mixing angles in the 3σ range, respectively. The blue line represents $\sin^2(\theta_{12})$. It grows consistently from $Y_{\nu 11} = 1.0 \times 10^{-6}$ to $Y_{\nu 11} = 1.3 \times 10^{-6}$, with rapid growth from $Y_{\nu 11} = 1.07 \times 10^{-6}$ to $Y_{\nu 11} = 1.136 \times 10^{-6}$, but it remains almost constant in the $Y_{\nu 11}$ region $[1.3 \times 10^{-6}, 1.5 \times 10^{-6}]$. The yellow line represents $\sin^2(\theta_{23})$. We can find that it keeps stable going from $Y_{\nu 11} = 1.0 \times 10^{-6}$ to $Y_{\nu 11} = 1.5 \times 10^{-6}$, which is always in the range of 3σ . The green line represents $10 \times \sin^2(\theta_{13})$. With the increase of $Y_{\nu 11}$, the $10 \times \sin^2(\theta_{13})$ grows faster and faster. From that all, we can see that in order to satisfy the mixing angle from the experiments $Y_{\nu 11}$ must be taken between two pink lines. So $Y_{\nu 11}$ should be one of the values in the range from 1.08902×10^{-6} to 1.9701×10^{-6} .

Using the Gaussian likelihood function, we construct a function combining three mixing angles and two mass variances:

$$p(\mathbf{y}) = \prod_{i=1}^5 \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(y_i - \mu_i)^2}{2\sigma_i^2}\right), \quad (23)$$

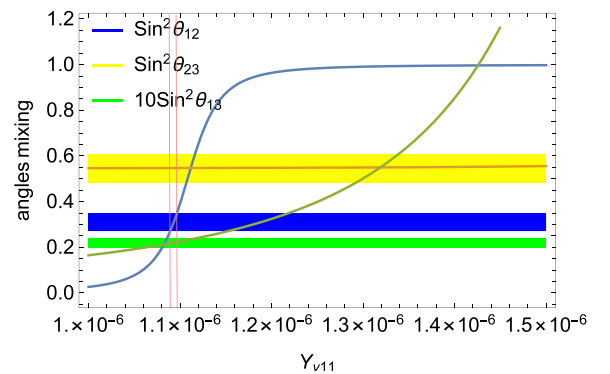


Fig. 5. (color online) $\sin^2(\theta_{12})$, $\sin^2(\theta_{23})$ and $10 \times \sin^2(\theta_{13})$ are plotted as $Y_{\nu 11}$ changes. The blue line represents $\sin^2(\theta_{12})$. The yellow line represents $\sin^2(\theta_{23})$. The green line represents $10 \times \sin^2(\theta_{13})$. The blue, yellow and green regions correspond to the values of the $\sin^2(\theta_{12})$, $\sin^2(\theta_{23})$ and $10 \times \sin^2(\theta_{13})$ mixing angles in the 3σ range. The pink lines represent the overlapping satisfaction interval.

where y_i and μ_i from 1 to 5 represent three mixing angles and two mass variances respectively. We take

$$\begin{aligned}\mu_1 &= \sin^2(\theta_{12}) = 0.307, \\ \mu_2 &= \sin^2(\theta_{23}) = 0.546, \\ \mu_3 &= \sin^2(\theta_{13}) = 0.022, \\ \mu_4 &= \Delta m_\odot^2 = 7.53 \times 10^{-5} \text{ eV}^2, \\ \mu_5 &= |\Delta m_A^2| = 2.453 \times 10^{-3} \text{ eV}^2.\end{aligned}\quad (24)$$

And σ_i correspond to their standard deviation. The extreme value of the ordinate in Fig. 6 corresponds to the value of our parameter. We can get the value of $Y_{\nu_{11}} = 1.092847 \times 10^{-6}$.

In summary, combining the five experimental constraints on neutrinos, we can determine the range of values for our selected parameters. Through our analysis, we can determine that the following parameters are reasonable:

$$\begin{aligned}Y_{\nu_{11}} &= 1.092847 \times 10^{-6}, \quad Y_{\nu_{22}} = 1.4000 \times 10^{-6}, \\ Y_{\nu_{33}} &= 1.352420 \times 10^{-6}, \quad Y_{\nu_{12}} = 7.604202 \times 10^{-8}, \\ Y_{\nu_{13}} &= 4.516926 \times 10^{-8}, \quad Y_{\nu_{23}} = 2.803229 \times 10^{-7}.\end{aligned}\quad (25)$$

B. The processes of $\nu_j \rightarrow \nu_i \gamma$

In this part, the objective of this study is to investigate the influence of certain sensitive parameters on the numerical results of neutrino transition magnetic moment μ_{ij}^M under experimental constraints. In the following discussion, we use the Eq. (25) to continue the numerical calculation. Besides, μ_{ij}^M is used to represent the transition magnetic moment of the Majorana neutrinos. We choose a number of parameters and investigate them to the extent allowed, such as g_X , λ_H , M_2 , μ .

g_X is the gauge coupling constant of the new gauge group $U(1)_X$. Besides, the mass matrixes of slepton and

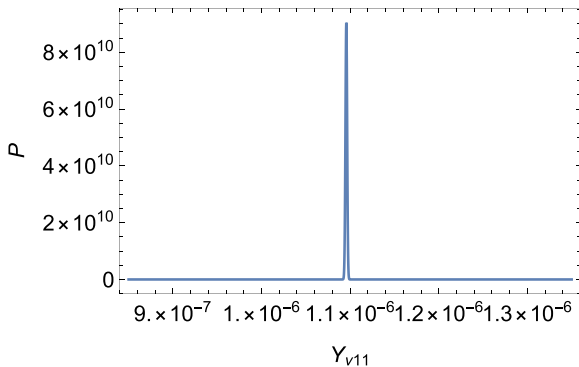


Fig. 6. (color online) The relationship between $Y_{\nu_{11}}$ and P .

coupling vertices $\nu_i \chi_j^- \tilde{e}_k^*$ all have the important parameter g_X , which can improve the new physics effect. We plot g_X and μ_{12}^M/μ_B in the Fig. 7 (a), in which the dashed line corresponds to $\lambda_H = 0.3$ and the solid line corresponds to $\lambda_H = 0.1$. Here, we take $\mu = 1000$ GeV and $M_2 = 1200$ GeV. We find that the both lines increase in the most region of g_X during the range of 0.3-0.51. And the solid line is larger than the dashed line. Generally speaking, larger g_X should lead to larger UMSSM contributions.

