Revisiting the alpha-decay reduced width of the lightest uranium isotope ^{214}U*

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Abstract: Background: The lightest uranium isotope ^{214}U has been produced using the Heavy Ion Research Facility in Lanzhou, China. It is found that the α-decay reduced width (δ^2) of ^{214}U is significantly larger than other nuclei by a factor of two. However, the extraction of δ^2 depends on the penetration probability (P) through the barrier, and P is related to the theoretical method of obtaining it and the form of the α-core potential. Purpose: The aim of the present study is to investigate whether the selections of the α-core potential and the method of calculating P can affect the above conclusion. Method: Four different phenomenological α-core potentials and two microscopic double-folding potentials, together with the Wentzel-Kramers-Brillouin (WKB) approximation and the transfer matrix (TM) approach are used to obtain P. Results: The value of P obtained with WKB is about 20% ~ 40% smaller than the one obtained with TM approach, and consequently the deduced δ^2 is overestimated. The choice of α-core potential can significantly affect the value of the δ^2. With spherical form for the α-core potentials, the δ^2 of ^{214}U obtained with both WKB and TM approaches are about twice as large as that of the surrounding nuclei. While with the deformed double-folding potential, the ratio between δ^2 of ^{214}U and that of the surrounding nuclei is found slightly below 2. Conclusions: Effects of nuclear deformation and the α-core potential should be considered when studying the α-decay reduced width in the N_pN_n systematic.

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I. INTRODUCTION

Alpha decay is a transformation that occurs when the nucleus spontaneously emits α particle, it is one of the main decay modes of heavy and superheavy nuclei (SHN). In the 1920s, Gamow[1] and Gurney and Condon[2], independently, have used quantum mechanics to describe α decay, the process by which a preformed α-particle tunneling through a barrier created by the α particle and the residual nucleus. Since then, many theoretical models have been proposed to describe the nuclear interaction potential between the α particle and the daughter nucleus and to calculate the penetration probability (P) in α decay [3−11]. The Wentzel-Kramers-Brillouin (WKB) approximation is a quasi-classical approximation for solving the one-dimensional Schrödinger equation proposed by Wenzel, Kramers and Brillouin, it has been very widely used to calculate the penetration probability of a particle across potential barrier in quantum tunneling process, such as α decay and heavy-ion fusion reaction in nuclear physics. The transport matrix (TM) approach uses multistep functions (multistep potential approximation) to approximate an arbitrary potential, then the wave function in each region can be calculated analytically and the penetration probability is calculated by connecting momentum eigenfunctions [12]. TM method can accurately calculate the penetration probability across arbitrary potential barriers when the number of segment is large enough. It has been found that the penetration penetrability obtained with WKB approximation is about 30-40% smaller than the accurate result obtained with the TM approach, in the case of α decay [13].

The synthesis of SHN has attracted a lot of attention in the field of nuclear physics since long time ago, in order to search the possible existence of "island of superheavy nuclei". α decay of SHN is of central interest for both experimental and theoretical studies. Not only because α decay is the main means of identifying SHN experimentally, but also due to the α-decay reduced width

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δ^2, which is deduced from the measured half-life and the calculated penetration probability, is believed to encode rich information on the structure of nuclei.

Uranium is the heaviest element in nature. ^{234}U, ^{235}U, and ^{238}U are the three isotopes of Uranium that can be found in nature. Recently, the lightest uranium isotope ^{214}U has been produced using the Heavy Ion Research Facility in Lanzhou, China, which has attracted widespread interest in the nuclear physics community [14]. With Rasmussen method, the δ^2 of ^{214,216}U are found to be significantly enhanced by a factor of two as compared with other nuclei in the N_pN_n systematic [14]. It is known that the extraction of δ^2 is model dependent, it relates to the choice of α-core potential and the method of obtaining the penetration probability. Therefore, it is necessary to investigate whether these two factors affect the conclusion that the δ^2 of ^{214,216}U are significantly larger than the surrounding nuclei. To do that, four different phenomenological α-core potentials and two microscopic double-folding potentials, together with the WKB approximation and the transfer matrix approach are used to obtain the penetration probability. It is known that the α-decay reduced width can be derived from the α-preformation probability. Various effects (e.g., nuclear deformation, isospin asymmetry, nuclear shells and pairing) on the α-preformation probability have been extensively studied, see e.g., Refs. [15−18]. In the present work, we only focus on the influences of nuclear deformation and the α-core potential on the α-decay reduced width in the N_pN_n systematic.

