The influence of low-energy supersymmetric vector-like quirk particles on the W mass increment and the muon g-2 anomaly*

Ping Zhou (周平)\textsuperscript{1,2,3}\textsuperscript{†}

\textsuperscript{1}National Space Science Center, Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{2}University of Chinese Academy of Sciences, Beijing 100040, China
\textsuperscript{3}Beijing Key Lab. of Space Environment Exploration, Beijing 100140, China

Abstract: In the low energy realization of quirk assisted Standard Model, the couplings between the exotic particles "quirks" and the gauge bosons may contribute to the W mass and muon $g-2$ anomaly reported by FermiLab. We calculate the contributions from supersymmetric quirk particles as an example. Imposing the theoretical constraints, we found that the CDF II $W$-boson mass increment constrains strictly the mixing and coupling parameters and the quirk mass $m_F$, while muon $g-2$ anomaly cannot be explained only via the exotic particles involved in view of their large masses.

Keywords: low-energy supersymmetric vector-like quirk models, W mass increment, the muon g-2 anomaly

DOI: 

I. INTRODUCTION

Since it contains the key information of EWSB, precision measurement of W boson mass can provide a stringent test of the SM, and constrain various new physics models. Recently, CDF II collaborators at the Fermilab Tevatron collider \cite{1}, using data corresponding to \(8.8 fb^{-1}\) of integrated luminosity collected in proton-antiproton collisions at a 1.96 TeV center-of-mass energy, obtain the new value of W boson mass as

\[
M_W = 80.433.5 \pm 6.4\,\text{(stat)} \pm 6.9\,\text{syst} \\
= 80.433.5 \pm 9.4\,\text{MeV/c}^2,
\]  
(1)

which is in significant tension with the standard model(SM) expectation which gives \cite{2}

\[
M_W = 80.357 \pm 4\,(\text{inputs}) \pm 4\,(\text{theory})\,\text{MeV/c}^2,
\]  
(2)

and the discrepancy is\cite{3}\textsuperscript{1)}

\[
\Delta M_W = 70 \pm 11\,\text{MeV/c}^2.
\]  
(3)

Such deviations, if got confirmed by other experiments, will strongly indicate the existence of new physics beyond SM \cite{6, 7}. So, it is interesting to survey what is the new constraint that the new CDF II data can impose on the new physics models in addition to the 125 GeV Higgs.

This W mass increment has great discrepancy from the SM prediction, which may imply the existence of new physics beyond SM. Many attempts have been made in new physics framework, in which the anomaly usually attributes to the deviation of oblique parameters, especially $\Delta T$\cite{8}.

On the other hand, with the current world-averaged result given by \cite{9}, the precision measurement of $a_\mu = (g-2)/2$ has been performed by the E821 experiment at Brookhaven National Laboratory \cite{10},

\[
a_\mu^{\text{exp}} = 116592091(54)(33) \times 10^{-11}. \]  
(4)

While the Standard Model (SM) prediction from the Particle Data Group gives\cite{9},

\[
a_\mu^{\text{SM}} = 116591803(1)(42)(26) \times 10^{-11}. \]  
(5)

\textsuperscript{1)} This estimate of discrepancy can be changed by all the variations at the level of 10%. For instance, in Refs\cite{3-5}, the global fit updated central values are $M_W^{\text{exp}} = 80.413\,\text{GeV}$ and $M_W^{\text{SM}} = 80.350\,\text{GeV}$. However, it can be seen that the anomaly in the W-boson mass is certainly present.

* This work was supported by the National Natural Science Foundation of China(NSFC) under grant 12075213

† E-mail: pzhou@nssc.ac.cn

©2024 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd
The difference between theory and experiment shows a $4.2\sigma$ discrepancy, hinting at tantalizing new physics beyond the SM.