In Fig. 7 (b), with $g_X = 0.5$ (dashed line) and $g_X = 0.3$ (solid line), we take $\lambda_H = 0.1$ and $M_2 = 1200$ GeV. As solid and dashed lines go from bottom to top, μ_{12}^M/μ_B increases as g_X increases. They are decreasing functions of μ . μ appears in the term $\frac{1}{\sqrt{2}} \lambda_H \nu_S + \mu$ in the mass matrix for the chargino, which may has influence on the result. As shown in Fig. 7 (b), with the increase of μ , the chargino mass becomes heavier, which suppresses the numerical results.

λ_H comes from the term $\lambda_H \hat{S} \hat{H}_u \hat{H}_d$ in the superpotential. The mass matrices of several particles (chargino, neutralino) all have the important parameter λ_H , which possibly produces complex effects on the numerical results. In Fig. 7 (c), we take $\mu = 1000$ GeV and $g_X = 0.3$. The solid and dashed lines respectively represent $M_2 = 1200$ GeV and $M_2 = 2400$ GeV. Both the dashed and solid lines are decreasing functions as λ_H turns large.

In Fig. 7 (d), we take $\lambda_H = 0.1$ and $g_X = 0.3$. The solid and dashed lines represent $\mu = 1000$ GeV and $\mu = 1200$ GeV respectively. Similarly, M_2 as the mass matrix element of chargino, has the similar effect to μ on μ_{12}^M/μ_B . We also can see that μ_{12}^M/μ_B decreases as M_2 increases.

The above discussion is about μ_{12}^M/μ_B . For μ_{13}^M/μ_B and μ_{23}^M/μ_B , the influence of certain sensitive parameters are very similar as the condition of μ_{12}^M/μ_B . Therefore, we only list some of the parameters and plot their effects.

Fig. 8 (a) and Fig. 8 (b) describe the relationship between g_X and μ_{13}^M/μ_B and μ_{23}^M/μ_B . Like the description of μ_{12}^M/μ_B , the dashed line corresponds to $\lambda_H = 0.3$ and the solid line corresponds to $\lambda_H = 0.1$. With $\mu = 1000$ GeV and $M_2 = 1200$ GeV, we can find that the effects of g_X on the different components of μ_{ij}^M/μ_B have similar trends. For Fig. 8 (a), within the value range, the maximum values for solid and dashed lines are respectively 3.27×10^{-19} and 1.92×10^{-19} . For Fig. 8 (b), the maximum values for solid and dashed lines are 7.04×10^{-20} and 4.12×10^{-20} .

Fig. 8 (c) and Fig. 8 (d) describe the influences of M_2 on μ_{13}^M/μ_B and μ_{23}^M/μ_B . We can also find the similar trends, and they are decreasing functions of M_2 . In Fig. 8 (c), the maximum values for the solid and dashed lines are 1.69×10^{-19} and 1.55×10^{-19} respectively. The maximum values for the dashed and solid lines in the Fig. 8 (d) are 3.10×10^{-19} and 4.03×10^{-20} .

From the above graphs we can conclude that μ_{ij}^M/μ_B increase with g_X and decrease with μ , λ_H and M_2 . Their