This paper is organized as follows: In Sec. II, the WKB approximation, the TM approach and the α-core potentials are introduced. In Sec. III, the ratios of the δ^2 of ^{214}U to that of the surrounding nuclei for different methods of calculating the P and for different α-core potentials are given and compared. The summary is given in Sec. IV

II. THEORETICAL FRAMEWORK

The α-decay reduced width δ^2 is expressed as [19] :

$$\delta^2 = \frac{hln2}{T_{1/2}P}, \tag{1}$$

where $T_{1/2}$ is the experimental half-life for α-decay and P is the penetration probability. In the present work, the half-life are taken from Refs. [14, 20].

A. WKB approximation

Using the WKB approximation, the probability of penetration of a particle penetrating a barrier V(r) at an incident energy of Q can be obtained:

$$P = \exp\left[-\frac{2}{\hbar}\left|\int_{r_2}^{r_3} \sqrt{2\mu|V(r)-Q|}\,dr\right|\right], \tag{2}$$

where μ is the mass of the particle. The turning points r_2 and r_3 are determined from the equation $V(r_2) = V(r_3) = Q$. This method for calculating barrier penetration probability has certain limitations. It is generally believed that WKB approximation can better calculate the penetration probability at energies well below the barrier peak and for the potentials which are slowly varying [21].

B. TM approach

The TM approach assumes that arbitrary potential barrier can be split into segments. When the number of segments is large enough, this method can reasonably describe arbitrary potential barrier. Assuming that the barrier is equally divided into N segments, the potential energy of each segment is expressed as

$$V_j = V\left(\frac{r_{j-1}+r_j}{2}\right) \tag{3}$$

with $r_{j-1} < r < r_j$ $(j = 0,1,2,...N,N+1)$. The wave function of a particle with energy Q in the jth region is given by

$$\Psi_j(r) = A_j\exp\left(ik_jr\right) + B_j\exp\left(-ik_jr\right) \tag{4}$$

Here $k_j = \sqrt{2\mu(Q-V_j)}/\hbar$ is the wave number. The coefficients A_j and B_j can be obtained from the continuity of ψ(r) and its derivative at each boundary. By setting $A_0 = 1$ and $B_{N+1} = 0$, the penetration probability can be calculated as follows,

$$P = \frac{k_{n+1}}{k_0}|A_{n+1}|^2, \tag{5}$$

here $A_{n+1} = \frac{k_0}{k_{n+1}}\frac{1}{M_{22}}$. And M_{22} is given by

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \prod_{l=0}^{n} M_l, \tag{6}$$

where,

$$M_l = \frac{1}{2}\begin{pmatrix} (1+S_l)e^{-i(k_{l+1}-k_l)r_l} & (1-S_l)e^{-i(k_{l+1}+k_l)r_l} \\ (1-S_l)e^{i(k_{l+1}+k_l)r_l} & (1+S_l)e^{i(k_{l+1}-k_l)r_l} \end{pmatrix}, \tag{7}$$

$$S_l = \frac{k_l}{k_{l+1}}. \tag{8}$$

The accuracy of TM approach can be validated with $V(x) = V_0 \cosh^{-2}(x/a)$ potential, for which the exact analytic transmission probability is known [22, 23]. Thus, the penetration probability obtained with the TM approach is quoted as the exact value throughout the paper.

C. Alpha-core potentials

Usually, the α-core potential is composed of the nuclear, Coulomb, and centrifugal terms,

$$V(r) = V_N(r) + V_C(r) + V_L(r). \tag{9}$$

In the present work, the following six α-core potentials are considered.