In models beyond the SM, there may exist a new confining unbroken non-abelian gauge interaction \[11\text{--}16\], in analogy to quantum chromodynamics (QCD) of strong interaction. In general, one can assume the new color group confinement scale $\Lambda_X$ is smaller than the QCD scale $\Lambda_{QCD}$ such that the new color degree of freedom bears the name infracolor (IC). We call this infracolor gluon fields igluons, and the fermions quirks\[15\]. It was called the quirk model, which is also regarded as a certain limit of QCD with some heavy quarks called quirks\[16\], and when QCD gets strong, the scale $\Lambda_X$ is much smaller than the quark masses \[12, 13\] with the light quarks removed from the particle list. Unlike the real world with light quarks, without worrying about the spontaneous chiral symmetry breaking, this hypothetical QCD can have drastically different phenomenology.

All kinds of possible new physics beyond the SM have been searched at the Large Hadron Collider (LHC), after the discovery of the SM Higgs boson. Solutions to the gauge hierarchy problem of the SM of particle physics such as supersymmetry and composite Higgs models usually predict a colored top partner with mass around TeV scale. They have been challenged by the null results of LHC searches so far. Theories of neutral naturalness \[17\] aim to address the gauge hierarchy problem without introducing colored states, thus relieve the tension with the LHC searches. This class of models include folded supersymmetry \[18, 19\], quirky little Higgs \[20\], twin Higgs \[21\text{--}23\], minimal neutral naturalness model \[24\] and so on. In those models, some new $SU(N)$ gauge symmetries are introduced in addition to the SM gauge group. Since under the framework of supersymmetry may all be quirk particles\[\text{[26]}\]. With the new color group $G_X$, the new fields are taken to transform in the $N = 2, 3, 3$ dimensional representations respectively for these three cases. Thus the new quirk chiral supermultiplets containing fermion multiplet $D, L, S$ and their partners $\bar D, \bar L, \bar S$ transform under $G_X \times SU(3)_c \times SU(2)_L \times U(1)_Y$ \[26\].

When $G_X = SU(2)_X$, \[\text{(6)}\]

$$D, \bar D = (2, 3, 1, -\frac{1}{3}) + (\bar 3, \bar 3, \bar 1, \frac{1}{3})$$

$$L, \bar L = (2, 1, 2, -\frac{1}{2}) + (2, 1, -2, \frac{1}{2})$$

$$S, \bar S = (2, 1, 1, 0) \times 2n_s,$$ \[\text{(7)}\]

and $G_X = SU(3)_X$, \[\text{(9)}\]

$$D, \bar D = (3, 3, 1, -\frac{1}{3}) + (\bar 3, \bar 3, \bar 1, \frac{1}{3})$$

$$L, \bar L = (3, 1, 2, \frac{1}{2}) + (3, 1, -2, \frac{1}{2})$$

$$S, \bar S = [(3, 1, 1, 0) + (\bar 3, \bar 1, 1, 0)] \times n_s,$$ \[\text{(10)}\]

and $G_X = SO(3)_X$, \[\text{(11)}\]

$$S, \bar S = [(3, 1, 1, 0) + (\bar 3, \bar 1, 1, 0)] \times n_s,$$

1) Quirks can have their scalar partner called squirks, and they can also exist in the same supermultiplets.

2) This particle was also called “thetons”[11] or “squark”[16]
D, L, S and their partners \( \tilde{D}, \tilde{L}, \tilde{S} \) may be assumed to get the masses the same as superpotential term of the minimum SUSY models (MSSM), \( \mu H_u H_d, \) where \( H_u \) and \( H_d \) are vector-like Higgs chiral supermultiplets in the SUSY models, with VEVs \( v_u, v_d \) ratio \( \tan \beta = v_u/v_d \) and \( v = \sqrt{v_u^2 + v_d^2} \approx 175 \) GeV. If one assume that the mass terms \( H_u H_d \) and \( D \bar{D} \) and \( L \bar{L} \) and \( S \bar{S} \) are forbidden at tree-level, the non-renormalizable superpotential terms may appear as [26–28]:

\[
W = \frac{1}{M_P^2} \left( \lambda_S H_u H_d + \lambda_D D \bar{D} + \lambda_L L \bar{L} + \lambda_i S \bar{S} \right),
\]

where \( i = 1, \ldots, n_S \) with \( n_S \) SM group singlets in the same representations of \( G_X \), and the reduced Planck mass \( M_P = 2.4 \times 10^{18} \) GeV. The fields \( X, \bar{X} \) will get VEVs roughly of order \( 10^{11} \) GeV, which are at intermediate-scale. These VEVs are natural, since there exist terms such as, a superpotential \( W \) and soft terms \( \mathcal{L}_{\text{soft}} \)[28],

\[
W = \frac{\lambda_{X}}{4M_p^2} X^{\dagger} \bar{X},
\]

\[
- \mathcal{L}_{\text{soft}} = m_{X}^{2} |X|^{2} + m_{\bar{X}}^{2} |\bar{X}|^{2} + \left( \frac{\alpha_{X}}{4M_p^2} X^{\dagger} \bar{X} + \text{c.c.} \right).
\]

Given that there may exist a minimum value of the potential, the vector-like mass terms in the low-energy effective superpotential can be written as [26]

\[
W = \mu H_u H_d + \mu_D D \bar{D} + \mu_L L \bar{L} + \mu_i S \bar{S},
\]

where \( \mu, \mu_D, \mu_L, \mu_S \) can be in order of 100 GeV to 1 TeV, only if the corresponding couplings \( \lambda_S, \lambda_D, \lambda_L, \lambda_i \), which are respectively absorbed into the \( \mu, \mu_D, \mu_L, \mu_S \) factors, are not too small.

For \( n_S > 0 \), the new chiral supermultiplets can have Yukawa couplings in addition to their mass terms in Eq. (18):

\[
W = k H_u L \bar{S} + k' H_d L \bar{S}.
\]

On the other side, if there exists the superpotential term such as,

\[
W = \lambda_{i} S \bar{L} \ell \ell
\]

with \( \ell \) an MSSM \( SU(2)_L \) doublet lepton, we may expect it would have some influence on the muon \( g - 2 \) discrepancy between the experiments and the theoretical calculation.

### III. THE S, T, U PARAMETERS AND W-MASS INCREMENT

The corrections to various electroweak precision observables can be obtained from the corresponding oblique parameters. The new physics contributions to the W-boson mass increment can embody in the Peskin’s \( S, T, U \) oblique parameters [8, 29, 30] by the following [8, 31, 32],

\[
\Delta m_W = \frac{\alpha m_W}{2(c_w^2 - s_w^2)} \left( -\frac{1}{2} S + c_w^2 T + \frac{c_w^2 - s_w^2}{4s_w^2} U \right),
\]

with \( \alpha = 4s_w^2 c_w^2 \left[ \Pi_{\gamma Z}(0) - c_w^2 - s_w^2 \Pi_{Z\gamma}(0) - \Pi_{\gamma\gamma}(0) \right], \)

\[
\alpha T = \frac{\Pi_{WZ}(0) - \Pi_{ZZ}(0)}{m_w^2},
\]

\[
\alpha U = 4s_w^2 \left[ \Pi_{WW}(0) - c_w^2 \Pi_{ZZ}(0) - 2s_w^2 \Pi_{\gamma Z}(0) - s_w^2 \Pi_{\gamma\gamma}(0) \right],
\]

and \( \alpha^{-1}(0) = 137.035999084, s_w^2 = 0.23126 \).