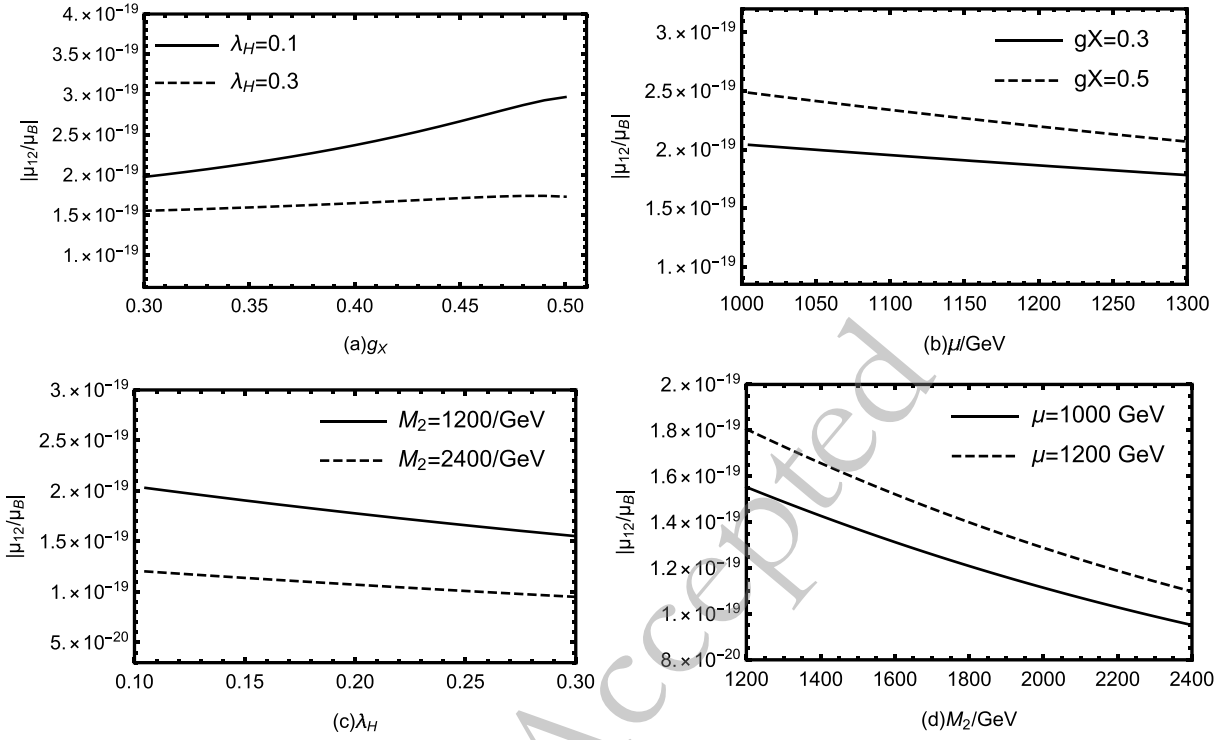


Fig. 7. The relationship between different parameters and μ_{12}^M/μ_B .

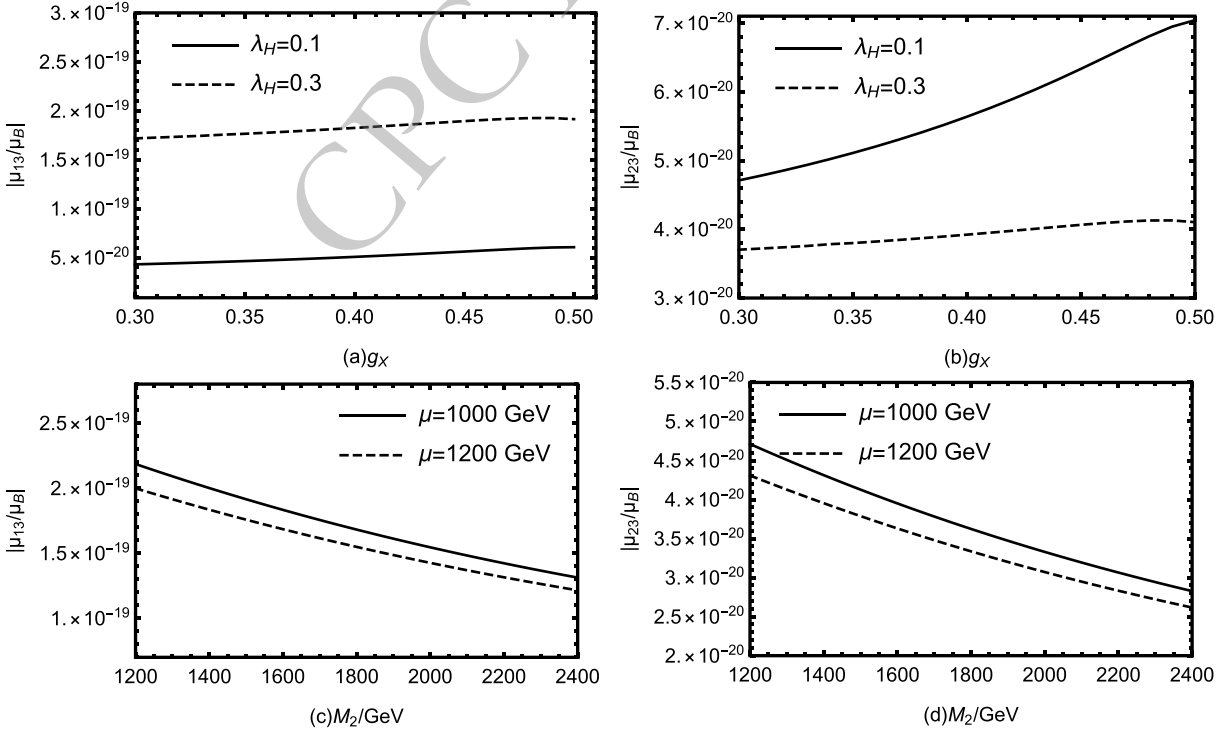


Fig. 8. The relationship between different parameters with μ_{13}^M/μ_B and μ_{23}^M/μ_B .

influences on μ_{ij}^M/μ_B tend to be similar. Overall, g_X , λ_H , μ and M_2 are indeed sensitive parameters that have obvious impacts on μ_{ij}^M/μ_B .

To explore the μ_{ij}^M parameter spaces well, we plot scatter diagrams for several parameters shown in Fig. 10.

The scanned parameters are listed in Table 2. Then, we use $\blacksquare$ ($\mu_{12}^M/\mu_B < 1.6 \times 10^{-19}$), $\blacklozenge$ ($1.6 \times 10^{-19} \leq \mu_{12}^M/\mu_B < 1.9 \times 10^{-19}$), $\blacktriangle$ ($1.9 \times 10^{-19} \leq \mu_{12}^M/\mu_B < 2.2 \times 10^{-19}$), $\bullet$ ($2.2 \times 10^{-19} \leq \mu_{12}^M/\mu_B < 3.2 \times 10^{-19}$) to represent the results of the transition magnetic moment.