(1) In a cluster model proposed by B. Buck et al. in Ref. [24], a cosh form of nuclear potential was used. It is given by,

$$V_N(r) = -V_0 \frac{1+\cosh\frac{R}{a}}{\cosh\frac{r}{a}+\cosh\frac{R}{a}} \tag{10}$$

of depth V_0, diffuseness parameter a, and radius R. A Langer modified centrifugal barrier $\frac{\hbar^2}{2\mu}\frac{(L+\frac{1}{2})^2}{r^2}$ is used instead of $\frac{\hbar^2}{2\mu}\frac{L(L+1)}{r^2}$. This modification is necessary when going from the one-dimensional problem to three-dimensional problems [25, 26]. The Coulomb potential $V_C(r)$ is taken as a form appropriate to a point α particle interacting with a uniformly charged spherical core of radius R:

$$V_C(r) = \begin{cases} \dfrac{Z_1Z_2e^2}{r} & r > R \\ \dfrac{Z_1Z_2e^2}{2R}\left[3-\left(\frac{r}{R}\right)^2\right] & r \le R \end{cases}. \tag{11}$$

The R in the above two equations is determined by the Bohr-Sommerfeld quantization condition

$$\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}|Q_\alpha - V(r)|}\,dr = (G+L-1)\frac{\pi}{2}. \tag{12}$$

Here G is the global quantum number, and Q_α is the α-decay energy. The classic turning points (r_1, r_2, and r_3 in order of increasing distance from the origin) are solved by $V(r) = Q_\alpha$. The three free parameters are set to $V_0 = 162.3$ MeV, a=0.40 fm, $G = 20$ (for N≤ 126) = 22 (for N≥ 126), they were determined by a best fit to the available data [24]. This potential is named as Pot1.

(2) In our recent work, an isospin-dependent depth parameter V_0=152.5(1+0.2 $\frac{N-Z}{N+Z}$) and an adjusted diffuseness parameter $a = 0.39$ fm in Eq.(10) are obtained by fitting the experimental half-lives of Uranium isotopes [27]. This potential is named as Pot2.

(3) In Ref. [13], a mass and charge number dependent α-core potential was obtained using the following expression:

$$V_N(r) = -\frac{A_1 U_0}{1+\exp\left(\frac{r-R_0}{a}\right)} \tag{13}$$

with $U_0 = [53-33(N-Z)/A]$ MeV, $R_0 = 1.27A^{1/3}$ fm, $a = 0.67$ fm. A_1 is the mass number of the emitted particle. N, Z, A are the neutron, proton, and mass number of the parent nucleus, respectively. The Coulomb potential is given by the Eq.(11) with radius $R = 1.28A^{1/3} - 0.76 + 0.8A^{-1/3}$ fm. This potential is named as Pot3.

(4) In the Rasmussen method, the α-core potential is taken from the real part of a potential deduced by Igo to fit alpha elastic-scattering data[28, 29]. It reads,

$$V_N(r) = -1100\exp\left[-\left(\frac{r-1.17A^{1/3}}{0.574}\right)\right] \text{MeV}, \tag{14}$$

where A is the mass number of the parent nucleus. This potential is named as Pot4.

(5) Unlike the previous phenomenological α-core potentials, the double-folding potential uses the microscopic nuclear potential and the realistic Coulomb potential[30]:

$$V_N(\mathbf{r}) = \lambda \int d\mathbf{r_1} d\mathbf{r_2} \rho_1(\mathbf{r_1})\rho_2(\mathbf{r_2}) g(E, |\mathbf{s}|), \tag{15}$$

$$V_C(\mathbf{r}) = \int d\mathbf{r_1} d\mathbf{r_2} \rho_1'(\mathbf{r_1})\rho_2'(\mathbf{r_2}) \frac{e^2}{|\mathbf{s}|}, \tag{16}$$

where λ is the renormalized factor. In the present work, λ=0.6 is used according to the values in Ref. [30]. In some works, the value of λ is determined with Bohr-Sommerfeld quantization condition, see e.g., [15, 31]. While there is also some works, a constant value of λ is used, for example in Refs. [6, 30, 32]. As discussed in Ref. [32], the variation of λ is small in both spherical and deformed cases for different nuclei. We have checked that the extracted δ^2 is influenced when varying λ, however its effect on the α-decay reduced width ratio is weak. Therefore a constant value of λ is ued in the present work for simplicity. $\boldsymbol{r}$ is the distance between the mass center of α-particle and the mass center of core. $\mathbf{r_1}$ and $\mathbf{r_2}$ are the nucleon coordinates belonging to the daughter nucleus and the α-particle, respectively. The quantity $|\mathbf{s}|$ is the distance between a nucleon in the core and a nucleon in the

α-particle ($\mathbf{s} = \mathbf{r}+\mathbf{r_2}-\mathbf{r_1}$). ρ_1 and ρ_2 are the mass density distributions of α-particle and the core.