The oblique parameters \( (S, T, U) \) [8], which represent radiative corrections to the two-point functions of gauge bosons, can describe most effects on precision measurements. As we know, the total size of the new physics sector can be measured by the oblique parameter \( S \), while the weakisospin breaking can be measured by \( T \) parameter. The new results of \( S, T, U \) can be given as [33],

\[
S = 0.14 \pm 0.08, \quad T = 0.26 \pm 0.06, \quad U = 0
\]

The most important electroweak precision constraints on quirk models comes from the electroweak oblique parameters \( S \) and \( T \) [8, 29, 30], and we will proceed to study the connection between the electroweak precision data with the W mass. The model can produce main corrections to the masses of gauge bosons via the self-energy diagrams exchanging the vector-like extra fermions.

With the Yukawa couplings \( k, k' \), the new contributions to the Peskin-Takeuchi \( S, T \) observables from the new fermions can be given as [26],

\[
S = 0.14 \pm 0.08, \quad T = 0.26 \pm 0.06, \quad U = 0
\]
\[
\Delta T = \frac{N\nu^4}{480\pi s_{\theta W} M^2_W M^2_T} [13(\hat{k}^4 + \hat{k}'^4) + 2(\hat{k}^3 \hat{k}' + \hat{k}\hat{k}'^3) + 18\hat{k}^2 \hat{k}'^2],
\]

\[
\Delta S = \frac{N\nu^2}{30\pi M^2_T} [4\hat{k}^2 + 4\hat{k}'^2 - 7\hat{k}\hat{k}'].
\]

where \(\hat{k} = k\sin\beta\) and \(\hat{k}' = k'\cos\beta\) and \(\nu \approx 175\) GeV.

In our analysis, we will perform a global fit to the predictions of \(S, T\) parameters in profiled 1\(\sigma\) favoured regions. We scan \(m_F, \tan\beta, k\) and \(k'\) parameters in the following ranges:

\[
100\text{ GeV} \leq m_F \leq 1100\text{ GeV}, \quad 1 \leq \tan\beta \leq 50, \quad 0.01 < k, k' < 1.
\]

(26)

In Fig. 1 and Fig. 2, we show the W mass increment varies with the ratio \(\tan\beta\), Fermion masses \(m_F\), and \(k, k'\), which are in the range of (1-50), (100–1100 GeV), and (0.01 – 1), respectively, with the fixed parameters shown in the figures. From the two figures, we can see that the W mass increment decreases monotonically with increasing \(m_F, \tan\beta\), while increases monotonously with increasing \(N, k, k'\). The dependence of \(N\) and \(m_F\) of the W mass increment is obvious.

But when \(\tan\beta\) gets larger, the influence will be smaller and smaller. That is because, from Eq.(24) and (25), the coefficient of \(\Delta T\) is much larger than that of \(\Delta S\) and the contributions of the following terms of the two are of the same size, so the contribution of \(\Delta T\) is primary. On the other hand, the coefficient is much smaller than the subsequent terms. Then from Eq. (21) we see that the factor that the coefficient of \(\Delta T\) is larger than that of \(\Delta S\), ensures that they together contribute positively to \(\Delta m_W\), and the coefficients are smaller than the subsequent terms, while \(m_F\) is a part of the coefficients. Therefore, as long as the result is positive, \(m_F\) hardly affects \(\Delta m_W\), that is to say, \(m_F\) is not restrained by the CDF data.

From the first diagram of Fig. 2, we can also see that the contributions from \(k\) and \(k'\) are not the same, just same as the discussed above, and \(k\) is bound as \(k > 0.4\), while \(k'\) ranges the whole space. The reason for the in-

![Fig. 1.](color online) The W mass increment varies with varying \(m_F\) and \(\tan\beta\).
sensitivity of $k'$ is that $k'$ is always multiplied by the factor $\cos\beta$, which is small with large $\tan\beta$, which starts from 1.

The third diagram of Fig. 2 shows that the constraints on $\tan\beta$ is also quite weak, which can also be seen in the right diagram of Fig. 1, that is because since $k$ and $\sin\beta$, $k'$ and $\cos\beta$ appear together, and the relation $\sin^2\beta + \cos^2\beta = 1$ will finally decrease the contribution with increasing $\tan\beta$.