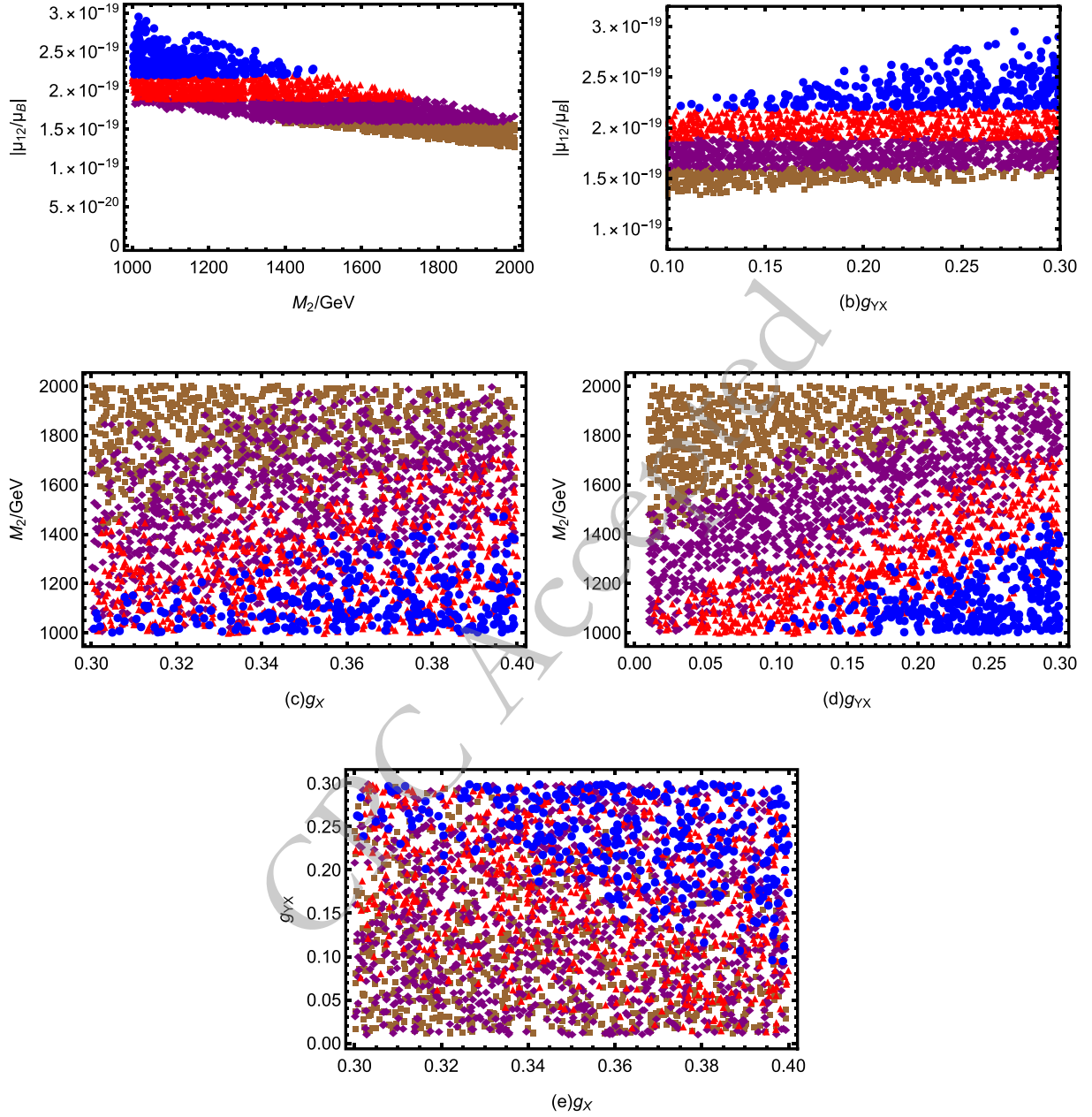


Fig. 9. (color online) The relationship between different parameters and μ_{12}^M/μ_B .

Table 2. Scanning parameters for Fig. 10

Parameters	g_X	g_{YX}	M_2/GeV
Min	0.3	0.01	1000
Max	0.4	0.3	2000

In Fig. 10(a), scatter plots of M_2 versus μ_{12}^M/μ_B are shown. The overall figure resembles a parallelogram. We can see that $\bullet$ is at the top, $\blacktriangle$ and $\blacklozenge$ are in the middle and $\blacktriangle$ is on top of $\blacklozenge$. Finally $\blacksquare$ is at the bottom. It can be seen that μ_{12}^M/μ_B decreases as M_2 increases. We can also see from the graph that its maximum value is 3.0×10^{-19} . This result is consistent with the result of line μ_{12}^M/μ_B . In Fig.

10(b), scatter plots of g_{YX} versus μ_{12}^M/μ_B are shown. It has the same graphic color layout as Fig. 10(a), just with a different trend. It can be gotten that the value of μ_{12}^M/μ_B increases as g_{YX} increases.

Fig. 10(c) shows the effects of g_X and M_2 on μ_{12}^M/μ_B . Viewed from the whole, different values of μ_{12}^M/μ_B in the parameter space have obvious stratification. The upper left corner is $\blacksquare$, followed by $\blacklozenge$, immediately followed by $\blacktriangle$, and to the bottom right by $\bullet$. From the trend in the graph, we get that μ_{12}^M/μ_B reaches its maximum value when $g_X = 0.4$ and $M_2 = 1000$ within the parameter space of Fig. 10(c). In Fig. 10(d), its graphical distribution is similar to that of Fig. 10(c). It proves that the effects of