$$\rho_1(r_1) = 0.4299\exp(-0.7024r_1^2), \tag{17}$$

$$\rho_2(r_2) = \frac{\rho_0}{1+\exp(\frac{r_2-c}{a})}, \tag{18}$$

where ρ_0 is fixed by the mass numbers of daughter nucleus (A_d). The $c = 1.07A_d^{1/3}$, $a = 0.54$ fm.

$$\int \rho_i(\mathbf{r})d\mathbf{r} = A_i. \tag{19}$$

ρ_1' and ρ_2' in Eq.(16) are the charge density distributions of α-particle and daughter nucleus.

$$\rho_1' = \rho_0'\exp(-0.7024r_1^2), \tag{20}$$

$$\rho_2' = \frac{\rho_0'}{1+\exp(\frac{r_2-c}{a})}. \tag{21}$$

The value of ρ_0' is fixed by the charge numbers of α particle and daughter nucleus.

$$\int \rho_i'(\mathbf{r})d\mathbf{r} = Z_i. \tag{22}$$

The $g(E,|\mathbf{s}|)$ in Eq.(15) is the microscopic M3Y nucleon-nucleon interaction potential,

$$g(E,|\mathbf{s}|) = 7999\frac{\exp(-4s)}{4s} - 2134\frac{\exp(-2.5s)}{2.5s} + J_{00}\delta(\mathbf{s}), \tag{23}$$

$$J_{00} = -276(1-0.005E_\alpha/A_\alpha). \tag{24}$$

E_α and A_α denote the energy and mass number of the cluster, respectively. This potential is named as Pot5.

(6) The vast majority of all known atomic nuclei have varying degrees of deformation. The deformed double-folding potential takes into account the axial deformation of the daughter nuclei [32]. The α-core potential is:

$$V(\boldsymbol{r},\beta) = \lambda V_N(\boldsymbol{r},\beta)+V_C(\boldsymbol{r},\beta), \tag{25}$$

where β is the orientation angle of the α particle relative to the symmetry axis of the daughter nucleus.

For the deformed residual nucleus, its density distribution is related to the deformation parameters:

$$\rho_2(r_2,\theta) = \frac{\rho_0}{1+\exp(\frac{r_2-R(\theta)}{a})}, \tag{26}$$

where the half-density radius $R(\theta)$ is given by :

$$R(\theta) = R_0(1+\beta_2Y_{20}(\theta)+\beta_4Y_{40}(\theta)). \tag{27}$$

Here $R_0 = 1.07A_d^{1/3}$ fm and $a = 0.54$ fm. In the present work, only the contribution of β_2 is considered for simplicity. This potential is named as Pot6.

In the Pot6, the P of α decay is

$$P_\beta = \exp\left[-2\int_{r_2(\beta)}^{r_3(\beta)}\sqrt{\frac{2\mu}{\hbar^2}|Q_\alpha - V(r,\beta)|dr}\right], \tag{28}$$

where the values of $r_2(\beta)$ and $r_3(\beta)$ can be calculated by $Q_\alpha = V(r,\beta)$. The total penetration factor P is obtained by

$$P = \frac{1}{2}\int_0^\pi P_\beta\sin(\beta)d\beta. \tag{29}$$

III. RESULTS AND DISCUSSION

The six potential barrier for the α decay of ^{214}U are shown in Fig. 1. The potentials for two different orientations β=0° and 90° in Pot6 are shown as red solid lines with triangles and blue solid lines with circles, respectively. It is quite different of these potentials, which could lead to a large difference in the penetration probability P of the α particle through each potential since P depends strongly on the height and width of the potential barrier.

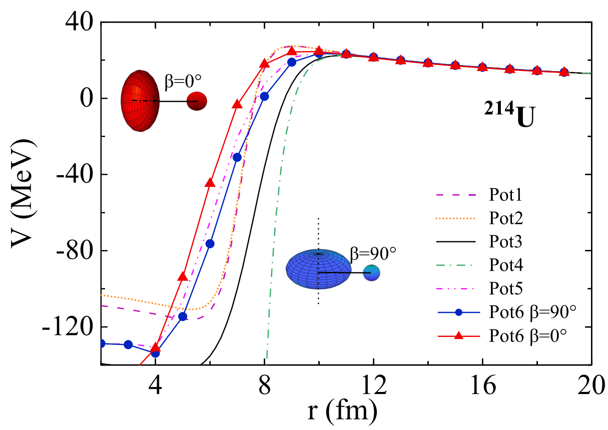

Fig. 1. (color online) Six α-core potentials of ^{214}U. $\beta = 0°$ and $\beta = 90°$ are two different orientations of the $\alpha + {}^{210}$Th system under Pot6.