Thus we can conclude that in the most of the parameter space, the parameters in supersymmetric quirk models can account for the CDF data of the $W$ mass increment, and only the constraints on $k$ is obvious, $k \geq 0.4$.

IV. THE $g-2$ ANOMALY OF THE NEW COUPLINGS

In quirk models, the muon $g-2$ contributions are mainly obtained via the one-loop diagrams induced by the couplings shown in Eq.(20) as in Fig. 4. Note that two-loop Barr-Zee diagrams disappear since there is no mixing between SM gauge bosons of the new quirk particles [26] and the $m^2_{\chi}/m^2_{\nu}$ suppression of the diagram containing such two couplings[35].

The one-loop contribution can be written as[36, 37]:

$$\Delta a^\mu_{\text{(1-loop)}} = \lambda^2 \int_0^1 \frac{x^3 - x^2}{m^2_{\chi}x + m^2_{\nu}(1-x)} \, dx$$

where $\lambda^2$ is the coupling shown in Eq.(20), on which the one-loop moment magnetic can be realized as Fig. 4.

Since $\xi(L)$ is the scalar(fermion) component of the supermultiplet, their masses should be in the same level as that of the fermion $F$, so we take them changing also in the range of $100 - 1100$ GeV. We scan coupling $\lambda^2$ from 0 to 1.

Fig. 3 shows that the contributions of scalar and fermion from the supermultiplet at one-loop level, and we find that the contribution is negative and even the absolute value is quite small, about $\sim 10^{-10}$, which can not possibly explain the discrepancy between the experiments and the theoretical prediction. The situation is not surprising, since it has been pointed out that only if the scalar masses are very small, such as several GeV, the contribution may be large[38], but our choice for the new particle masses is larger than 100 GeV.

Hence, at the one-loop level, it is difficult to fill the gap between the experiments and the theoretical prediction in the supersymmetric quirk models. Therefore, with the missing two-loop Barr-Zee diagram, we can conclude that the supersymmetric quirk models can not account for the muon $g-2$ anomaly.

V. CONCLUSIONS

In this paper, we firstly show the $W$ mass increment varies with the parameters $\tan\beta$, $m_F$, and $k$, $k'$ with different new color group representations $N$, and we find the dependence on the parameters of the $W$ mass increment is obvious. We then scan the allowed points possible to exist for the mass increment in the $1\sigma$ range of the experimental bound and find that there are almost no constraints on $m_F$, and the contributions from $k$ and $k'$ are not the same, with $k > 0.4$, while $k'$ ranges the whole space, and the constraints on $\tan\beta$ is also quite weak. In a word, in the most of the parameter space, supersymmetric quirk models can account for the CDF data of the $W$ mass increment, and only the constraints on $k$ is obvious, $k \geq 0.4$.

We also calculate contribution from the vector-like fermions and the scalars in the supermultiplet to the muon $g-2$ anomaly at the one-loop level, and find that it is difficult to account for the gap between the experiments and the theoretical prediction in the supersymmetric quirk models.

Thus, we conclude that in the parameter space of supersymmetric quirk models, the $W$ mass increment of the CDF data may be accounted for, while the contribution...
from the vector-like fermions and the scalars in the supermultiplet to the muon $g-2$ anomaly at the one-loop level is not possible to remedy the discrepancy between experiment and theory.
The influence of low-energy supersymmetric vector-like quirk particles on the W mass...

Chin. Phys. C 48, (2024)

References

[3] S. Afonin, W-boson mass anomaly as a manifestation of spontaneously broken additional SU(2) global symmetry on a new fundamental scale, Universe 8, 627 (2022)
[37] Guo-Li Liu, Ping Zhou, Universe 8, 12 (2022)