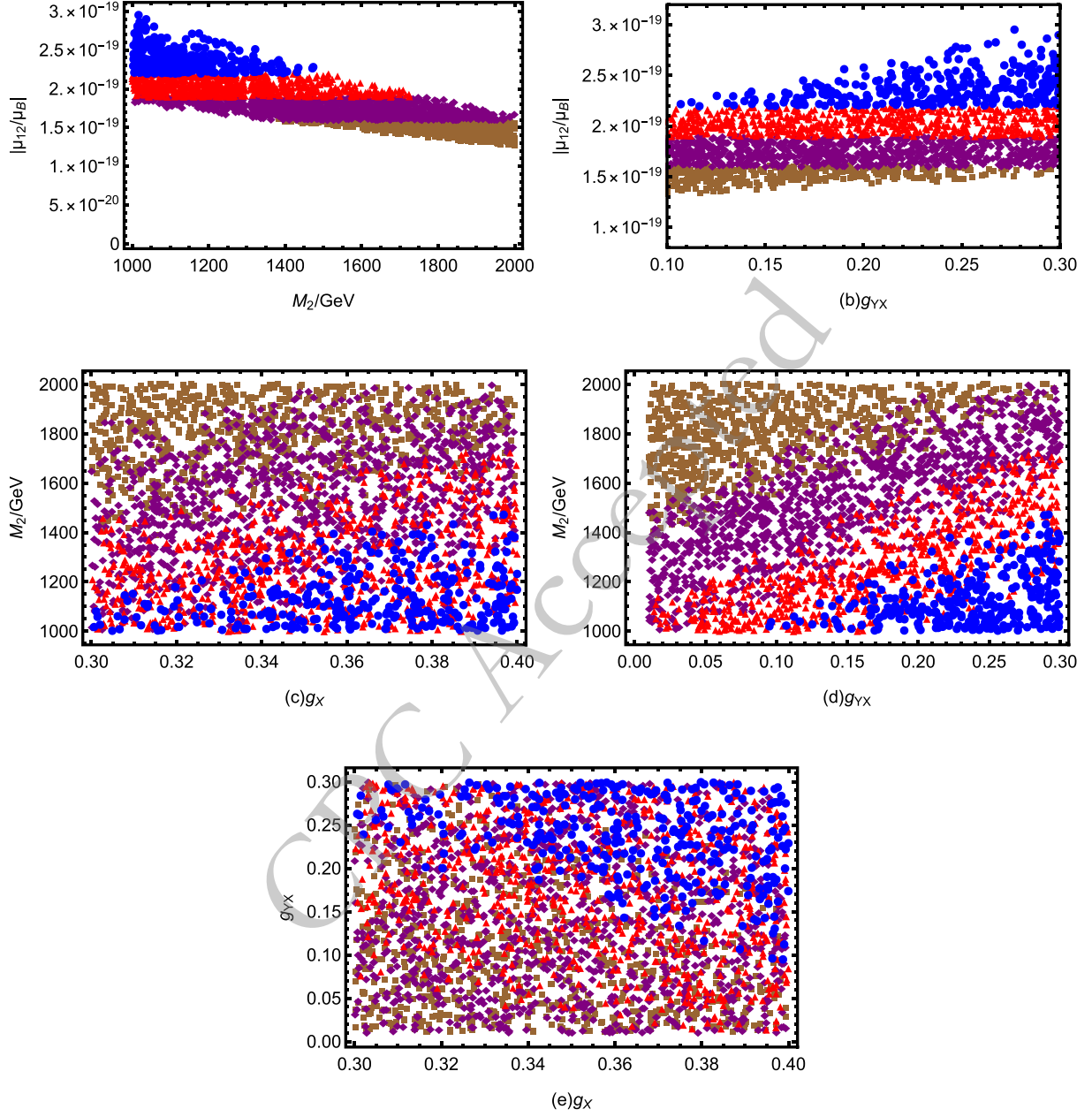


Fig. 10. (color online) The relationship between different parameters and μ_{12}^M/μ_B .

Table 3. The limits on neutrino transition magnetic moment at XENONnT experiment

XENONnT	90% C.L	99% C.L
$ \mu_{12}/\mu_B $	$< 6.77 \times 10^{-12}$	$< 9.63 \times 10^{-12}$
$ \mu_{13}/\mu_B $	$< 6.98 \times 10^{-12}$	$< 9.94 \times 10^{-12}$
$ \mu_{23}/\mu_B $	$< 9.04 \times 10^{-12}$	$< 12.9 \times 10^{-12}$

g_{YX} and g_X on μ_{12}^M/μ_B are similar.

From Fig. 10(e), we can derive the effects of g_X and g_{YX} on μ_{12}^M/μ_B . The color distribution is obvious from the overall view, the upper right corner is a mix of blue, red and purple colors with some brown. The lower left corner

is a mix of red, brown and purple colors. The red color is less distributed in the lower left corner. From the figure we can notice that the larger values of μ_{12}^M/μ_B are concentrated in the upper right corner, which means that an increase in g_X and g_{YX} will promote its increase.

V. CONCLUSION

In this article, we first introduce $U(1)_X$ SSM and then analyze the neutrino transport magnetic moment on this basis. We study the transition magnetic moment of the Majorana neutrinos by applying the effective Lagrangian method and the on-shell scheme. We derive the Feynman

diagrams and calculate the neutrino transport moment by combining the operators. We do a theoretical analysis of neutrino mixing. Based on the five bounds of the neutrino experiment, we filter for the right effective light neutrino mass matrix element. Besides we perform a large number of numerical calculations and plot lines with different parameters versus μ_{ij} according to the experimental limits, followed by a large scan that yields rich numerical results. In the numerical calculation, at first we fit the experimental data on neutrino mass variance and mixing angle for the normal order condition. Then we select some sensitive parameters, including g_X , λ_H , M_2 , μ and g_{YX} . In the one dimensional plot, we analyze the parameters including g_X , λ_H , M_2 and μ versus μ_{ij}^M/μ_B . In the scatter plot, we select three variants in Table 2 and study them. By analysing the numerical results, we understand the relationship between the selected parameters and μ_{ij}^M/μ_B , and they are indeed sensitive parameters.

Besides, we conclude that the order of magnitude of μ_{ij}^M/μ_B is between 10^{-20} and 10^{-19} . From the diagrams, one can find that the numerical result of μ_{12}^M/μ_B is at the order of 10^{-19} . The better limits on neutrino transition magnetic moment come from the recent XENONnT experiment[28]. We show the bounds at 90% and 99% C.L in the following Table 3. In the whole, the experimental sensitivity for $|\mu_{ij}/\mu_B|$ with $i \neq j$ is a little smaller than 10^{-11} . In a Type-II radiative seesaw scenario[29], the authors investigate neutrino magnetic moment, and the obtained numerical results($|\mu_{ij}/\mu_B|$) are large and can reach 10^{-12} . Our corresponding results are at the order of 10^{-19} , which are much smaller than their results[29].