Consequently, the α-decay reduced width δ^2 could be quite different correspondingly.

A. Influence of the method used in calculating the penetration probability

It is known that the WKB approximation is more reliable with gentle variation of potential, i.e., only slightly change over the de Broglie wavelength. Its computational accuracy in the studies of heavy-ion fusion, alpha decay, proton and cluster radioactivity has been discussed [13, 33, 34]. In Ref. [13], with Pot3, it is found that the penetration probability P of the α decay obtained with WKB approximation is about 30%-40% smaller than the exact one which can be obtained with the transfer matrix approach. In the present work, the accuracy of WKB approximation on the calculation of P is examined with different α-core potentials. The relative deviation of the WKB approximation can be examined by using $RD=(P_{WKB}-P_{TM})/P_{TM}\times 100\%$. P_{WKB} and P_{TM} denote the penetration probability obtained with WKB approximation and the transfer matrix approach, respectively. In the present work, the α decay of nuclei around ^{214}U are considered. The experimental Q_α values are taken from Refs. [14, 35]. The RD values obtained with different α-core potentials are plotted as a function the mass number of parent nuclei in Fig. 2. It can be seen that, for Pot1 and Pot2, the RD values are almost constant (about -40%) While for Pot3 and Pot4, the RD values are varied from about -20% to -40%. This is because both Pot3 and Pot4 are dependent on the mass number while Pot1 and Pot2 are not.

On the basis of the WKB approximation and the TM approach, we extracted the α-decay reduced width δ^2 of the ground state to ground state decays of ^{214}U and the surrounding nuclei with Pot1-4, which is shown in Fig. 3 as function of N_pN_n. Here, N_p and N_n are the proton and neutron numbers relative to the nearest closed shell Z = 82 and N = 126, respectively. In this work, the error in the δ^2 is only caused by the uncertainty of the experimental half-life [14]. Figure 3 clearly illustrates the α-decay reduced width δ^2 is model-dependent. Nevertheless, as discussed in Ref. [36], rich information about the structural properties of nuclei can be gained from the trend of δ^2 of different isotopes, rather than δ^2 itself. Comparing the results in the left column of Fig. 3, it can be seen that the values of the δ^2 for different α-core potentials vary by almost three orders of magnitude, from 10^2 to 10^5. According to Eq. (1), for the same nucleus, the only factor that affects the value of the δ^2 is P. In Fig. 1, Pot1 has the largest barrier width, resulting in a lower probability of α particle crossing this barrier. Therefore, the value of the δ^2 obtained with Pot1 is the largest one. If one compares the results in the left panels to that in the right panels, with the same α-core potential, it is found that the values of δ^2 obtained with TM approach are

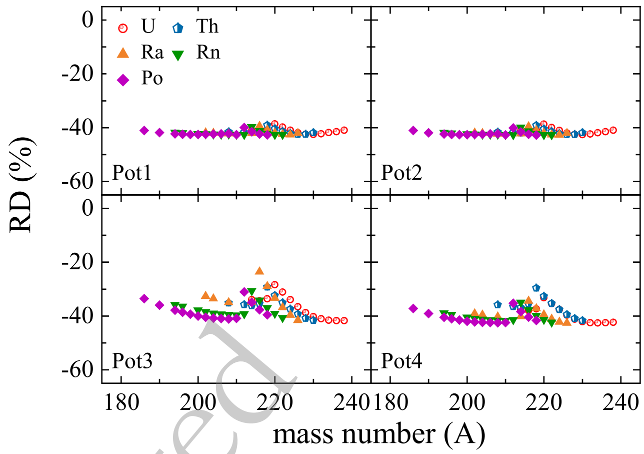

Fig. 2. (color online) Relative deviation of penetration probability caused by the WKB approximation for α decay for Pot1-4.