Compared with other conclusions[10], our results are two orders of magnitude larger than them. The reason is that $U(1)_X$ SSM has new gauge couplings g_X and g_{YX} . The

vertices of $\nu_i\text{-}\chi_j^--\tilde{e}_k^*$ are in Eq.(37). On the face of it, Yukawa couplings($Y_{e,a}$ and $Y_{\nu,a}$) and gauge coupling g_2 are obviously shown in the above equation. $Y_{\nu,a}$ are very tiny, and $Y_{e,a}$ are small. Such as $Y_{e,2}$ for muon, with $\tan\beta = 10$ the value of $Y_{e,2}$ is about 0.006, which is much smaller than g_2 . g_X and g_{YX} appear in the mass squared matrix of scalar lepton, and their effects embody in the rotation matrix Z^E . So the new gauge couplings g_X and g_{YX} can produce new effects. Furthermore, the right-handed neutrinos and three Higgs singlets are added. They can produce new effects and improve the numerical results.

APPENDIX A

The mass matrix for chargino reads:

$$m_{\tilde{\chi}^-} = \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}}g_2v_\mu \\ \frac{1}{\sqrt{2}}g_2v_d & \frac{1}{\sqrt{2}}\lambda_H v_s + \mu \end{pmatrix}. \quad (A1)$$

This matrix is diagonalized by U and V :

$$U^* m_{\tilde{\chi}^-} V^\dagger = m_{\tilde{\chi}^-}^{dia}, \quad (A2)$$

with

$$\begin{aligned} \tilde{W}^- &= \sum_{t_2} U_{j1}^* \lambda_{j1}^-, & \tilde{H}_d^- &= \sum_{t_2} U_{j2}^* \lambda_{j2}^-, \\ \tilde{W}^+ &= \sum_{t_2} V_{1j}^* \lambda_j^+, & \tilde{H}_u^- &= \sum_{t_2} V_{2j}^* \lambda_j^+. \end{aligned} \quad (A3)$$

The mass matrix for slepton reads:

$$m_{\tilde{e}}^2 = \begin{pmatrix} m_{\tilde{e}_L \tilde{e}_L^*} & \frac{1}{2}(\sqrt{2}v_d T_e^\dagger - v_u(\lambda_H v_s + \sqrt{2}\mu)Y_e^\dagger) \\ \frac{1}{2}(\sqrt{2}v_d T_e - v_u Y_e(\sqrt{2}\mu^* + v_s \lambda_H^*)) & m_{\tilde{e}_R \tilde{e}_R^*} \end{pmatrix}. \quad (A4)$$

$$m_{\tilde{e}_L \tilde{e}_L^*} = M_L^2 + \frac{1}{8}((g_1^2 + g_{YX}^2)(-v_u^2 + v_d^2) + g_{YX}g_X(-2v_\eta^2 + 2v_\eta^2 - v_u^2 + v_d^2); + g_2^2(-v_d^2 + v_u^2)) + \frac{1}{2}v_d^2 Y_e^\dagger Y_e, \quad (A5)$$

$$m_{\tilde{e}_R \tilde{e}_R^*} = M_E^2 - \frac{1}{8}(2(g_1^2 + g_{YX}^2)(-v_u^2 + v_d^2) + g_{YX}g_X(3v_d^2 - 3v_u^2 - 4v_\eta^2 + 4v_\eta^2) + g_X^2(-2v_\eta^2 + 2v_\eta^2 - v_u^2 + v_d^2)) + \frac{1}{2}v_d^2 Y_e^\dagger Y_e. \quad (A6)$$

This matrix is diagonalized by Z^E :

$$Z^E m_{\tilde{e}}^2 Z^{E,\dagger} = m_{\tilde{e}}^{dia}, \quad (A7)$$

with

$$\tilde{e}_{L,i} = \sum_j Z_{ji}^{E,*} \tilde{e}_j, \quad \tilde{e}_{R,i} = \sum_j Z_{ji}^{E,*} \tilde{e}_j. \quad (A8)$$

The mass matrix for neutrino reads:

$$M_\nu = \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} Y_\nu^T \\ \frac{v_u}{\sqrt{2}} Y_\nu & \sqrt{2} v_{\tilde{\eta}} Y_X \end{pmatrix}. \quad (\text{A9})$$

This matrix is diagonalized by U_V :

$$U^{V,*} m_\nu U^{V,\dagger} = m_\nu^{dia}, \quad (\text{A10})$$

with

$$\nu_{L,i} = \sum_j U_{ji}^{V,*} \lambda_{\nu,j}, \quad \nu_{R,i}^* = \sum_j U_{ji}^V \lambda_{\nu,j}^*. \quad (\text{A11})$$

Here, we show some needed couplings in this model. We derive the vertices of $\nu_i - \chi_j^- - \tilde{e}_k^*$

$$\begin{aligned} \mathcal{L}_{\nu_i \chi_j^- \tilde{e}_k^*} = & \bar{\nu}_i \left\{ \sum_{a=1}^3 \left(U_{j2}^* U_{ia}^{V,*} Y_{e,a} Z_{k3+a}^E - g_2 U_{j1}^* U_{ia}^{V,*} Z_{ka}^E \right) P_L \right. \\ & \left. + \sum_{a,b=1}^3 Y_{\nu,ab}^* U_{i,3+a}^V Z_{kb}^E V_{j2} P_R \right\} \chi_j^- \tilde{e}_k^*. \end{aligned} \quad (\text{A12})$$

APPENDIX B

The expressions of $C_2^R, C_2^L, C_6^R, C_6^L$ are

$$\begin{aligned} C_2^R &= \sum_{j=1}^2 \sum_{k=1}^6 |A_L^{ijk}|^2 F(X_{L_k}, X_{X_j^\pm}), \\ C_2^L &= \sum_{j=1}^2 \sum_{k=1}^6 |A_R^{ijk}|^2 F(X_{L_k}, X_{X_j^\pm}), \\ C_6^R &= \sum_{j=1}^2 \sum_{k=1}^6 A_R^{ijk} B_R^{ijk} G(X_{L_k}, X_{X_j^\pm}), \\ C_6^L &= \sum_{j=1}^2 \sum_{k=1}^6 A_L^{ijk} B_L^{ijk} G(X_{L_k}, X_{X_j^\pm}). \end{aligned} \quad (\text{B1})$$

with the functions

$$\begin{aligned} F(x, y) &= \frac{1}{384\pi^2 \Lambda^2} \left(\frac{x^2 - 5xy - 2y^2}{(x-y)^3} + \frac{6xy^2(\log x - \log y)}{(x-y)^4} \right), \\ G(x, y) &= \frac{1}{64\pi^2 \Lambda^2} \frac{x^2 - y^2 + 2xy(\log y - \log x)}{(x-y)^3}. \end{aligned} \quad (\text{B2})$$

The used couplings are

$$A_L = \sum_{a=1}^3 (U_{j2}^* U_{ia}^{V,*} Y_{e,a} Z_{k3+a}^E - g_2 U_{j1}^* U_{ia}^{V,*} Z_{ka}^E),$$

$$A_R = \sum_{a,b=1}^3 Y_{\nu,ab}^* U_{i3+a}^V Z_{kb}^E V_{j2},$$

$$B_R = A_L^*, \quad B_L = A_R^*. \quad (\text{B3})$$

APPENDIX C.

The effective light neutrino mass matrix can be written as

$$\mathcal{M}_\nu^{eff} \approx - \left(\frac{v_u Y_\nu}{\sqrt{2}} \right) \left(\sqrt{2} v_{\tilde{\eta}} Y_X \right)^{-1} \left(\frac{v_u Y_\nu}{\sqrt{2}} \right)^T. \quad (\text{C1})$$

Using the "top-down" method[30], we get the Hermitian matrix

$$\mathcal{H} = (\mathcal{M}_\nu^{eff})^\dagger \mathcal{M}_\nu^{eff}. \quad (\text{C2})$$

Besides we can diagonalize the 3×3 matrix $\mathcal{H}$ to gain three eigenvalues

$$\begin{aligned} m_1^2 &= \frac{a}{3} - \frac{1}{3} p (\cos \phi + \sqrt{3} \sin \phi), \\ m_2^2 &= \frac{a}{3} - \frac{1}{3} p (\cos \phi - \sqrt{3} \sin \phi), \\ m_3^2 &= \frac{a}{3} + \frac{2}{3} p \cos \phi. \end{aligned} \quad (\text{C3})$$

These parameters can be given by

$$\begin{aligned} p &= \sqrt{a^2 - 3b}, \quad \phi = \frac{1}{3} \arccos \left(\frac{1}{p^3} (a^3 - \frac{9}{2} ab + \frac{27}{2} c) \right), \\ a &= \text{Tr}(\mathcal{H}), \\ b &= \mathcal{H}_{11} \mathcal{H}_{22} + \mathcal{H}_{11} \mathcal{H}_{33} + \mathcal{H}_{22} \mathcal{H}_{33} - \mathcal{H}_{12}^2 - \mathcal{H}_{13}^2 - \mathcal{H}_{23}^2, \\ c &= \text{Det}(\mathcal{H}). \end{aligned} \quad (\text{C4})$$

We take the normal ordering (NO), so:

$$\begin{aligned} m_{\nu_1} &< m_{\nu_2} < m_{\nu_3}, \quad m_{\nu_1}^2 = m_1^2, \quad m_{\nu_2}^2 = m_2^2, \quad m_{\nu_3}^2 = m_3^2, \\ \Delta m_{\odot}^2 &= m_{\nu_2}^2 - m_{\nu_1}^2 = \frac{2}{\sqrt{3}} p \sin \phi > 0, \\ \Delta m_A^2 &= m_{\nu_3}^2 - m_{\nu_1}^2 = p (\cos \phi + \frac{1}{\sqrt{3}} \sin \phi) > 0. \end{aligned} \quad (\text{C5})$$

From the mass squared matrix $\mathcal{H}$, one gets the normalized eigenvectors

$$\begin{pmatrix} (U_\nu)_{11} \\ (U_\nu)_{21} \\ (U_\nu)_{31} \end{pmatrix} = \frac{1}{\sqrt{|X_1|^2 + |Y_1|^2 + |Z_1|^2}} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix},$$

$$\begin{pmatrix} (U_\nu)_{12} \\ (U_\nu)_{22} \\ (U_\nu)_{32} \end{pmatrix} = \frac{1}{\sqrt{|X_2|^2 + |Y_2|^2 + |Z_2|^2}} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix},$$

$$\begin{pmatrix} (U_\nu)_{13} \\ (U_\nu)_{23} \\ (U_\nu)_{33} \end{pmatrix} = \frac{1}{\sqrt{|X_3|^2 + |Y_3|^2 + |Z_3|^2}} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix}. \quad (C6)$$