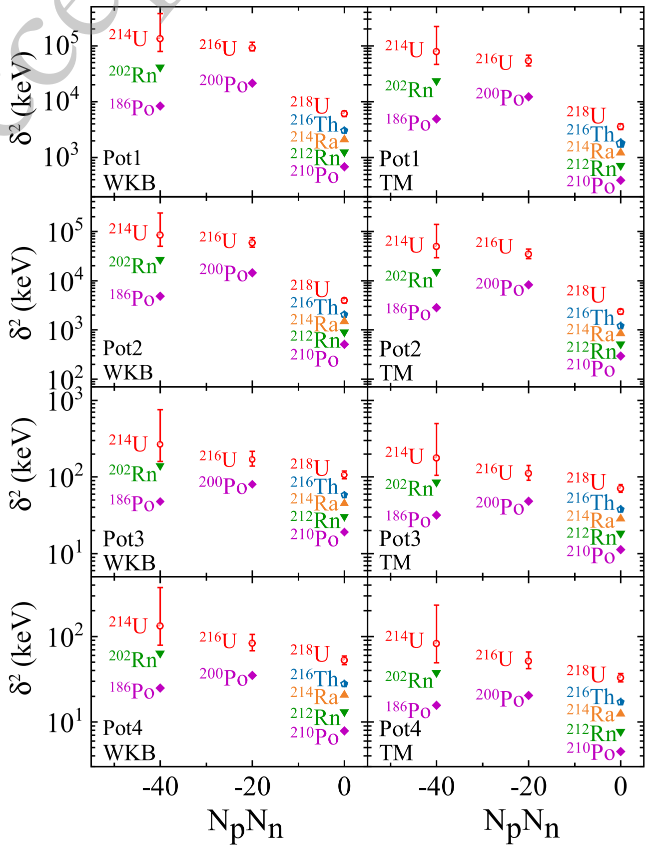

Fig. 3. (color online) The α-decay reduced width δ^2 of ^{214}U and the surrounding nuclei obtained by WKB approximation (left panels) and the TM method (right panels) with different α-core potentials. The N_p and N_n values are proton and neutron number relative to $Z=82$ and $N=126$ closed shells, respectively.

smaller than that obtained with WKB approximation. This is due to the underestimation of P in the WKB approximation. In addition, in all cases, the δ^2 of ^{214,216}U are larger than that of surrounding nuclei. This is consistent with the results presented in the experimental paper Ref. [14] in which the Pot4 and WKB approximation were used.

To quantitatively study whether δ^2 of ^{214,216}U are significantly enhanced by a factor of two as compared to that of the surrounding nuclei in the N_pN_n systematic, the ratio of the δ^2 of the ^{214,216,218}U to that of surrounding nuclei with the same N_pN_n is listed in Table 1. First, the δ^2 ratio between ^{214}U (^{216}U) and ^{202}Rn (^{200}Po) is larger than 2 for Pot1-4. Where in Pot1, the δ^2 ratio of ^{214}U to ^{202}Rn and ^{216}U to ^{200}Po are the largest,3.29 and 4.33, respectively. It seems that the conclusion that δ^2 of ^{214,216}U are enhanced holds despite the value of δ^2 may vary several orders of magnitude for different α-core potentials. Second, it can be found that, the values of δ^2 ratio obtained with WKB approximation are quite close to that obtained with TM approach, which means both WKB approximation and TM approach can be used in the study of α-decay reduced width. By considering the fact that TM approach is much more time consuming than WKB approximation, in the following discussions we only concentrate on the results obtained with WKB approximation.

Table 1. Under different α-core potentials and different theoretical methods to obtain the penetration probability, the ratio of the δ^2 of the ^{214,216,218}U to that of surrounding nuclei with the same N_pN_n.