The concrete forms of X_I, Y_I, Z_I for $I = 1, 2, 3$ are shown here

$$\begin{aligned} X_1 &= (\mathcal{H}_{22} - m_{\nu_1}^2)(\mathcal{H}_{33} - m_{\nu_1}^2) - \mathcal{H}_{23}^2, \\ Y_1 &= \mathcal{H}_{13}\mathcal{H}_{23} - \mathcal{H}_{12}(\mathcal{H}_{33} - m_{\nu_1}^2), \\ Z_1 &= \mathcal{H}_{12}\mathcal{H}_{23} - \mathcal{H}_{13}(\mathcal{H}_{22} - m_{\nu_1}^2), \\ X_2 &= \mathcal{H}_{13}\mathcal{H}_{23} - \mathcal{H}_{12}(\mathcal{H}_{33} - m_{\nu_2}^2), \end{aligned} \quad (C7)$$

$$\begin{aligned} Y_2 &= (\mathcal{H}_{11} - m_{\nu_2}^2)(\mathcal{H}_{33} - m_{\nu_2}^2) - \mathcal{H}_{13}^2, \\ Z_2 &= \mathcal{H}_{12}\mathcal{H}_{13} - \mathcal{H}_{23}(\mathcal{H}_{11} - m_{\nu_2}^2), \\ X_3 &= \mathcal{H}_{12}\mathcal{H}_{23} - \mathcal{H}_{13}(\mathcal{H}_{22} - m_{\nu_3}^2), \\ Y_3 &= \mathcal{H}_{12}\mathcal{H}_{13} - \mathcal{H}_{23}(\mathcal{H}_{11} - m_{\nu_3}^2), \\ Z_3 &= (\mathcal{H}_{11} - m_{\nu_3}^2)(\mathcal{H}_{22} - m_{\nu_3}^2) - \mathcal{H}_{12}^2. \end{aligned} \quad (C7)$$

The mixing angles among three tiny neutrinos can be defined as follows

$$\begin{aligned} \sin\theta_{13} &= \left| (U_\nu)_{13} \right|, & \cos\theta_{13} &= \sqrt{1 - \left| (U_\nu)_{13} \right|^2}, \\ \sin\theta_{23} &= \frac{\left| (U_\nu)_{23} \right|}{\sqrt{1 - \left| (U_\nu)_{13} \right|^2}}, & \cos\theta_{23} &= \frac{\left| (U_\nu)_{33} \right|}{\sqrt{1 - \left| (U_\nu)_{13} \right|^2}}, \\ \sin\theta_{12} &= \frac{\left| (U_\nu)_{12} \right|}{\sqrt{1 - \left| (U_\nu)_{13} \right|^2}}, & \cos\theta_{12} &= \frac{\left| (U_\nu)_{11} \right|}{\sqrt{1 - \left| (U_\nu)_{13} \right|^2}}. \end{aligned} \quad (C8)$$

References

- [1] S.L. Glashow, H. Georgi, *Phys. Rev. Lett.* **32**, 438 (1974)
- [2] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967)
- [3] S. Weinberg, *Phys. Rev. D* **19**, 1277 (1979)
- [4] A. Salam, J. C. Ward, *Phys. Rev. Lett.* **30**, 1268 (1973)
- [5] H.P. Nilles, *Phys. Rept.* **110**, 1 (1984)
- [6] H.E. Haber, G.L. Kane, *Phys. Rept.* **117**, 75 (1985)
- [7] Rosiek, *Phys. Rev. D* **41**, 3464 (1990)
- [8] U. Ellwanger, C. Hugonie, A.M. Teixeira, *Phys. Rep.* **496**, 1 (2010)
- [9] B. Yan, S.M. Zhao, T.F. Feng, *Nucl. Phys. B* **975**, 115671 (2022)
- [10] H.B. Zhang, T.F. Feng, Zhao Feng Ge, *et al.*, *JHEP* **02**, 012 (2014)
- [11] A.D. Gouvea, S. Shalgar, *JCAP* **04**, 018 (2013)
- [12] V. Brdar, A. Greljo, J. Kopp, *et al.*, *JCAP* **01**, 039 (2021)
- [13] S.M. Zhao, T.F. Feng, M.J. Zhang, *et al.*, *JHEP* **02**, 130 (2020)
- [14] G. Belanger, J.D. Silva, H. M. Tran, *Phys. Rev. D* **95**, 115017 (2017)
- [15] V. Barger, P. F. Perez, S. Spinner, *Phys. Rev. Lett.* **102**, 181802 (2009)
- [16] P. H. Chankowski, S. Pokorski, J. Wagner, *Eur. Phys. J. C* **47**, 187 (2006)
- [17] J. L. Yang, T. F. Feng, S. M. Zhao, *et al.*, *Eur. Phys. J. C* **78**, 714 (2018)
- [18] T.F. Feng, L. Sun, X.Y. Yang, *Nucl. Phys. B* **800**, 221 (2008)
- [19] R.L. Workman, *et al.*, *Prog. Theor. Exp. Phys.* **2022**, 083C (2022)
- [20] CMS Collaboration, *Phys. Lett. B* **716**, 30 (2012)
- [21] ATLAS Collaboration, *Phys. Lett. B* **716**, 1 (2012)
- [22] M. Carena, J.R. Espinosa, M. Quirós, *et al.*, *Phys. Lett. B* **355**, 209 (1995)
- [23] M. Carena, S. Gori, N.R. Shah, *et al.*, *JHEP* **03**, 014 (2012)
- [24] F.P. An, *et al.*, *Phys. Rev. Lett.* **130**, 161802 (2023)
- [25] R. Abbasi, *et al.*, *Phys. Rev. D* **108**, 012014 (2023)
- [26] K. Abe, *et al.*, *Phys. Rev. D* **108**, 072011 (2023)
- [27] Marcela Carena, Alejandro Daleo, *et al.*, *Phys. Rev. D* **70**, 093009 (2004)
- [28] E. Aprile, *et al.*, *Phys. Rev. Lett.* **129**, 161805 (2022)
- [29] S. Singirala, D.K. Singha, R. Mohanta, *Phys. Rev. D* **109**, 075031 (2024)
- [30] B.Dziewit, S.Zajac, M. Zralek, *Acta Phys. Pol. B* **42**, 2509 (2011)