α–core potential	N_pN_n	δ^2 ratio	WKB method	TM approach
		^{214}U / ^{202}Rn	$3.22^{+5.89}_{-1.30}$	$3.29^{+6.01}_{-1.33}$
	−40	^{214}U / ^{190}Po	$5.97^{+10.91}_{-2.41}$	$6.04^{+11.04}_{-2.44}$
		^{214}U / ^{186}Po	$15.96^{+29.15}_{-6.44}$	$15.92^{+29.09}_{-2.44}$
	−20	^{216}U / ^{200}Po	$4.24^{+1.19}_{-0.44}$	$4.33^{+1.21}_{-0.77}$
Pot1		^{218}U / ^{216}Th	$2.00^{+0.24}_{-0.22}$	$2.02^{+0.25}_{-0.21}$
		^{218}U / ^{214}Ra	$2.87^{+0.35}_{-0.31}$	$2.94^{+0.36}_{-0.32}$
	0	^{218}U / ^{212}Rn	$4.83^{+0.59}_{-0.52}$	$4.98^{+0.61}_{-0.54}$
		^{218}U / ^{210}Po	$8.82^{+1.09}_{-0.95}$	$9.10^{+1.12}_{-0.98}$
		^{214}U / ^{202}Rn	$3.13^{+5.71}_{-1.26}$	$3.19^{+5.84}_{-1.29}$
	−40	^{214}U / ^{190}Po	$6.25^{+11.41}_{-2.52}$	$6.32^{+11.55}_{-2.55}$
		^{214}U / ^{186}Po	$17.41^{+31.82}_{-7.03}$	$17.37^{+31.73}_{-7.01}$
	−20	^{216}U / ^{200}Po	$4.07^{+1.14}_{-0.72}$	$4.15^{+1.16}_{-0.74}$
Pot2		^{218}U / ^{216}Th	$1.94^{+0.24}_{-0.21}$	$1.96^{+0.24}_{-0.21}$
		^{218}U / ^{214}Ra	$2.69^{+0.33}_{-0.29}$	$2.75^{+0.34}_{-0.30}$
	0	^{218}U / ^{212}Rn	$4.37^{+0.54}_{-0.47}$	$4.51^{+0.55}_{-0.49}$
		^{218}U / ^{210}Po	$7.72^{+0.95}_{-0.83}$	$7.96^{+0.98}_{-0.86}$
		^{214}U / ^{202}Rn	$1.90^{+3.46}_{-0.77}$	$2.04^{+3.73}_{-0.83}$
	−40	^{214}U / ^{190}Po	$2.33^{+4.26}_{-0.94}$	$2.40^{+4.39}_{-0.97}$
		^{214}U / ^{186}Po	$5.55^{+10.13}_{-2.24}$	$5.51^{+10.07}_{-2.23}$
	−20	^{216}U / ^{200}Po	$2.10^{+0.59}_{-0.37}$	$2.29^{+0.64}_{-0.41}$
Pot3		^{218}U / ^{216}Th	$1.82^{+0.22}_{-0.20}$	$1.87^{+0.23}_{-0.20}$
		^{218}U / ^{214}Ra	$2.35^{+0.29}_{-0.25}$	$2.49^{+0.31}_{-0.27}$
	0	^{218}U / ^{212}Rn	$3.50^{+0.43}_{-0.38}$	$3.82^{+0.47}_{-0.41}$
		^{218}U / ^{210}Po	$5.58^{+0.69}_{-0.60}$	$6.27^{+0.77}_{-0.67}$
		^{214}U / ^{202}Rn	$2.08^{+3.79}_{-0.83}$	$2.17^{+4.00}_{-0.89}$
	−40	^{214}U / ^{190}Po	$2.35^{+4.30}_{-0.95}$	$2.41^{+4.40}_{-0.97}$
		^{214}U / ^{186}Po	$5.30^{+9.68}_{-2.14}$	$5.26^{+9.61}_{-2.12}$
	−20	^{216}U / ^{200}Po	$2.36^{+0.66}_{-0.42}$	$2.52^{+0.70}_{-0.45}$
Pot4		^{218}U / ^{216}Th	$1.89^{+0.23}_{-0.20}$	$1.93^{+0.24}_{-0.21}$
		^{218}U / ^{214}Ra	$2.54^{+0.31}_{-0.27}$	$2.66^{+0.33}_{-0.29}$
	0	^{218}U / ^{212}Rn	$3.98^{+0.49}_{-0.43}$	$4.26^{+0.52}_{-0.46}$
		^{218}U / ^{210}Po	$6.71^{+0.83}_{-0.72}$	$7.30^{+0.90}_{-0.79}$

B. Influence of the nuclear deformation

The value of quadrupole deformation β_2 of the daughter nuclei of ^{214}U and surrounding nuclei are listed in Table 2. These values are taken from Ref. [37]. Clearly, the value of β_2 varies from positive to negative for different nuclei. One may expect that the δ^2 ratio between them can be influenced by the nuclear deformation, especially for ^{214}U and ^{202}Rn. The nuclear deformation effect can be considered within the double-folding potential. Besides, based on the nuclear proximity potential [38] and deformed Woods-Saxon type potential [39−41], the nuclear deformation effects on the α decay also have been studied. In this work, double-folding potentials without (Pot5) and with (Pot6) deformation effect are considered. The calculated δ^2 ratio with WKB approximation under Pot5 and Pot6 are listed in Table 3. Comparing the results obtained under Pot5 to that under Pot6, one sees that the ratio is reduced when the nuclear deformation effect is taken into account. The δ^2 ratio between ^{214}U and ^{202}Rn is found slightly below 2, implying that the nuclear deformation effect should be considered in the study of the trend of the α-decay reduced width.

We note that in Ref. [42], within the generalized liquid drop model, the δ^2 ratio between ^{214}U and ^{202}Rn is smaller than 2. In Ref. [36], by considering a shell-dependent α-core potential, the δ^2 ratio between ^{214}U and ^{202}Rn is found larger than 10. While with other phenomenological α-core potentials, this ratio is typically smaller than 3, as listed in the Table 1. Together with the present analysis, manifesting that the influences of α-core potential and the nuclear deformation should be discussed in the study of the α-decay reduced width. It is well known from studies (e.g., Refs. [15, 16]) on the α-preformation probability that shell closures in both parent and daughter nuclei are very important, consequently, the reduced width is also closely related to shell closures, more detailed study about the effects of shell closures is certainly needed to understand fully the α-decay reduced

Table 2. The quadrupole deformation β_2 for daughter nuclei is theoretical value taken from Ref. [37], while the α-decay energy Q_α is taken from experimental data [14, 20].

Parent nuclei	Q_α/MeV	Daughter nuclei	β_2 [37]
^{186}Po	8.501	^{182}Pb	0.011
^{200}Po	5.9816	^{196}Pb	0
^{202}Rn	6.7738	^{198}Po	0.075
^{216}Th	8.072	^{212}Ra	−0.053
^{214}U	8.696	^{210}Th	−0.135
^{216}U	8.531	^{212}Th	−0.094
^{218}U	8.775	^{214}Th	−0.063

Table 3. The same as Table 1 but under Pot5 and Pot6.

α−core potential	N_pN_n	δ^2 ratio	WKB method
Pot5	−40	^{214}U/^{202}Rn	$2.22^{+4.05}_{-0.90}$
		^{214}U/^{186}Po	$6.92^{+12.64}_{-2.79}$
	−20	^{216}U/^{200}Po	$2.59^{+0.73}_{-0.46}$
	0	^{218}U/^{216}Th	$1.92^{+0.24}_{-0.21}$
Pot6	−40	^{214}U/^{202}Rn	$1.78^{+23.25}_{-0.72}$
		^{214}U/^{186}Po	$5.35^{+9.78}_{-2.16}$
	−20	^{216}U/^{200}Po	$2.30^{+0.64}_{-0.41}$
	0	^{218}U/^{216}Th	$1.86^{+0.23}_{-0.20}$

width in the N_pN_n systematic.

IV. SUMMARY

In summary, by using the WKB approximation and the transfer matrix approach to obtain the penetration probability, we revisit the α-decay reduced width δ^2 of ^{214,216}U and their surrounding nuclei under four different phenomenological α-core potentials, as well as spherical and deformed double-folding potentials. It is found that δ^2 is very sensitive to the α-core potential, its value can vary by almost three orders of magnitude when different α-core potential is considered. And the values of δ^2 obtained with the WKB approximation are about 20%-40% larger than that obtained with the transfer matrix approach, because of the underestimation of penetration probability in the WKB approximation. This underestimation is found to be related to the choice of α-core potential, as well as the mass and charge number of parent nuclei. However, this underestimation only have a small effect on the ratio of the δ^2 of uranium isotopes to the surrounding nuclei under N_pN_n systematics. With spherical form for the α-core potentials, the δ^2 ratio is about twice as large as that of the surrounding nuclei. While with the deformed double-folding potential, the ratio between δ^2 of ^{214}U and that of the surrounding nuclei is found slightly below 2. The present work indicates that the influences of α-core potential and the nuclear deformation should be discussed in the study of the α-decay reduced width in the N_pN_n systematic.

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